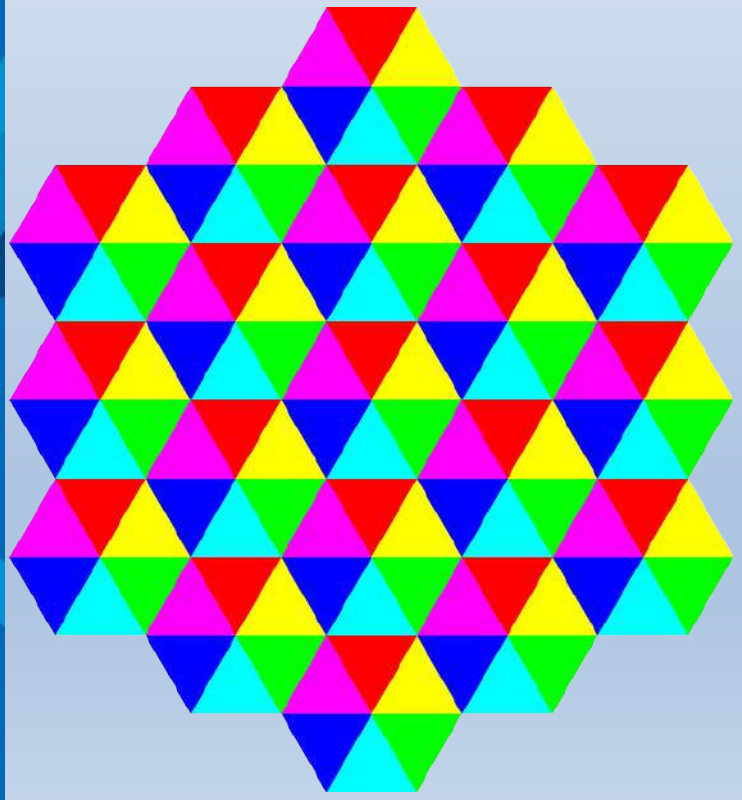
Principles to Actions: Effective Mathematical Teaching Practices



Tessellate This:

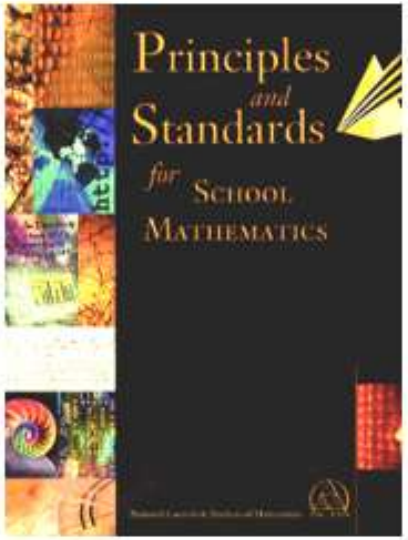
A WV Classroom Video Experience Grade 8

PLEASE STAND UP IF:

A 25-year History of Standards-Based Mathematics Education Reform

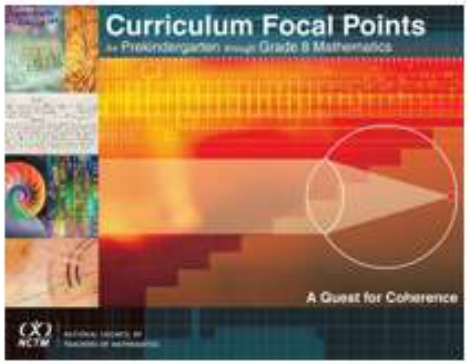
1989 *Curriculum and Evaluation Standards for School Mathematics*

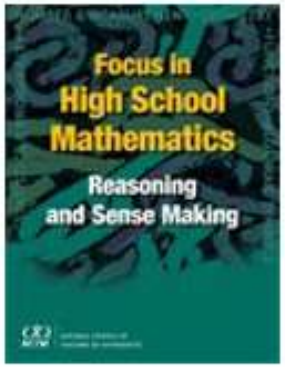
2000 *Principles and Standards for School Mathematics*



