LOGARITHMIC FUNCTION

The function $f(x) = \log_a x$ is the **logarithmic function** with base a, where $a > 0, x > 0$, and $a \neq 1$.

$$y = \log_a x \text{ is equivalent to } x = a^y$$

$f(x) = \log_a x$

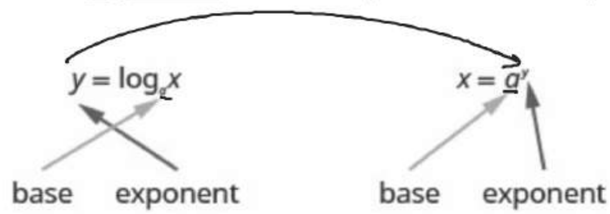
Read log base a of x

$y = a^x$

$x = a^y$

Convert Between Exponential and Logarithmic Form

Since the equations $y = \log_a x$ and $x = a^y$ are equivalent, we can go back and forth between them. This will often be the method to solve some exponential and logarithmic equations. To help with converting back and forth let's take a close look at the equations. See Figure 10.3. Notice the positions of the exponent and base.



$y = \log_a x \qquad x = a^y$

Convert to logarithmic form: ⓐ $2^3 = 8$, ⓑ $5^{\frac{1}{2}} = \sqrt{5}$, and ⓒ $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$.

a) $\underset{a}{2}^{\overset{y}{3}} = \underset{x}{8}$ $\qquad 3 = \log_2 8$

c) $\left(\frac{1}{2}\right)^{4^{y}} = \underset{x}{\frac{1}{16}}$

b) $\underset{a}{5}^{\overset{y}{1/2}} = \underset{x}{\sqrt{5}}$ $\qquad \frac{1}{2} = \log_5 \sqrt{5}$ $\qquad 4 = \log_{\frac{1}{2}} \frac{1}{16}$

Convert to logarithmic form: ⓐ $3^2 = 9$ ⓑ $7^{\frac{1}{2}} = \sqrt{7}$ ⓒ $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

a) $3^2 = 9$

$2 = \log_3 9$

b) $7^{1/2} = \sqrt{7}$

$\frac{1}{2} = \log_7 \sqrt{7}$

c) $\frac{1}{3}^x = \frac{1}{27}$

$x = \log_{\frac{1}{3}} \frac{1}{27}$

$y = \log_a x \quad x = a^y$

Convert to exponential form: ⓐ $2 = \log_8 64$, ⓑ $0 = \log_4 1$, and ⓒ $-3 = \log_{10} \frac{1}{1000}$.

a) $2 = \log_8 64$

$8^2 = 64$

b) $0 = \log_4 1$

$4^0 = 1$

c) $-3 = \log_{10} \frac{1}{1000}$

$10^{-3} = \frac{1}{1000}$

Convert to exponential form: ⓐ $3 = \log_4 64$ ⓑ $0 = \log_x 1$ ⓒ $-2 = \log_{10} \frac{1}{100}$

a) $3 = \log_4 64$

$4^3 = 64$

b) $0 = \log_x 1$

$x^0 = 1$

c) $-2 = \log_{10} \frac{1}{100}$

$10^{-2} = \frac{1}{100}$

Convert to exponential form: ⓐ $3 = \log_3 27$ ⓑ $0 = \log_3 1$ ⓒ $-1 = \log_{10} \frac{1}{10}$

a) $3 = \log_3 27$

$3^3 = 27$

b) $0 = \log_3 1$

$3^0 = 1$

c) $-1 = \log_{10} \frac{1}{10}$

$10^{-1} = \frac{1}{10}$

Find the value of x: ⓐ $\log_x 36 = 2$, ⓑ $\log_4 x = 3$, and ⓒ $\log_{\frac{1}{2}} \frac{1}{8} = x$.

a) $\log_x 36 = 2$

$x^2 = 36$

$x = \pm 6$

$\boxed{x = 6}$

b) $\log_4 x = 3$

$4^3 = x$

$x = 64$

c) $\log_{\frac{1}{2}} \frac{1}{8} = x$

$\frac{1}{2}^x = \frac{1}{8}$

$x = 3$

Find the exact value of each logarithm without using a calculator: ⓐ $\log_5 25$, ⓑ $\log_9 3$, and ⓒ $\log_2 \frac{1}{16}$.

