

# How is logic applied in mathematical arguments?


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**Week 13, Lesson 1**

1. Warm-Up
2. Logic Statements
3. ICA
4. HW

**Logic Statements**

How is logic applied in mathematical arguments?



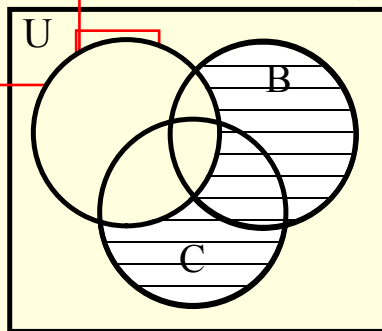
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## Warm-Up:

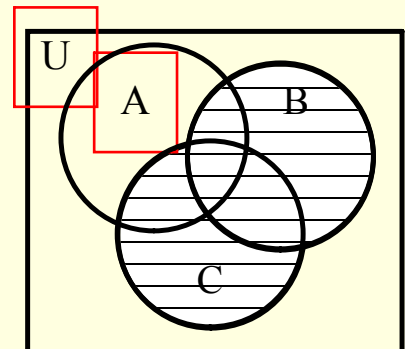
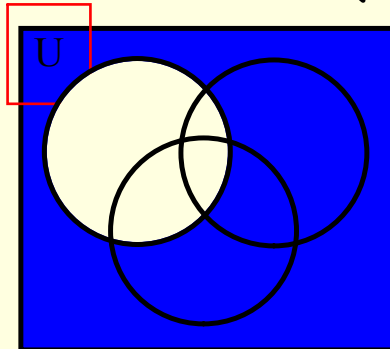
Assume  $A = \{1, 3, 5, 7\}$ ,  $B = \{2, 3, 5, 7, 8\}$  and  $C = \{1, 2, 3, 7\}$  are subsets of the universal set  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Determine each of the following sets.

1.  $A \cap B$   
 $\{3, 5, 7\}$
2.  $A \cup B$   
 $\{1, 2, 3, 5, 7, 8\}$
3.  $A' \cap (B \cup C)$   
 $\{2, 4, 6, 8, 9\} \cap \{1, 2, 3, 5, 7, 8\}$   
 $\{2, 8\}$

4. Name the shaded region using the letters A, B, C and the set Operations



Answer:  $A' \cap (B \cup C)$





Important Vocabulary

Know

Need to Know

Logic  
&  
Mathematical  
Statements



# 15 Week Study Guide

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MS 3.2/Unit 15 Assessment Name: \_\_\_\_\_ Pd \_\_\_\_\_

1. Three propositions  $p$ ,  $q$  and  $r$  are defined as follows:  
 $p$ : the water is cold.  $q$ : the water is boiling.  $r$ : the water is warm.

(a) Write one sentence, in words, for the following logic statement:  
 $(\neg p \wedge r) \Rightarrow \neg q$

(b) Write the following in words:  
 "The water is  $p$  and  $q$  but not  $r$ ."  
MS 3.2/Unit 15 Assessment Form 15.1  
 1. In a given situation, students must choose to show one of three options in logic and psychology, or three.

2. Police in a town are "Diner", "Sarah's Diner". They interviewed two.  
 Let  $p$ ,  $q$  and  $r$  be the following:  
 $p$ : Interviewed Sarah  
 $q$ : Interviewed Diner  
 $r$ : Interviewed both

(a) Write down  $\neg p$ .

(b) Write down  $\neg q$ .

(c) Write down  $\neg r$ .

(d) Write down  $\neg(p \wedge q)$ .

(e) Write down  $\neg(p \vee q)$ .

(f) Write down  $\neg(p \wedge q \wedge r)$ .

(g) Write down  $\neg(p \vee q \vee r)$ .

(h) Write down  $\neg(p \wedge q \wedge r \wedge s)$ .

(i) Write down  $\neg(p \vee q \vee r \vee s)$ .