2006 *Curriculum Focal Points*

2010 *Focus in High School Mathematics*



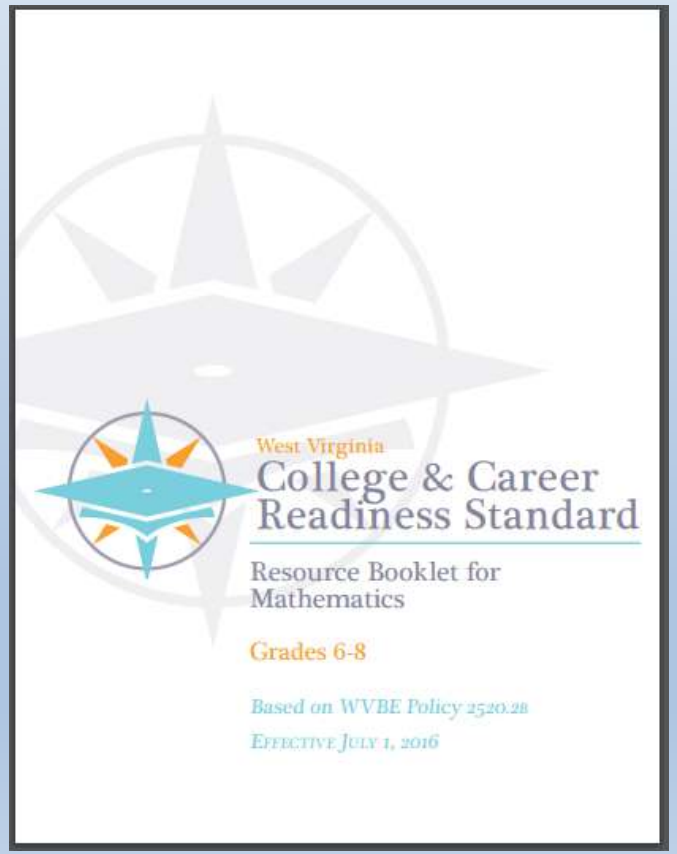


A 25-year History of Standards-Based Mathematics Education Reform

2014 *Principles to Actions: Ensuring Mathematical Success for All*

The overarching message of *Principles to Actions* is that **effective teaching** is the non-negotiable core necessary to ensure that all students learn mathematics.

West Virginia College- and Career-Readiness Standards for Mathematics (2016)

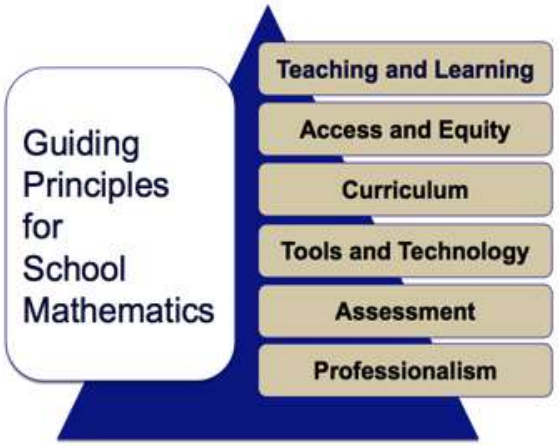


The West Virginia College- and Career-Readiness Standards for Mathematics define what students should understand and be able to do in their study of mathematics. However, the Standards do not describe or prescribe the essential conditions required to make sure mathematics works for all students.

Although We Have Made Progress, Challenges Remain

- The average mathematics NAEP score for eighth grade students has been essentially flat since 2009.
- Among 79 countries participating in the 2018 Programme for International Student Assessment (PISA) of 15-year-olds, the U.S. ranked 37th in mathematics.
- Significant learning differentials remain.

Teaching and Learning Principle



"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically."

Principles to Actions (NCTM, 2014, p.7)

Beliefs About Teaching and Learning Mathematics - Activity

DIRECTIONS

In a group or with a partner:

1. Open the packet and remove the cards.
2. Find the header cards: **Productive** and **Unproductive.** Place these cards on the table.
3. Each of the remaining cards identifies a belief about teaching and learning. Read each of the belief cards.
4. Identify which belief cards are Productive and which belief cards are Unproductive.
5. Place each belief card under the header card to which it was matched.
6. Be prepared to defend your decisions.

Beliefs about teaching and learning mathematics	
Unproductive beliefs	**Productive beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

We Must Focus on Instruction

- Student learning of mathematics "depends fundamentally on **what happens inside the classroom** as teachers and learners interact over the curriculum."

  (Ball & Forzani, 2011)

- Teaching has **6 to 10 times as much impact** on achievement as all other factors combined…Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points."

  (Schmoker, 2005)

Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Effective Mathematics Teaching Practices Look Fors

2.8 EFFECTIVE TEACHING LOOK FORS

As you watch the video, check each "Look For" for each Teaching Practice that is observed. Provide evidence (situation/time stamp) for each Teaching Practice observed.

