Misc Classwork

AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim_{x \to 0^-} (1 - 2\sin x) = 1$

$\lim_{x \to 0^+} e^{-4x} = 1$

$f(0) = 1$

So, $\lim_{x \to 0} f(x) = f(0)$.

Therefore f is continuous at $x = 0$.

2 : analysis

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$-2\cos x \ne -3$ for all values of $x < 0$.

$-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$.

Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

(c) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

$= \int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx$

$= \Big[x + 2\cos x\Big]_{x=-1}^{x=0} + \Big[-\frac{1}{4}e^{-4x}\Big]_{x=0}^{x=1}$

$= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)$

Average value $= \frac{1}{2}\int_{-1}^{1} f(x)\,dx$

$= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$

$4 : \begin{cases} 1 : \int_{-1}^{0} (1 - 2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

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Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

(a) Profit $= 120 \cdot 25 - \int_0^{25} 6\sqrt{x}\,dx = 2500$ dollars	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) $\int_{25}^{30} 6\sqrt{x}\,dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.	1 : answer with units
(c) Profit $= 120k - \int_0^{k} 6\sqrt{x}\,dx$ dollars	$2: \begin{cases} 1: \text{integral} \\ 1: \text{expression} \end{cases}$
(d) Let $P(k)$ be the profit for a cable of length k. $P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$. This is the only critical point for P, and P' changes from positive to negative at $k = 400$. Therefore, the maximum profit is $P(400) = 16{,}000$ dollars.	$4: \begin{cases} 1: P'(k) = 0 \\ 1: k = 400 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

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2008 SCORING GUIDELINES (Form B)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\frac{dy}{dx} = -2x - 2$

$\frac{dy}{dx} = \frac{-2(x+1)}{4\left(y^3+1\right)} = \frac{-(x+1)}{2\left(y^3+1\right)}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(-2,1)} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$

Tangent line: $y = 1 + \frac{1}{4}(x+2)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$