Illustrative Mathematics

Mathematical Language Routines

Developing Students' Voices and Sense Making

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Let’s get to know each other!

Who teaches or works with teachers…

- PK - 2?
- 3 - 5?
- 6 - 8?
- 9 - 12?
- Post secondary?

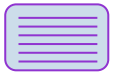
Why are you here?

Learning Goals

- Understand that learning mathematics is a language-demanding activity for all students
- Understand how MLRs support mathematics sense-making and language development
- Engage in and prepare MLRs

Let's Do Math

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?



Handout 1

Source: Illustrative Mathematics 6-8 Math (Grade 7 Unit 4, Lesson 7, Cool Down)

Language demands?

Prompts:

- How did you use language to engage in this task?
- How did you make your thinking visible?

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?

Mathematical Language Demands

It matters for all students

READING

WRITING

REPRESENTING

CONVERSING

LISTENING

SPEAKING

(Adapted from Aguirre & Bunch, 2012)

UL/SCALE: Mathematical Language Routines

1. Stronger and Clearer Each Time
2. Collect and Display
3. Clarify, Critique, Correct
4. Information Gap
5. Co-Craft Questions
6. Three Reads
7. Compare and Connect
8. Discussion Supports

MLRs are structured but adaptable formats for amplifying, assessing, and developing students' language.

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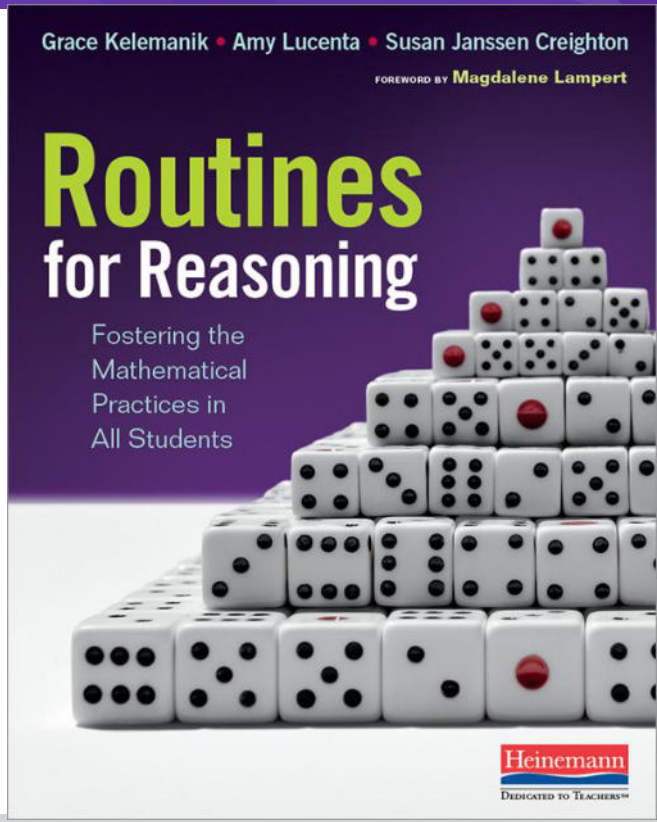
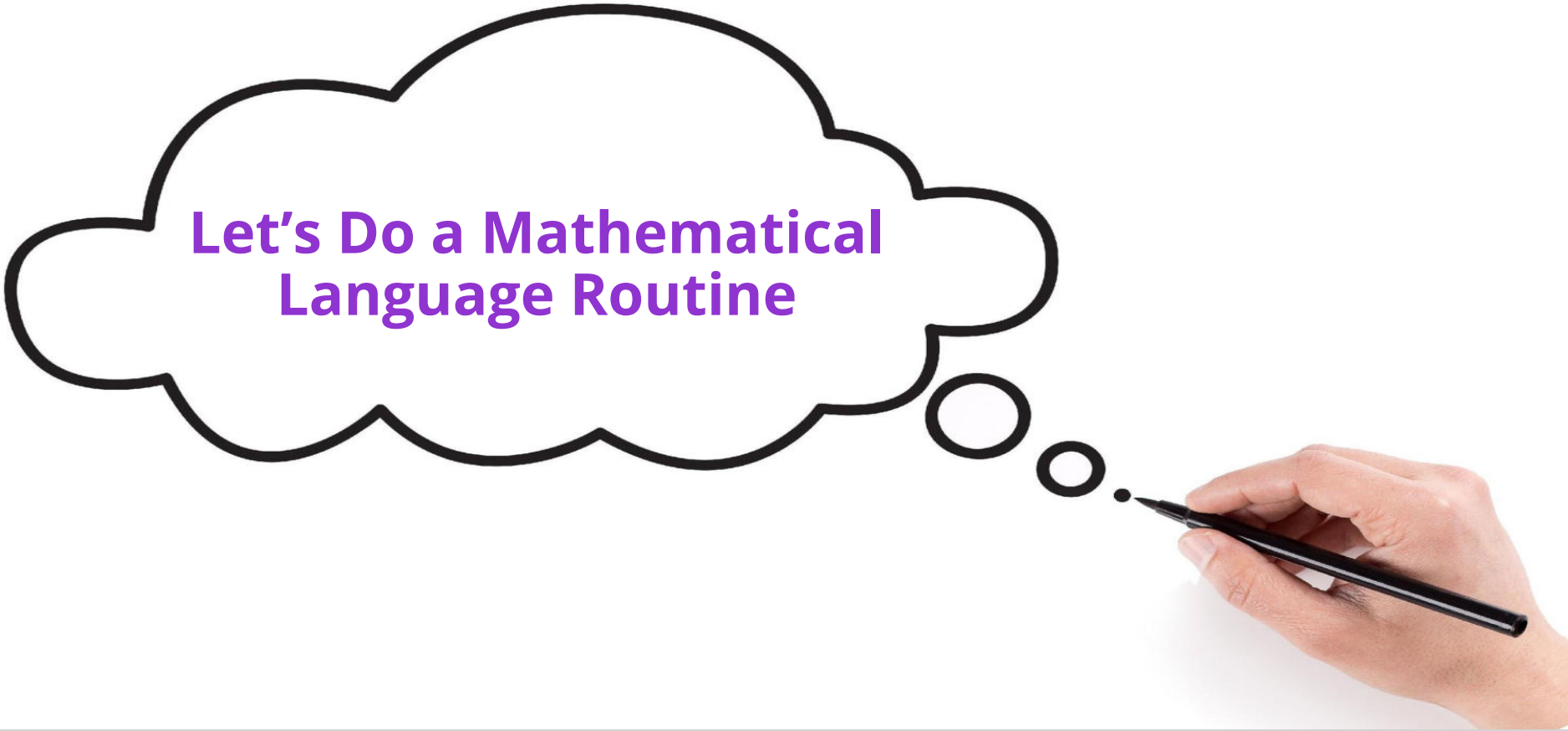
What are instructional routines?