Teaching Practice (NCTM, 2014) Look Fors	Evidence
Establish mathematics goals to focus learning. ☐ Goals are appropriate, challenging, and attainable. ☐ Goals are specific to the lesson and clear to students. ☐ Goals are connected to other mathematics. ☐ Goals are revisited throughout the lesson.	
Implement tasks that promote reasoning and problem-solving. ☐ Chooses engaging, high-cognitive-demand tasks with multiple solution pathways. ☐ Chooses tasks that arise from home, community, and society. ☐ Uses how, why, and when questions to prompt students to reflect on their reasoning.	
Use and connect mathematical representations. ☐ Uses tasks that lend themselves to multiple representations. ☐ Selects representations that bring new mathematical insights. ☐ Gives students time to select, use, and compare representations. ☐ Connects representations to mathematicsconcepts.	
Facilitate meaningful mathematical discourse. ☐ Helps students share, listen, honor, and critique each other's ideas. ☐ Helps students consider and discuss each other's thinking. ☐ Strategically sequences and uses student responses to highlight mathematical ideas and language.	
Pose purposeful questions. ☐ Questions make the mathematicsvisible. ☐ Questions solidify and extend student thinking. ☐ Questions elicit student comparison of ideas and strategies. ☐ Strategies are used to ensure every child is thinking of answers.	
Build procedural fluency from conceptual understanding. ☐ Gives students time to think about different ways to approach a problem. ☐ Encourages students to use their own strategies and methods. ☐ Asks students to compare different methods. ☐ Asks why a strategy is a good choice.	
Support productive struggle in learning mathematics. ☐ Provides ample wait time. ☐ Talks about the value of making multiple attempts and persistence. ☐ Facilitates discussion on mathematical error(s), misconception(s), or struggle(s) and how to overcome them.	
Elicit and use evidence of student thinking. ☐ Identifies strategies or representations that are important to look for as evidence of student understanding. ☐ Makes just-in-time decisions based on observations, student responses to questions, and written work. ☐ Uses questions or prompts that probe, scaffold, or extend students' understanding.	

Source: Previously published by Bay-Williams, J., McGatha, M., Calcot, B., and Wray, J. (2014). Mathematics Coaching: Resources and Tools for Coaches and Leaders, K–12. *New York, NY: Pearson Education, Inc.*

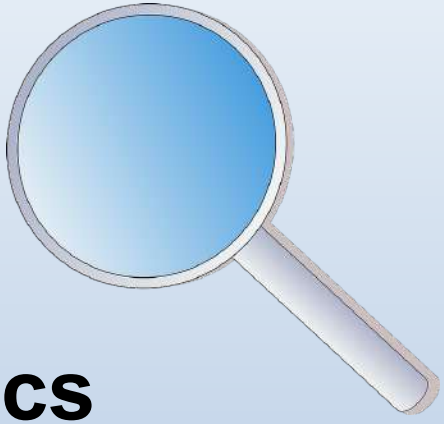
West Virginia Classroom Video - *Tessellate This* Overview

Students utilize a set of geometric shapes to create their own tessellations. They analyze a partner's tessellation and describe the transformations employed to take the pattern to itself.

Upon successful task completion, students will:

- Create tessellations and designs with rotational symmetry using rigid transformations.
- Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.
- Determine a sequence of rigid motion transformation that maps a figure onto a congruent figure.
- Repeatedly use rigid transformation to make interesting repeating patterns of figures.

A Closer Look



Effective Mathematics Teaching Practice:

Establish Mathematics Goals to Focus Learning

Establish Mathematics Goals to Focus Learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction.
- Be situated within learning progressions.
- Frame the decisions that teachers make during a lesson.

Establish Mathematics Goals to Focus Learning

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk & Janssen, 2007)

WATCH VIDEO

NCTM Principles to Action:

Establish Mathematics Goals to Focus Learning

Video

Establish Mathematics Goals to Focus Learning

Establish mathematics goals to focus learning Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

Tessellate This

Lesson Alignment to the West Virginia College- and Career-Readiness Standards

M.8.16
Verify experimentally the properties of rotations, reflections and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

M.8.17
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.18
Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

M.8.19
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

WV Classroom Video - *Tessellate This*

Goals to focus learning should be written in student-friendly language

Upon successful completion, students will:

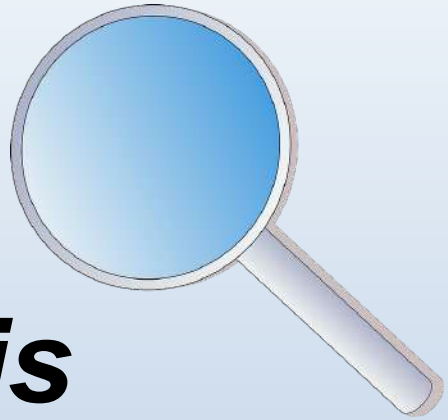
- Create tessellations and designs with rotational symmetry using rigid transformations.
- Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.
- Determine a sequence of rigid motion transformations that maps a figure onto a congruent figure.
- Repeatedly use rigid transformation to make interesting repeating patterns of figures.