Consider the following propositions:  
 $p$ : Interviewed Sarah  
 $q$ : Interviewed Diner  
 $r$ : Interviewed both

(a) Write, in words, the compound propositions:  
 $(p \wedge q) \wedge r$

(b) Complete the truth table for  $p \Rightarrow q$ .

$p$	$q$	$p \Rightarrow q$
T	T	
T	F	
F	T	
F	F	

(c) Does it matter to you if  $p$  is a tautology, a contradiction or neither, based on your answer?

(d) Complete the truth table above below:

$p$	$q$	$p \Rightarrow q$	$(p \wedge q) \Rightarrow p$	$(p \wedge q) \Rightarrow q$
T	T			
T	F			
F	T			
F	F			

(e) Does it matter to you if  $(p \wedge q) \Rightarrow p$  is a tautology, a contradiction or neither?

Consider the following propositions:  
 $p$ : Interviewed Sarah  
 $q$ : Interviewed Diner  
 $r$ : Interviewed both

(a) Write, in words, the following compound propositions:  
 $(p \wedge q) \wedge r$

(b) Write, in words, the following compound propositions:  
 $(p \wedge q) \wedge r$

(c) Write, in words, the following compound propositions:  
 $(p \wedge q) \wedge r$

(10 Marks Std. 3.2)

# Show Your Work

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MS 3.2/Unit 15 Assessment Name: \_\_\_\_\_ Pd \_\_\_\_\_

2. Consider the following statements:  
 $p$ : The hike is difficult.  $q$ : Bill exercises.  $r$ : Bill finishes the hike.

(a) If the hike is difficult, then Bill will not finish the hike.

(b) If Bill exercises, then the hike is not difficult.

(c) Bill will finish the hike if the hike is not difficult.

(d) Bill will not finish the hike if Bill does not exercise.

(e) If the hike is not difficult and Bill does exercise, then Bill will finish the hike.

6. Consider the two propositions  $p$  and  $q$ .  
 $p$ : The sun is shining.  $q$ : I will go swimming.

Write in words the compound propositions

(a)  $p \Rightarrow q$

(b)  $\neg q \vee p$

(c) Complete the truth table for these compound propositions below:

$p$	$q$	$p \Rightarrow q$	$\neg q$	$\neg q \vee p$
T	T			
T	F			
F	T			
F	F			

(d) Does the relationship between the compound propositions  $p \Rightarrow q$  and  $\neg q \vee p$  matter?

(10 Marks Std. 3.2)

## Statements

### Standard 3.2b

#### Statements-

A statement is a sentence or phrase and must have a precise mathematical meaning.

-A simple statement has a value of true or false (not both)

-Opinions can never be a statement

i.e.-  
Obama is the President of the United States.  
 $2 + 2 = 5$   
I do not swim.  
To enter the race, you must be over 45 years old.

#### *Classify as T or F*

Determine if the following are statements or not. If they are a statement, try to determine if it is true or false.

- |                               |                         |
|-------------------------------|-------------------------|
| 1. Pizza is good              | 4. The Lakers Suck      |
| 2. A triangle has 3 sides     | 5. The Hobbit was great |
| 3. There are 5 days in a week | 6. $7 < 3$              |

## Statements

### Compound Statements-

Made up of simple statements joined together by connectives.

### Connectives-

The five common connectives with symbols:

not	$\neg$	Negation
and	$\wedge$	Conjunction
or	$\vee$	<i>Inclusive</i> Disjunction
<u>OR</u>	$\underline{\vee}$	<i>Exclusive</i> Disjunction
If...Then	$\Rightarrow$	Implication

i.e.- The sun is shining and it is below 75 degrees

### or VS. OR-

or            Do you put milk or sugar in your coffee?  
OR            You may have a sandwich OR pizza.

### Quick Write

In your own words, explain the difference between or and OR

*or*- Either one, both

OR- One or the other, but *NOT* both

### Example 1-

Write your own example compound statement using one of the connectives

### Example 2-

Determine which "or" is being used, the inclusive or exclusive

- You either walk or bike to school
- The Cardinals or Broncos will win the Superbowl
- He is a captain in the Army or Navy
- x is odd or x is even
- Can you speak Japanese or Korean?