"Instructional routines are specific and repeatable designs for learning that support both the teacher and students in the classroom."



Let's Do a Mathematical Language Routine



Mathematical Language Routine:
Clarify, Critique, Correct

PROMPT partners to:

- **Clarify**: Describe what the author did
- **Critique**: Analyze the author's reasoning
- **Correct**: Improve the response

Incomplete Response

Handout 2

Clarify, Critique, Correct

Partners Clarify (1 minute)

- “I notice … “ “What do you notice?”
- “I wonder…” “What do you wonder?”

Partners Critique (2 minutes)

- “This reasoning is strong because …”
- “What might make this reasoning stronger is …”

Individuals Correct (2 minutes)

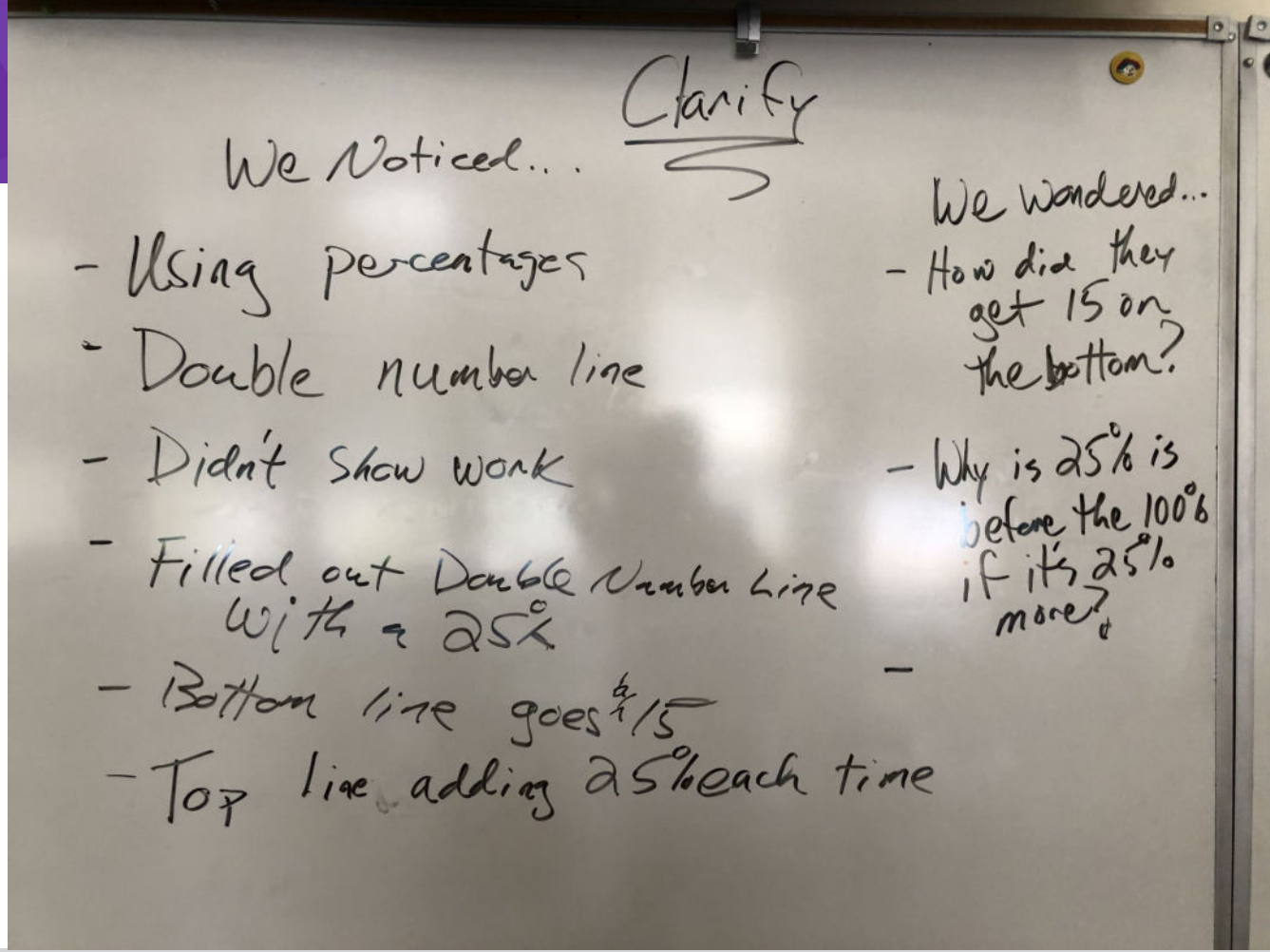
- “I can build onto this response by …”
- “What else could I do/show?”