Establish Mathematics Goals to Focus Learning

1. Why is it important to establish math expectations for student learning?

2. In what ways will the math goals focus the teacher's interactions with students throughout the lesson?

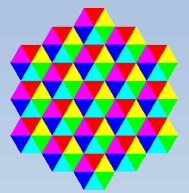
A Closer Look



Tessellate This

WV Classroom Video Experience and the Effective Mathematics Teaching Practice:

Implement Tasks That Promote Reasoning and Problem Solving



Implement Tasks That Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to understanding concepts
- Build on students' current understanding and experiences
- Have multiple entry points
- Allow for varied solution strategies

Why Are Tasks So Important?

Mathematical tasks:

- Represent the meat of instruction
- Are how we engage students and support the development of mathematical understanding
- Connect learning goals to the actual classroom opportunity to learn
- Use procedures to get answers to simple problems BUT are opportunities to develop deeper and broader understanding and application of mathematics

What Makes a GOOD Task?

GOOD Mathematical Tasks Are:

Accessible – Have clear directions and multiple entry points

Fair – All students are able to complete the task

Reasonable – Not too complex and have familiar context

Aligned – Matches standards and current learning goals

Comprehensive – Integrate key understandings and big enough bang for the time commitment

Engaging – Use graphics and have an intriguing or familiar context

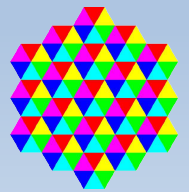
Divergent – Have multiple pathways to solve

Implement Tasks That Promote Reasoning and Problem Solving

Based on the WV Classroom Video:

Video Clip

In what ways did the implementation of the task allow for multiple entry points and engage students in reasoning and problem solving?

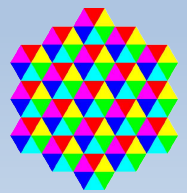


A Closer Look

Tessellate This

WV Classroom Video Experience and the Effective Mathematics Teaching Practice:

Use and Connect Mathematical Representations



Use and Connect Mathematical Representations

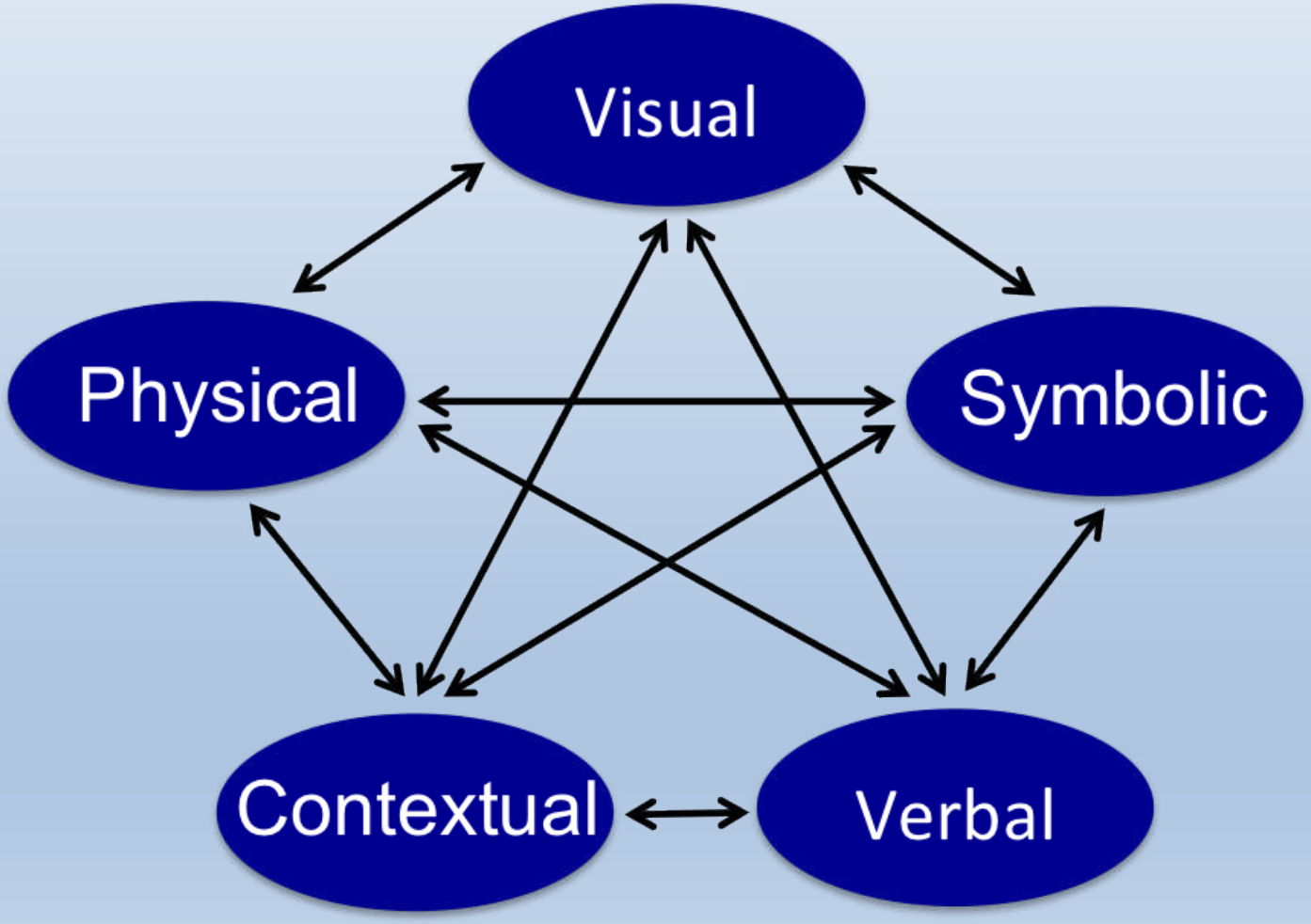
Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas.

(National Research Council, 2001, p.94)

Use and Connect Mathematical Representations

Representations embody critical features of mathematical constructs and actions, such as **drawing pictures, creating tables, or using manipulatives** to show and explain mathematical understanding. When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate enhanced problem-solving ability.

Different Types of Mathematical Representations



Use and Connect Mathematical Representations

Teachers should:

- Allocate instructional time for students to use, discuss, and make connections among representations
- Encourage students to explain, elaborate or clarify their thinking
- Ask students to use pictures to explain and justify their reasoning

Rich Mathematical Task Rubric – Representations and Connections

TASK LEVEL	DESCRIPTION OF USE AND CONNECTION OF REPRESENTTIONS
ADVANCED	• Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding
PROFICIENT	• Uses a representation or multiple representations to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem
DEVELOPING	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem
EMERGING	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

Use and Connect Mathematical Representations

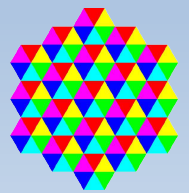
Based on the WV Classroom Video:

Video Clip

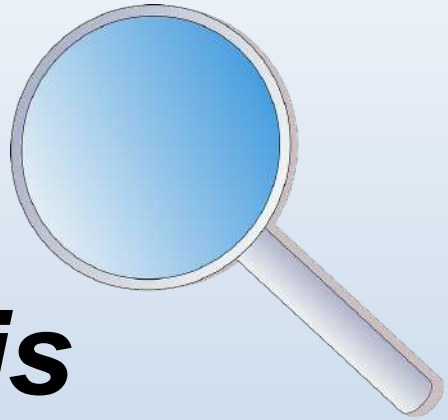
1. What mathematical representations were students using in the lesson?

2. How did the teacher support students in making connections between and within different types of representations?

3. What is the Task Level for the *Tessellate This* lesson? Explain your rating.



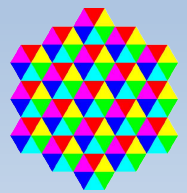
A Closer Look



Tessellate This

WV Classroom Video Experience and the Effective Mathematics Teaching Practice:

Facilitate Meaningful Mathematical Discourse



Facilitate Meaningful Mathematical Discourse

Mathematical discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the math learning of the whole class.

Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual and written communication. The discourse in the mathematics classroom gives students opportunities to share ideas, clarify understandings, construct convincing arguments, develop language for expressing mathematical ideas, and learn to see things from other perspectives.

