Precalc takeaways Name:

Math 3 Unit 6: Modeling Periodic Behavior

Standards

M3 6.1 I can find coordinates of points on a circle centered at the origin.

M3 6.2 I can calculate the length of an arc that subtends an angle and convert between degrees and radians.

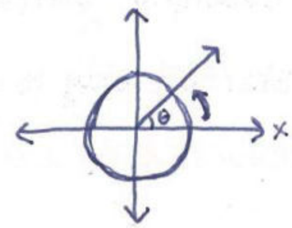
M3 6.3 I can graph points using polar coordinates and find coterminal angles.

M3 6.4 I can model circular motion with a transformed sine or cosine function and graph a transformed sine or cosine function.

M3 6.5 I can relate sine and cosine to points on a unit circle and use special right triangles to find certain points on the unit circle.

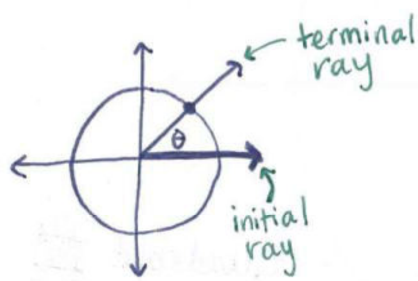
Angle of rotation in standard position

standard position of an angle:
angle starts at positive x-axis
and then rotates counter-clockwise
(negative angles rotate clockwise)



we often name angles with lowercase Greek letters, especially θ (theta)

Calculating the point (x, y) where a terminal ray of an angle of rotation in standard position intersects a circle centered at the origin $(0,0)$



$(x,y) = (r\cos\theta,\ r\sin\theta)$

r= radius of circle

Coterminal angles

angles that have the same terminal ray
example: $2°$, $362°$, $722°$, $-358°$

start with angle θ. you can add or subtract $360°$ (or 2π radians) a whole number of times to get a coterminal angle.

$\theta \pm 360° n$, where n is a whole number

or $\theta \pm 2\pi n$, where n is a whole number

$\cos(-\theta) = \cos(\theta)$
$\sin(-\theta) = -\sin(\theta)$

angles that have the same cosine:

$\theta, -\theta, \theta \pm 360°n, -\theta \pm 360°n$

example: $\cos(2°) = \cos(-2°) = \cos(362°) = \cos(-362°) = \ldots$

shortest way to write: $\pm 2° \pm 360°n$, where n is a whole number

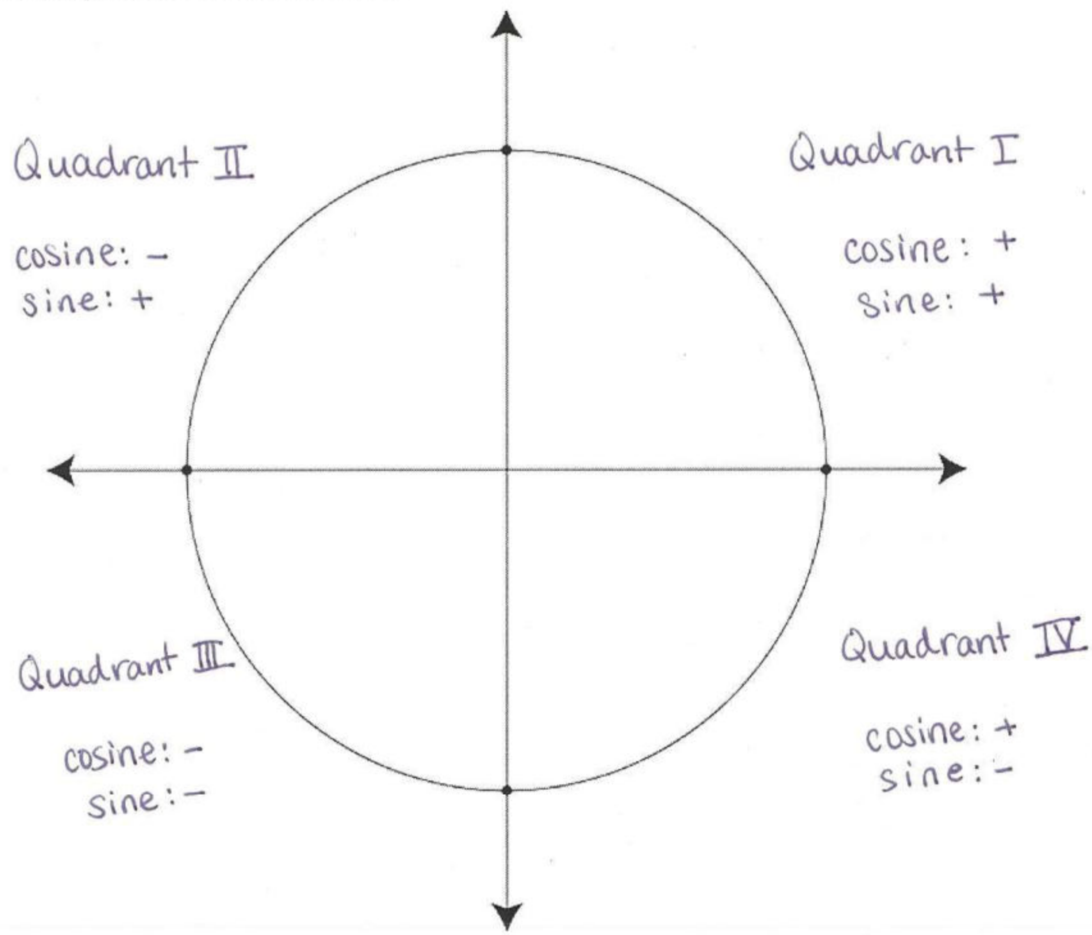
angles that have the same sine:

$\theta, 180° - \theta, \theta \pm 360°n, 180° - \theta \pm 360°n$

example: $\sin(2°) = \sin(178°) = \sin(362°) = \sin(538°) = \ldots$

shortest way to write: $2° \pm 360°n, 178° \pm 360°n$, where n is a whole number

positive and negative sines and cosines:

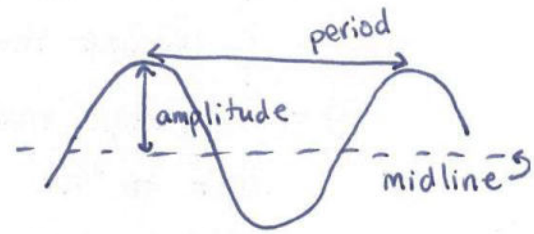


Features of the sine graph

shape: wave

midline: average height of the sine function

midline of $f(x) = \sin(x)$ is $y = 0$



amplitude: distance from midline to maximum (or minimum)

amplitude of $f(x) = \sin(x)$ is 1

period: distance from one peak to the next

period of $f(x) = \sin(x)$ is 360° or 2π radians

anchor points: of $f(x) = \sin(x)$

$(0,0)$, $(90°, 1)$, $(180°, 0)$, $(270°, -1)$, $(360°, 0)$

$h(t) = a\sin(bt) + d$

a = amplitude (or radius of circle)

b = degrees per second

d = height of midline

period = $360° \div b$ (or $2\pi \div b$)

Features of the cosine graph

shape: Same as sin, but y-intercept is $(0,1)$
(y-intercept of sin is $(0,0)$)

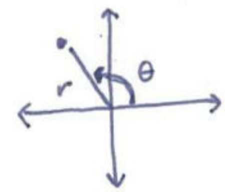
anchor points:

$(0,1)$, $(90°, 0)$, $(180°, -1)$, $(270°, 0)$, $(360°, 1)$

polar coordinates a different way to graph!

points are (r, θ)

$r =$ how far the point is from the origin
(r is like the radius)



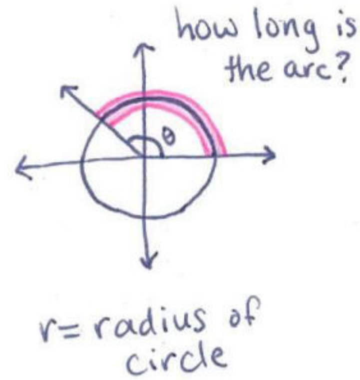
$\theta =$ the angle measure you turn to get to the point
(angle in standard position)

convert to rectangular coordinates: $x = r \cdot \cos\theta$, $y = r \cdot \sin\theta$

also, if your rectangular coordinates are (x, y) then
$r^2 = x^2 + y^2$

the length of an arc subtended by an angle

$$\begin{pmatrix}\text{circumference}\\ \text{of circle}\end{pmatrix} \cdot \begin{pmatrix}\text{fraction of circle}\\ \text{that the angle is}\end{pmatrix}$$



in degrees:

$$\text{arc length} = 2\pi r \cdot \frac{\text{angle in degrees}}{360^\circ}$$

in radians:

$$\text{arc length} = r \cdot \text{angle in radians}$$

radians

a different way to measure angles!