Handout 2

Clarify

"We noticed … "

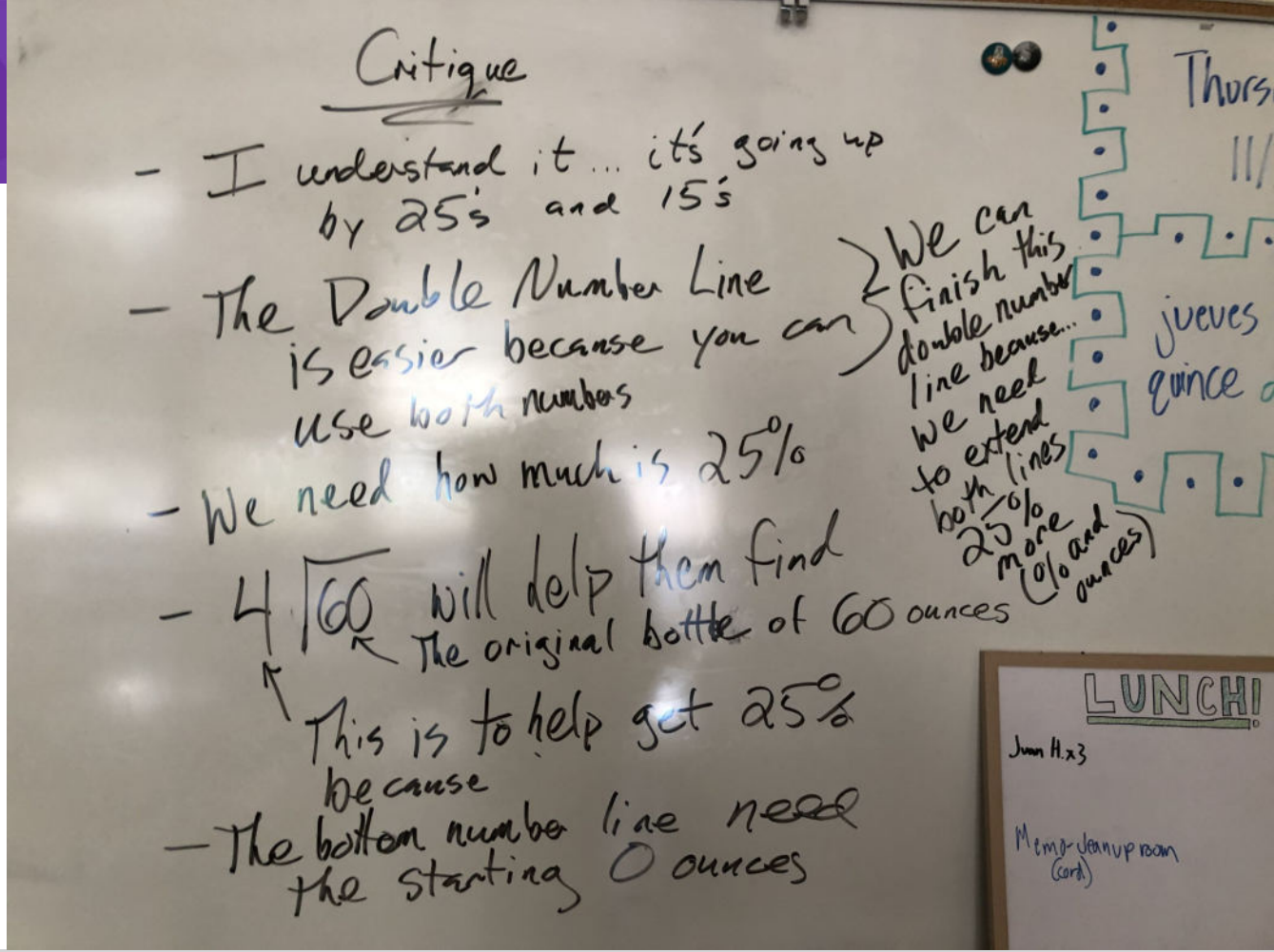
"We wondered…"



Critique

"This reasoning is strong because …"

"What might make this reasoning stronger is …"