a) $\log_5 25 = 2$

$5^x = 25$

$x = 2$

b) $\log_9 3 = \frac{1}{2}$

$9^x = 3$

$\sqrt{9} = 3$

$\frac{1}{2}$

c) $\log_2 \frac{1}{16} = -4$

$2^x = \frac{1}{16}$

-4

Find the exact value of each logarithm without using a calculator: ⓐ $\log_{12}144$ ⓑ $\log_4 2$ ⓒ $\log_2 \frac{1}{32}$

a) $\log_{12} 144 = 2$

$12^x = 144$

2

b) $\log_4 2 = \frac{1}{2}$

$4^x = 2$

$\sqrt{4} = 2$

$\frac{1}{2}$

c) $\log_2 \frac{1}{32} = -5$

$2^x = \frac{1}{32}$

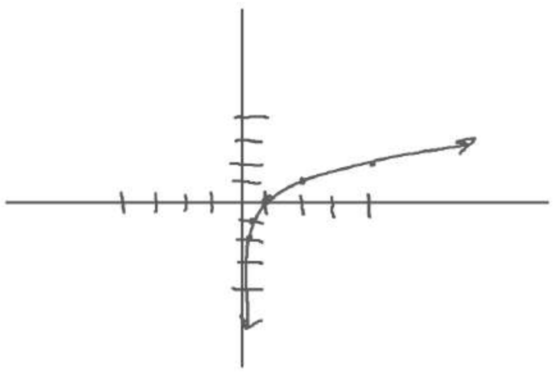
-5

$2^{-5} = \frac{1}{32}$

Graph $y = \log_2 x$.

Graph the log swith x + y on exponetial

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



Inverse

$y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

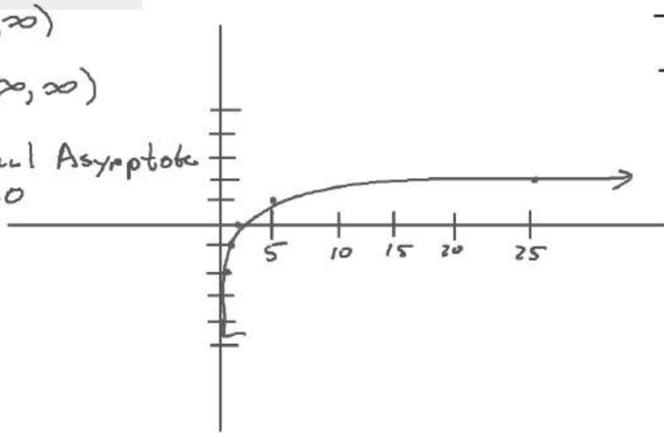
Graph: $y = \log_5 x$.

X	Y
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2

D $(0, \infty)$

R $(-\infty, \infty)$

Vertical Asymptote
Y = 0



$Y = 5^x$

X	Y
-2	$\frac{1}{25}$
-1	$\frac{1}{5}$
0	1
1	5
2	25

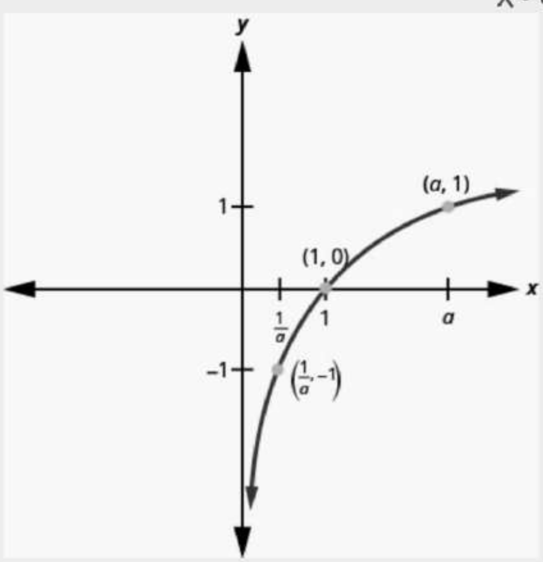
D: $(-\infty, \infty)$

R: $(0, \infty)$

PROPERTIES OF THE GRAPH OF $y = \log_a x$ WHEN $a > 1$

Domain	$(0, \infty)$
Range	$(-\infty, \infty)$
x-intercept	$(1, 0)$
y-intercept	None
Contains	$(a, 1), \left(\frac{1}{a}, -1\right)$
Asymptote	y-axis

$X=0$



$y = \log_2 x$

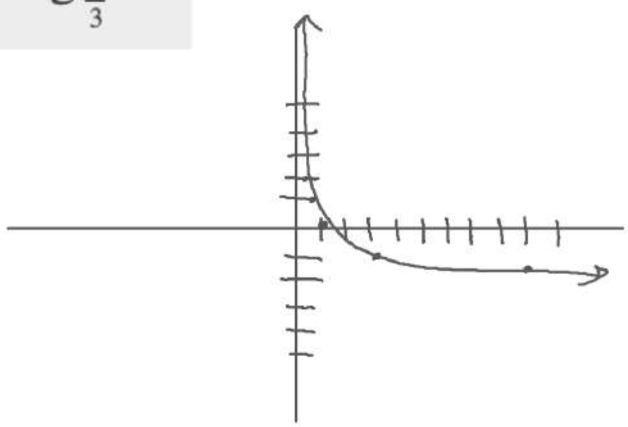
$(2,1)$ $\left(\frac{1}{2}, -1\right)$

$y = \log_5 x$

$(5,1)$ $\left(\frac{1}{5}, -1\right)$

Graph $y = \log_{\frac{1}{3}} x$.