one rotation is 2π radians, so $360^\circ = 2\pi$ radians

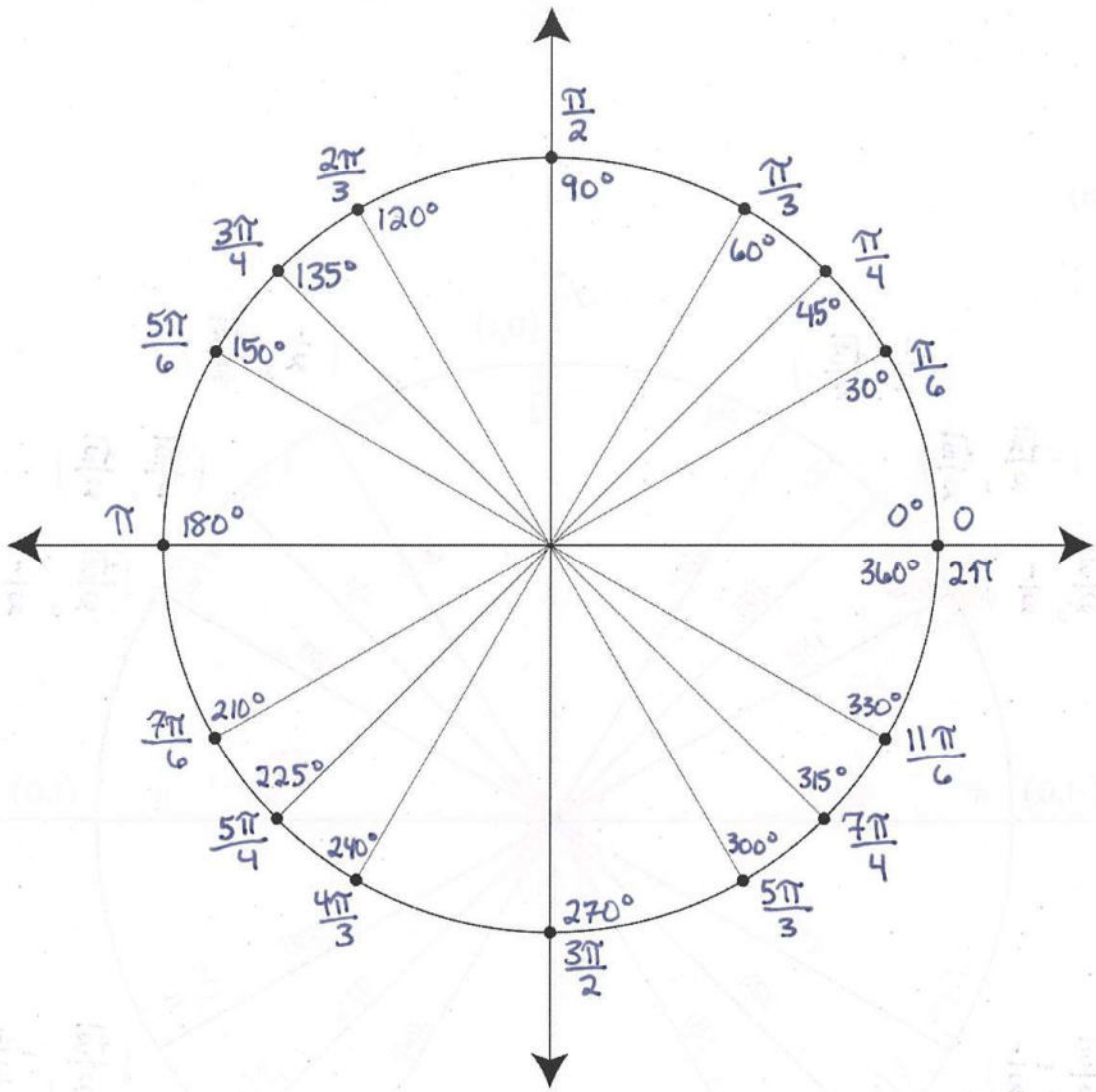
also, arc length $\div$ radius $=$ angle measure in radians

converting between degrees and radians

$$\text{angle in degrees} = \text{angle in radians} \cdot \frac{360^\circ}{2\pi}$$

$$\text{angle in radians} = \text{angle in degrees} \cdot \frac{2\pi}{360^\circ}$$

angles in standard position (degrees and radians)



rationalizing a denominator

when simplifying radicals, you are not allowed to have a square root symbol in the denominator of a fraction

if you do, correct it by multiplying by a clever form of 1

example:

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

original answer

clever form of 1

simplified answer

example:

$$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Unit circle

(cos, sin)