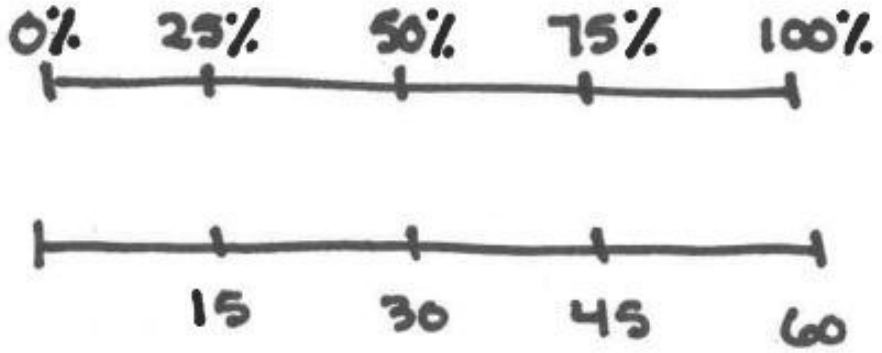


Initial Draft

60 fluid ounces

+

25% more

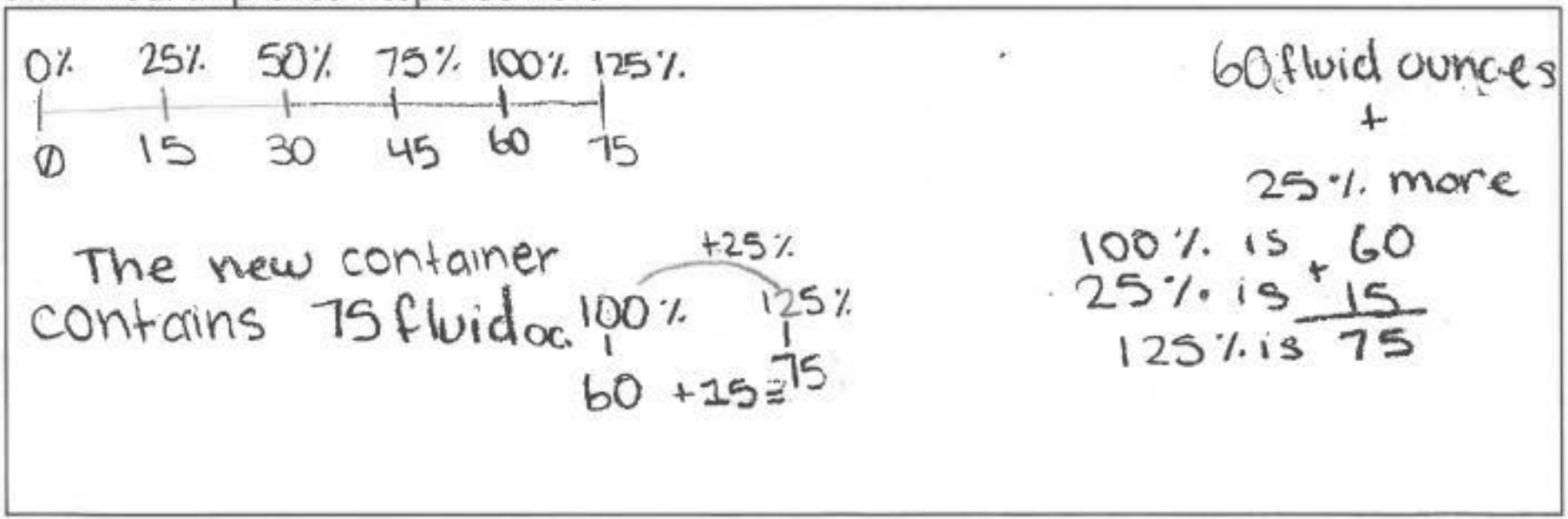


$4\overline{)60}$

is

Revised Response

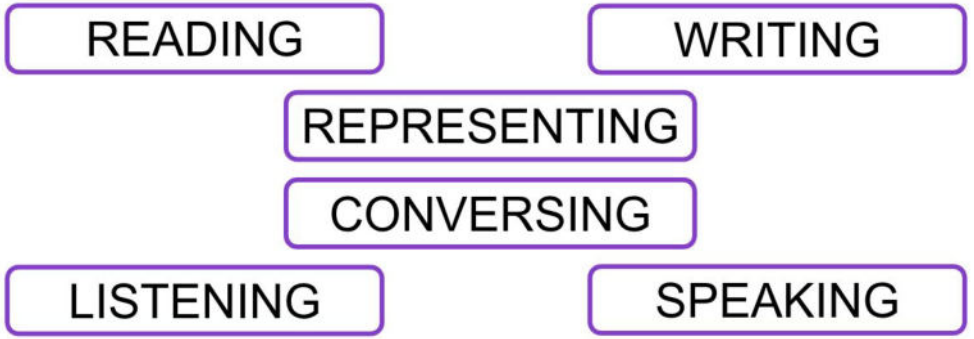
Show Your Improved Response Here



Unpack the Routine

How does “Clarify, Critique, Correct”

- support language demands of the task?
- support development of mathematical understanding?



Unpack the Routine

In this routine:

- What is the teacher doing?
- What are students doing?

Handout 3

Mathematical Language Routine: Clarify, Critique, Correct

Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect or incomplete written argument, and for students to improve upon the written work by clarifying thinking, offering critique and then correcting the initial response.

Handout 4

Clarify, Critique, Correct

1. **PRESENT** a partial or broken argument, explanation, or solution method.
2. **PROMPT** partners to:

   Clarify: the reasoning in the initial response

   Critique: Analyze the response in light of partner's own understanding of the problem

   Correct: Partners work collaboratively / individually to improve the initial response
3. **SHARE:** Partners share out their drafts of an improved response and refine as needed during whole group discussion.

Handout 4

A Partnership

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Design Principles

To Promote Mathematical Language Use and Development in Curriculum and Instruction