x	y
9	-2
3	-1
1	0
$\frac{1}{3}$	1
1/9	2

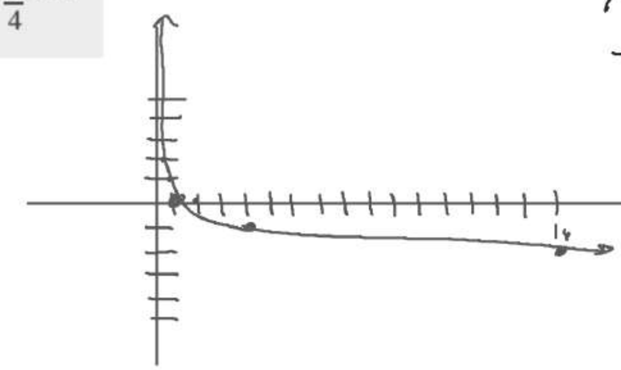


$y = \frac{1}{3}^x$

x	y
-2	9
-1	3
0	1
1	1/3
2	1/9

Graph: $y = \log_{\frac{1}{4}} x$.

x	y
16	-2
4	-1
1	0
$\frac{1}{4}$	1
$\frac{1}{16}$	2

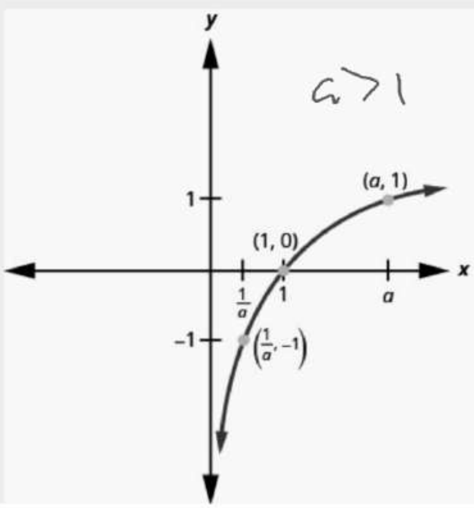


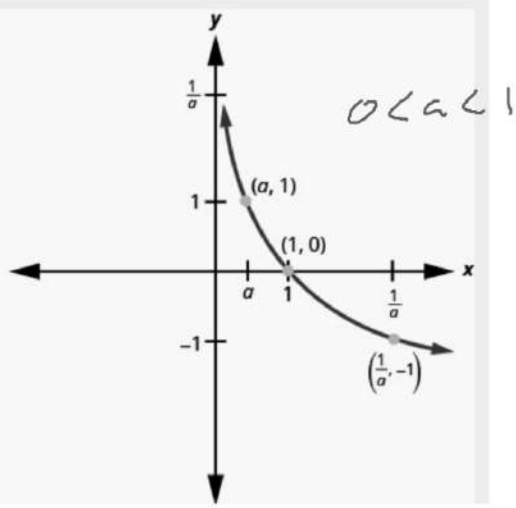
$y = \frac{1}{4}^{x}$

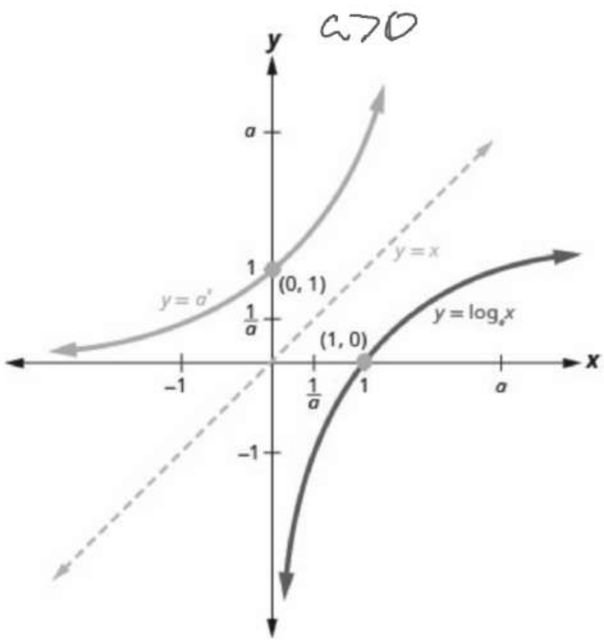
x	y
-2	16
-1	4
0	1
1	1/4
2	1/16

PROPERTIES OF THE GRAPH OF $y = \log_a x$

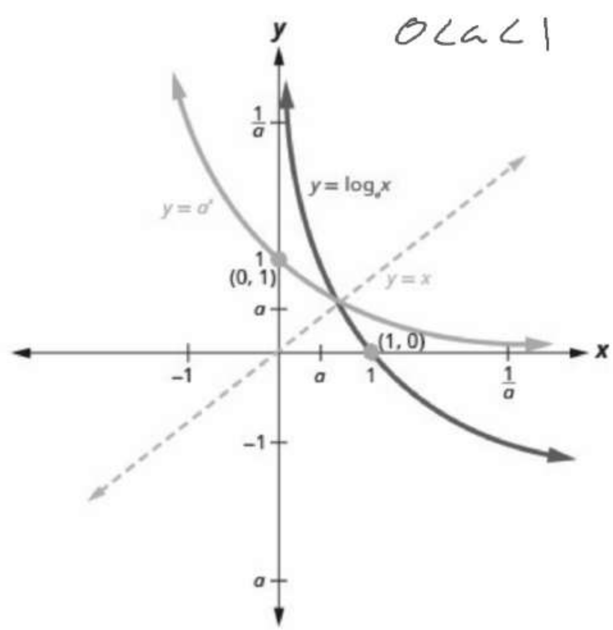
when $a > 1$		**when $0 < a < 1$**	
Domain	$(0, \infty)$	Domain	$(0, \infty)$
Range	$(-\infty, \infty)$	Range	$(-\infty, \infty)$
x-intercept	$(1, 0)$	x-intercept	$(1, 0)$
y-intercept	none	y-intercept	None
Contains	$(a, 1), \left(\frac{1}{a}, -1\right)$	Contains	$(a, 1), \left(\frac{1}{a}, -1\right)$
Asymptote	y-axis	Asymptote	y-axis
Basic shape	increasing	Basic shape	Decreasing







a>0
y
a
1
(0, 1)
y = a^x
1/a
y = x
(1, 0)
y = log_a x
–1
1/a
1
a
x
–1



0<a<1
y
1/a
y = log_a x
y = a^x
1
(0, 1)
y = x
a
(1, 0)
–1
a
1
1/a
x
–1
a

NATURAL LOGARITHMIC FUNCTION

The function $f(x) = \ln x$ is the **natural logarithmic function** with base e, where $x > 0$.