Summary:

Identify the common connectives that have been used.

She does not do her homework

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If the test is easy, then Sam will pass the test

Would you like Tea or Coffee

Maria is on the softball team and Luis is on the soccer team

There are not 100 cents in a dollar

The number 11 is prime and number 23 is prime

Would you like cream or sugar?

If it rains, then the game is cancelled

The product of two negative numbers is not a positive number

**Homework:** Use the five common connectives discussed in class and create an example of each.

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_



## How can we visually negate a statement?

Essential Question Essential Question Essential Question Essential Question Essential Question Essential Question Essential Question

### Week 13, Lesson 2

1. Warm-up
2. Truth Tables  
Negation
3. ICA- Matching

### Truth Tables Negation

How can we visually negate a statement?



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Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up Warm-up

Warm Up: Write an example of a true statement. Then write an example of a false statement. Share these with a neighbor. See if they know which is true or false.



## Statements and Negation

In logic, we use symbols to represent statements  
For instance,

$p$  represents the 1<sup>st</sup> statement

$q$  represents the 2<sup>nd</sup> statement

$r$  represents the 3<sup>rd</sup> statement

If you are using  $p$  and  $q$  and  $r$ , then you must use the connective symbols

Example 1-  $p$ : It is raining outside

$q$ : The streets are wet

Write out the following-

$p \wedge q$      $p \Rightarrow q$

It is raining outside and The streets are wet

If It is raining outside then The streets are wet

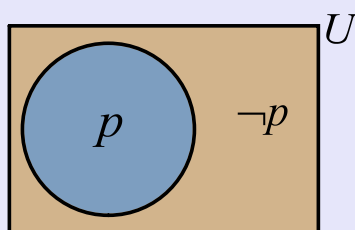
Negations- The negation of a statement  $p$  is written  $\neg p$   
This is pronounced "not  $p$ "

This DOES NOT mean opposite

i.e.-  $p$ : It is raining outside

Therefore,  $\neg p$  would say "It is NOT raining outside"

Negations are very similar to compliments



Example 2- Write the negations of the following statements

$p$ : John is a student counsel member  
John is not a student counsel member.

$q$ : ABCD is a parallelogram

ABCD is not a parallelogram.

$r$ : Barack Obama is the U.S. President

Barack Obama is not the U.S President.

## Truth Tables

We can actually organize statements into organized tables

We call them truth tables

$p$	$\neg p$
T	F
F	T

If a statement is true, what does that mean about the negation?

It will be ~~false~~

If a statement is false, what does that mean about the negation?

It will be ~~true~~

Example 3 Identify if the two statements are negations. If they are not negations, give an explanation why

$p$ : James is older than me      Not a negation  
 $q$ : James is younger than me      because it uses opp

$r$ : Meghan has 2 sisters  
 $s$ : Meghan does not have 2 sisters

Summary:





## How are simple conjunctions used in truth tables?

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### Week 13, Lesson 3

1. Warm-up
2. Conjunction
3. ICA- Is It True?
4. H.W

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### Conjunction

How are simple conjunctions used in truth tables?



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Warm-up: Make up your own statement for  $p$   
Now negate that statement:

$$\neg p :$$

Now negate THAT statement:

$$\neg(\neg p) :$$

## Conjunction

Conjunction is the connection of two simple statements using "and"

-This implies both simple statements at the same time

*i.e.*- I ate pizza and I drank a Coke

### Conjunction Truth Table

*p*: the grass is green

*q*: the sky is purple

<i>p</i>	<i>q</i>	<i>p</i> ∧ <i>q</i>
T	T	T
T	F	F
F	T	F
F	F	F

$p \wedge q$  is only true when BOTH statements are true

## Negated Conjunction Truth Tables

The truth tables will change when negating one of the simple statements

i.e.-  $\neg p \wedge q$

$\neg p \wedge q$

P	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

4x5

$p \wedge \neg q$

P	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

4x5

$\neg p \wedge \neg q$

P	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

5x5

$\neg(p \wedge q)$

P	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

4x5







## Attachments

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