1. Support sense-making
2. Optimize output
3. Cultivate conversation
4. Maximize linguistic and cognitive meta-awareness

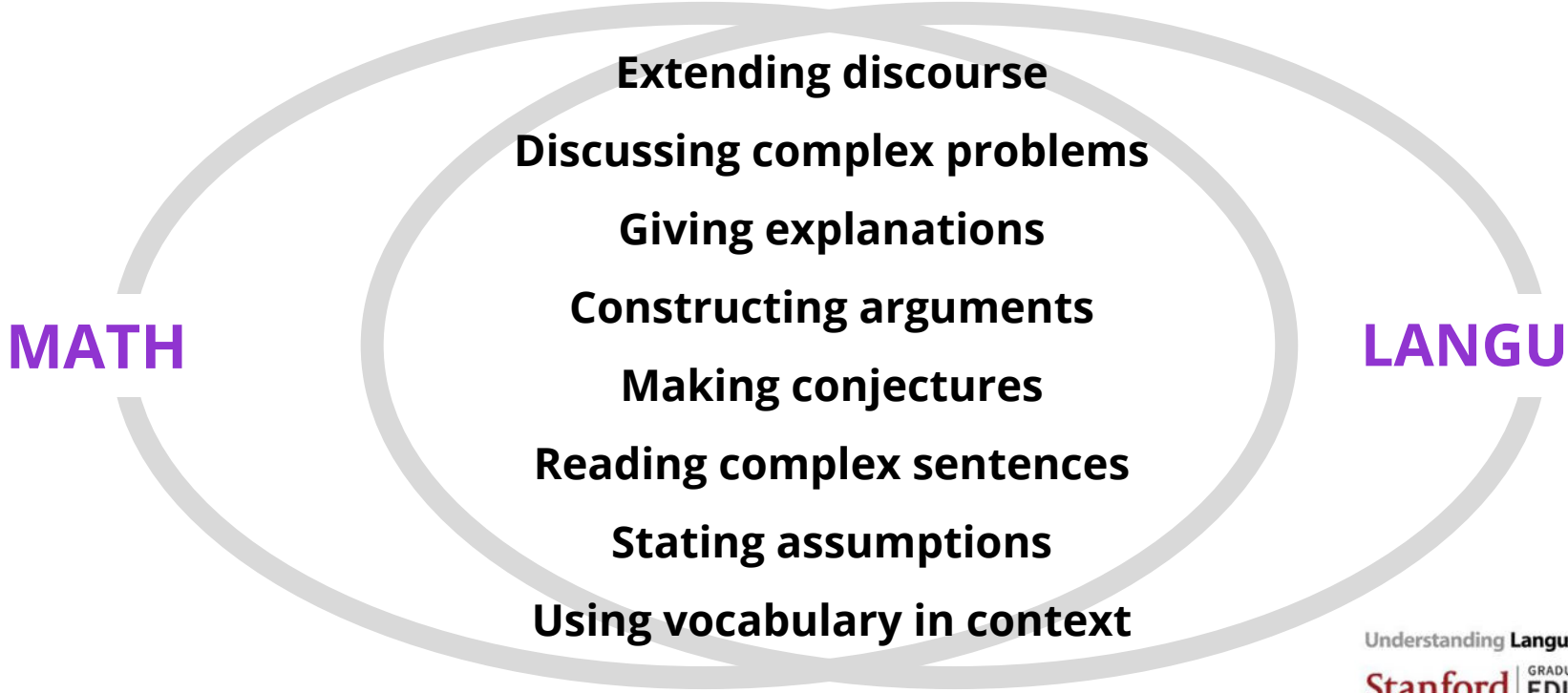
Connecting to the UL/SCALE Guiding Principles

Principle 3 CULTIVATE CONVERSATION: Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class).

Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Mercer & Howe, 2012; Zwiers, 2011). They also allow students to hear how other students express their understandings. When students have a reason or purpose to talk and listen to each other, interactive communication is more authentic. For example, when there is an "information gap," in which students need or want to share their thoughts (which are not the same), students have a reason or purpose in talking and listening to each other.

During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including within conversations, to make claims, justify claims with evidence, make conjectures, communicate reasoning, critique the reasoning of others, and engage in other mathematical practices – and above all, to make mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.

Students are Engaged in Mathematical Language Development When…



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Routines in the Mathematics Lessons Support Students'...