$f(x) = \ln x$ $\qquad$ $y = \ln x$ is equivalent to $x = e^y$

$= \log_e x$

COMMON LOGARITHMIC FUNCTION

The function $f(x) = \log x$ is the **common logarithmic function** with base 10, where $x > 0$.

$f(x) = \log x$ $\qquad$ $y = \log x$ is equivalent to $x = 10^y$

$\log_{10} x$

Solve: ⓐ $\log_a 49 = 2$ and ⓑ $\ln x = 3$.

a) $\log_a 49 = 2$

$a^2 = 49$

$a = 7$

b) $\ln x = 3$

$e^3 = x$

$x \approx 20.0855$

Solve: ⓐ $\log_a 121 = 2$ ⓑ $\ln x = 7$

$\log_a 121 = 2$

$a^2 = 121$

$a = 11$

b) $\ln x = 7$

$e^7 = x$

$x \approx 1096.633$

Solve: ⓐ $\log_a 64 = 3$ ⓑ $\ln x = 9$

a) $\sqrt[3]{a^3} = \sqrt[3]{64}$

$a = 4$

b) $\ln x = 9$

$e^9 = x$

$x = 8103.084$

Solve: ⓐ $\log_2(3x - 5) = 4$ and ⓑ $\ln e^{2x} = 4$.

a) $\log_2(3x-5) = 4$

$2^4 = 3x-5$

$16 = 3x-5$

$21 = 3x$

$x = 7$

b) $\ln e^{2x} = 4$

$e^4 = e^{2x}$

$4 = 2x$

$x = 2$

Solve: ⓐ $\log_2(5x-1)=6$ ⓑ $\ln e^{3x}=6$

a) $\log_2(5x-1)=6$

$2^6 = 5x-1$

$64 = 5x-1$

$65 = 5x$

$x = 13$

b) $\ln e^{3x} = 6$

$e^6 = e^{3x}$

$3x = 6$

$x = 2$

Solve: ⓐ $\log_3 (4x + 3) = 3$ ⓑ $\ln e^{4x} = 4$

a) $\log_3 (4x+3) = 3$

$3^3 = 4x+3$

$27 = 4x+3$

$24 = 4x$

$x = 6$

b) $\ln e^{4x} = 4$

$e^4 = e^{4x}$

$4x = 4$

$x = 1$