Mathematical Sense Making

Mathematical Language Development

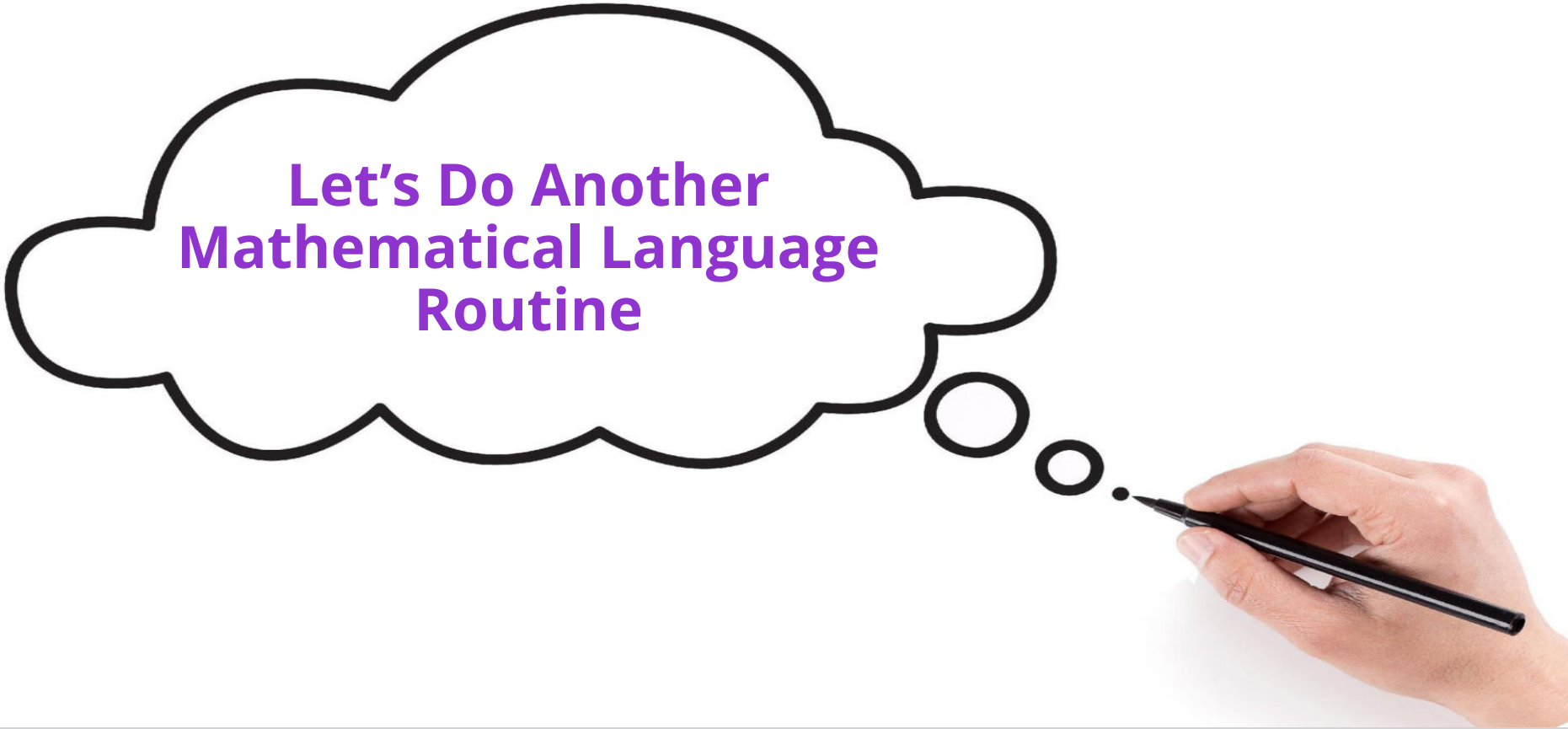
Simultaneously

A Double Challenge for English Learners

"(The English learner must) learn how to effectively employ a new language in an academic setting, while learning through that language the knowledge and skills in multiple disciplines."

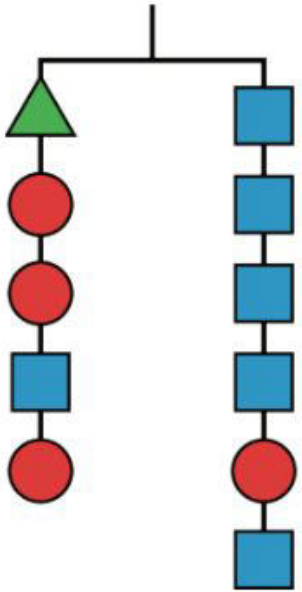
(UL/SCALE 2014)

Let's Do Another Mathematical Language Routine



What do you notice?
What do you wonder?

A



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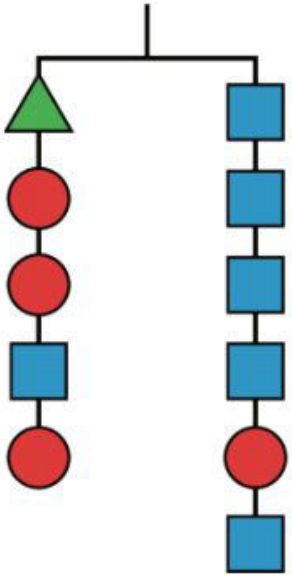


$3+6+6+6=4x+6$

$21+x=6+5x$

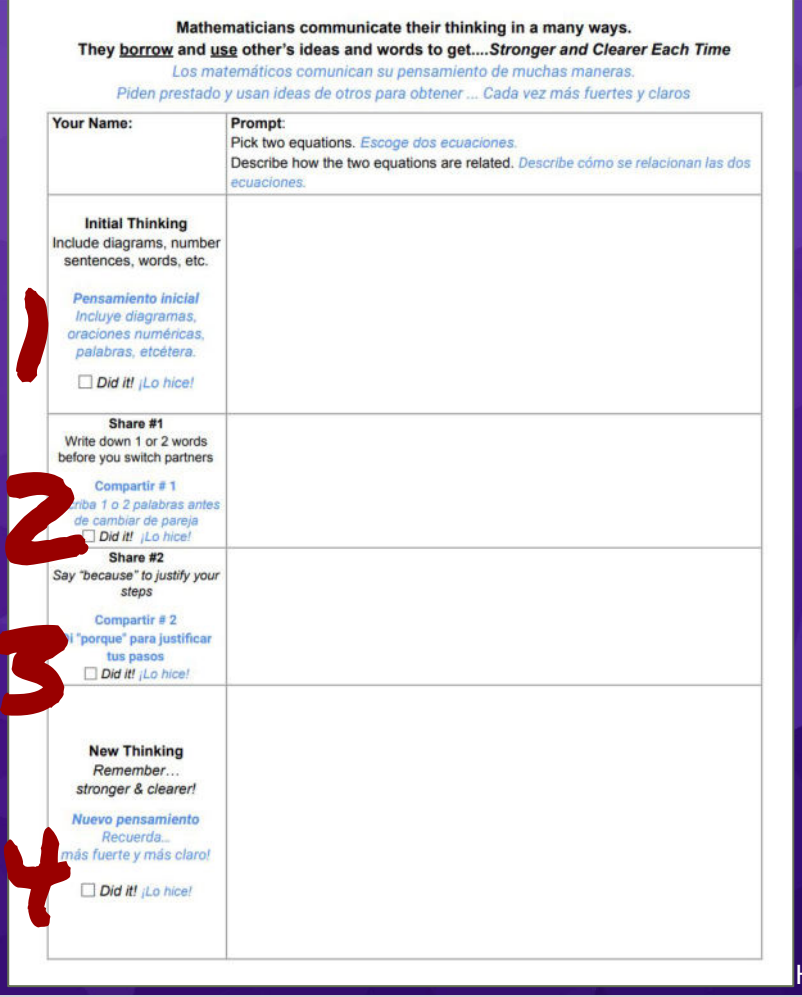
$x=3.75$

Pick 2 equations.
Describe how the two equations are related

What do you notice? What do you wonder?

Mathematical Routine: Stronger and Clearer Each Time

1. Pre-write
2. Think time
3. Pair share
4. Repeat with new partners
5. Revise pre-write



Mathematicians communicate their thinking in a many ways.
They borrow and use other's ideas and words to get....*Stronger and Clearer Each Time*
Los matemáticos comunican su pensamiento de muchas maneras.
Piden prestado y usan ideas de otros para obtener ... Cada vez más fuertes y claros

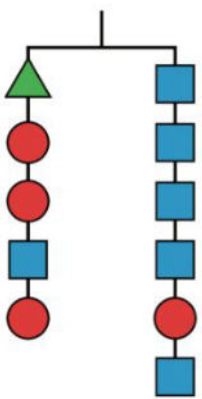
Your Name:	**Prompt**: Pick two equations. *Escoge dos ecuaciones.* Describe how the two equations are related. *Describe cómo se relacionan las dos ecuaciones.*
Initial Thinking Include diagrams, number sentences, words, etc. ***Pensamiento inicial*** *Incluye diagramas, oraciones numéricas, palabras, etcétera.* ☐ *Did it! ¡Lo hice!*	
Share #1 Write down 1 or 2 words before you switch partners **Compartir # 1** *Escriba 1 o 2 palabras antes de cambiar de pareja* ☐ *Did it! ¡Lo hice!*	
Share #2 *Say "because" to justify your steps* **Compartir # 2** *Di "porque" para justificar tus pasos* ☐ *Did it! ¡Lo hice!*	
New Thinking *Remember… stronger & clearer!* ***Nuevo pensamiento*** *Recuerda... más fuerte y más claro!* ☐ *Did it! ¡Lo hice!*	

1
2
3
4

Handout 5

Pick 2 equations. Describe how the two equations are related.

A



$3+6+6+6=4x+6$

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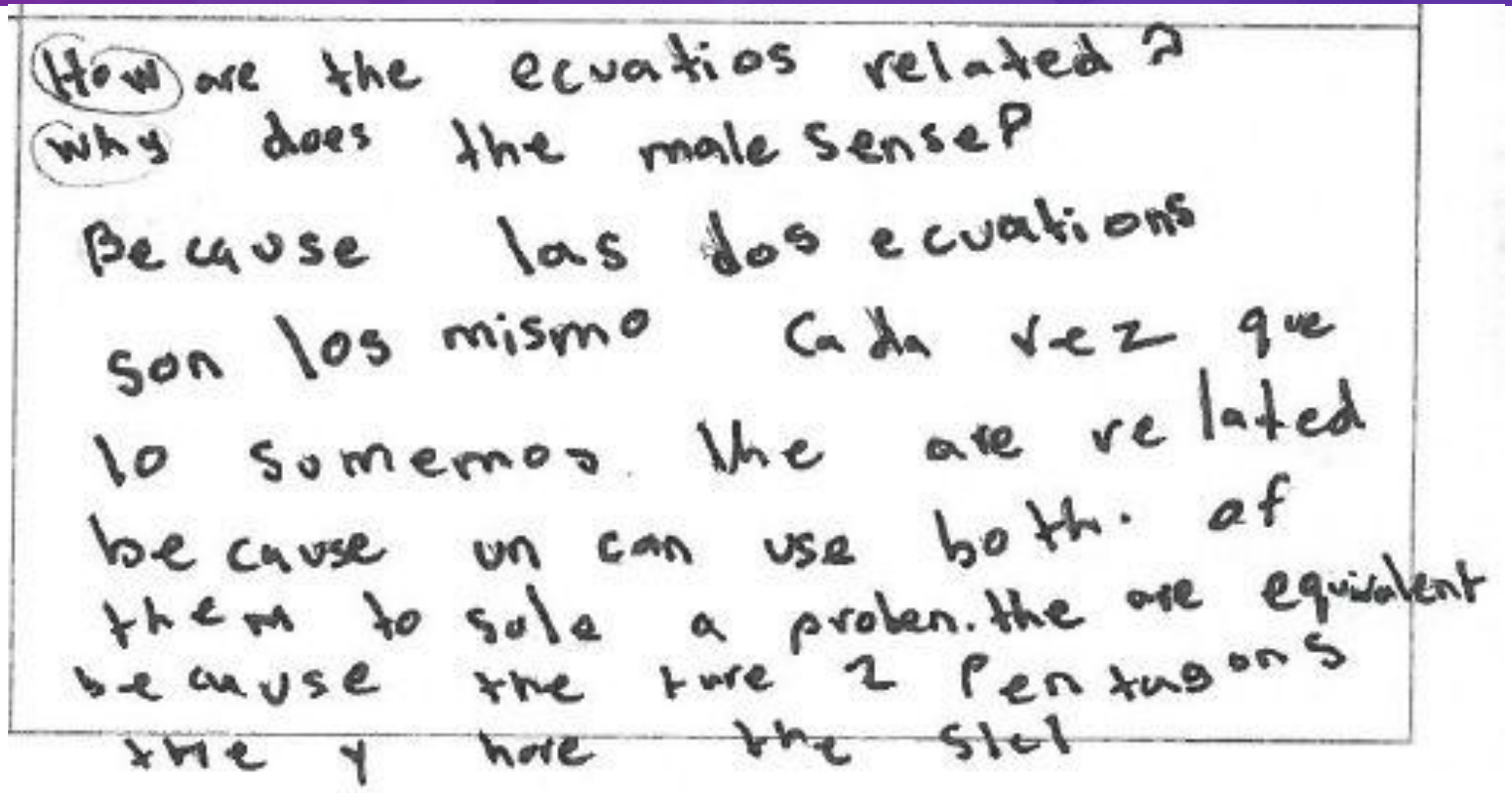
$x=3.75$

Initial Draft

Initial Thinking Include diagrams, number sentences, words, etc. ***Pensamiento inicial*** *Incluye diagramas, oraciones numéricas, palabras, etcétera.* ☐ *Did it! ¡Lo hice!*	Se relacionan Sumando 3 veces 6 para Sumarlo el 3+6+6+6=4x6 por que cuentas los numeros elena take triangles of the left side
Share #1 Write down 1 or 2 words before you switch partners **Compartir # 1** *Escriba 1 o 2 palabras antes de cambiar de pareja* ☐ *Did it! ¡Lo hice!*	la ecuacion es la misma que la otra.
Share #2 *Say "because" to justify your steps* **Compartir # 2** **Di "porque" para justificar tus pasos** ☐ *Did it! ¡Lo hice!*	por que unos son igvales

Revised Response



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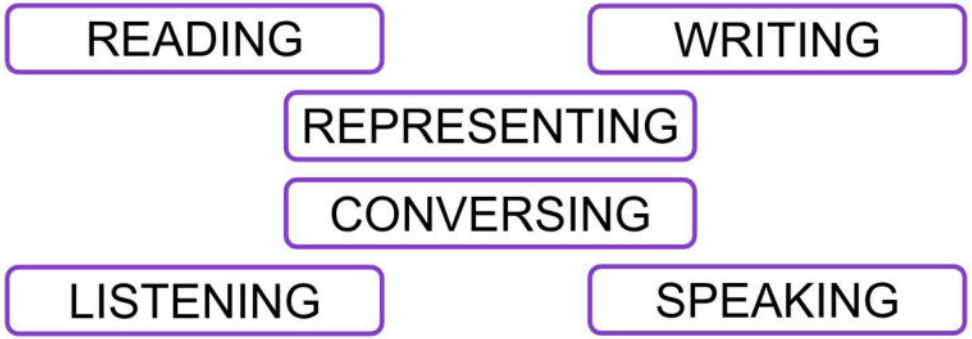


Unpack the Routine

In this routine:

- What is the teacher doing?
- What are students doing?

Handout 3

Mathematical Language Routine: Stronger and Clearer Each Time

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). Pairs borrow and use the language, ideas, and justifications each time. Responses become:

- Stronger (often longer) with better justifications and examples,
- Clearer with more precise terms and linked, organized, complete sentences.

Handout 6

Language Routines in General

After experiencing both routines:

- How is learning mathematics a language demanding activity for all students?
- How do these routines support mathematical sense making and language development simultaneously?
- How do these routines empower students?

Let's Prepare a Mathematical Language Routine

Implementing Mathematical Language Routines

Which of these routines will I try with my students?

- 3 Reads
- Clarify, Critique, Correct
- Stronger and Clearer Each Time

I want to try ____ because...

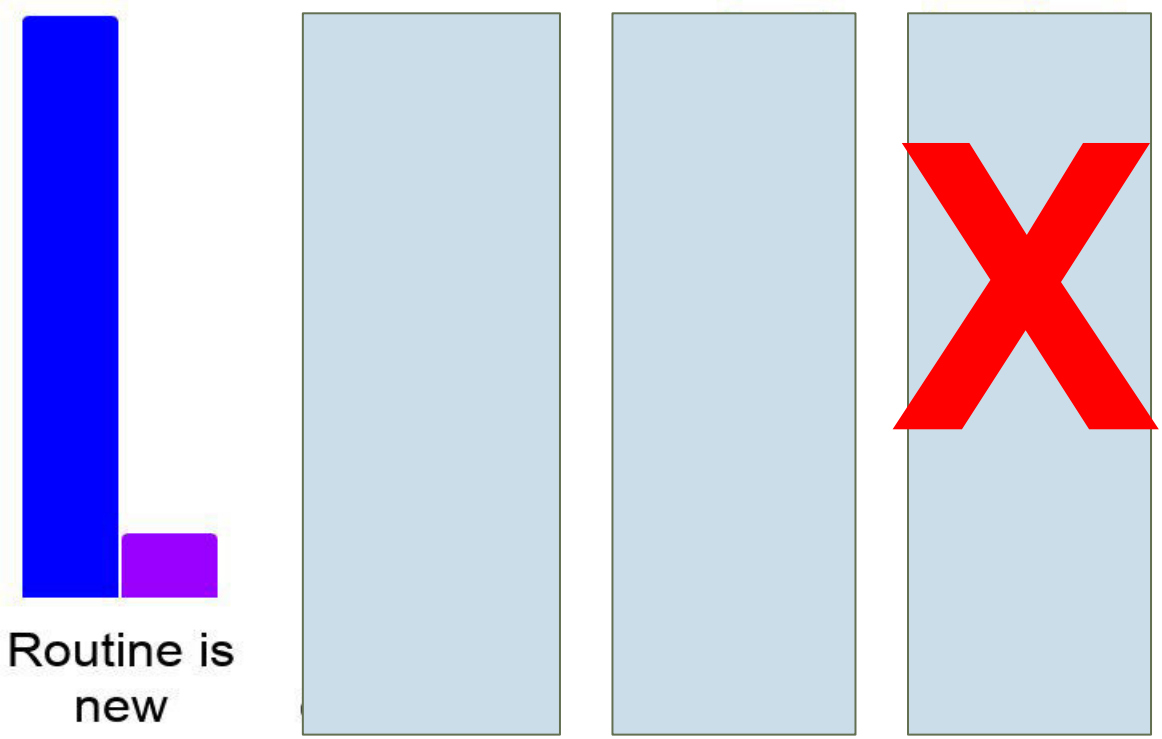
Handout 7

Implementing Mathematical Language Routines

As you prepare to develop your own students' voices and sense making, what are some things you might consider as you prepare to facilitate the routine with your students?

Handout 7

Consider Students' (and your) Cognitive Load



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Algebra 1, Geometry, Algebra 2	IM Certified™ 6-8 Math	K–5 Math
Available for fall implementation	Available Version 3.1415 available for fall implementation including Spanish translations and EL supports for teachers	Coming July 2021

Illustrative Mathematics CERTIFIED™

Perfect Score	*Perfect Score*	*Top Rated*
Focus & Coherence	Rigor and Mathematical Practices	Usability
14/14 6th, 7th & 8th	18/18 6th, 7th & 8th	37/38 6th, 7th & 8th

Scores for the LearnZillion instance of the IM 6-8 curriculum

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THANK YOU

For Exploring Mathematical Language Routines

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