Facilitate Meaningful Mathematical Discourse

Set up classroom norms so that everyone knows their role in the classroom.

The teacher's role includes orchestrating discourse by:

- Posing questions to challenge student thinking;
- Listening carefully and monitoring understanding; and
- Encouraging each student to participate - even if it means asking, "Who can repeat what Andrew said?" or "Who can explain in another way what Bailey did?"

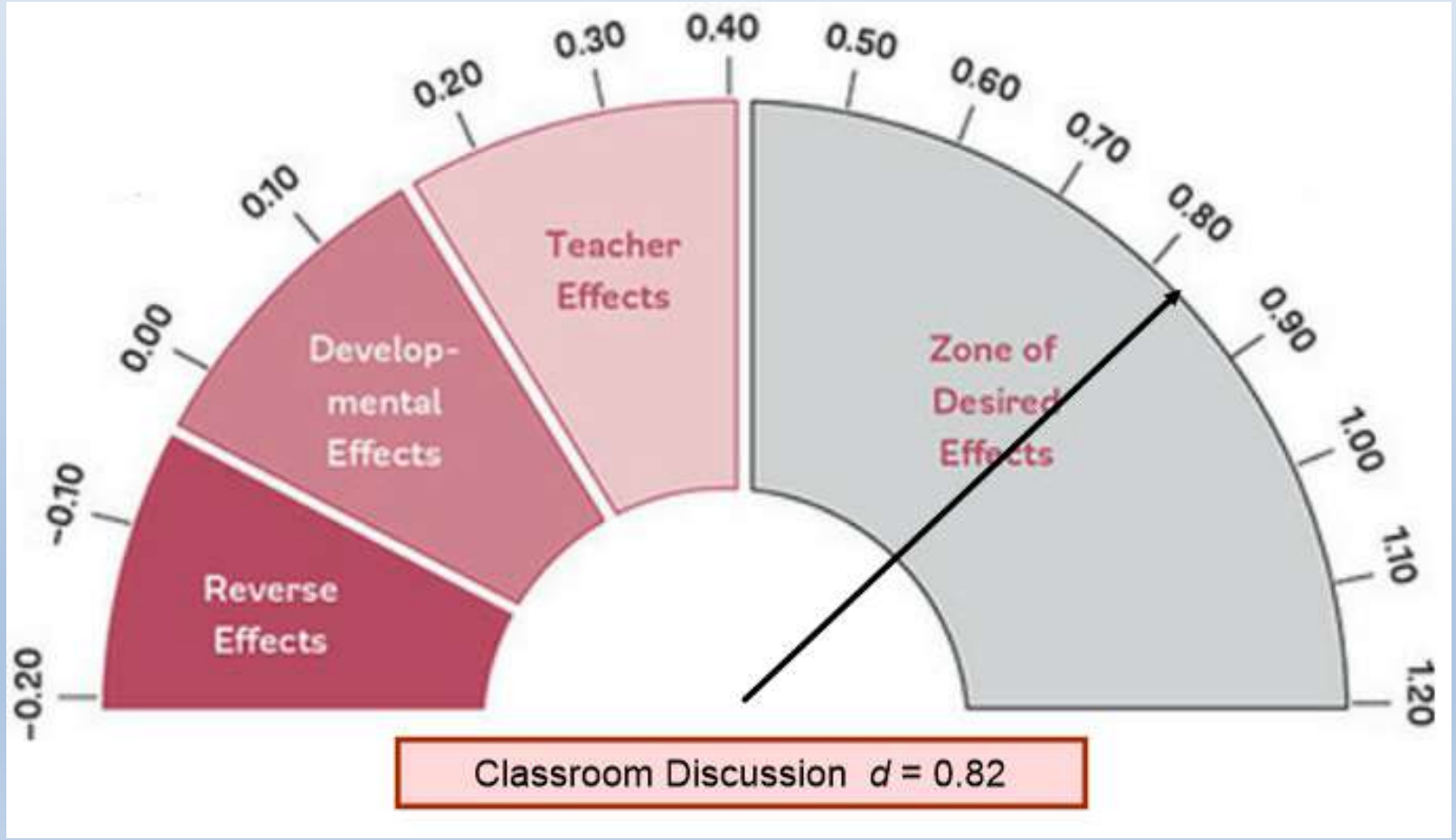
The student's role includes:

- Listening and responding to the teacher and one another;
- Using a variety of tools to reason, make connections, solve problems; and
- Communicating, and make convincing arguments of particular representations, procedures, and solutions.

Facilitate Meaningful Mathematical Discourse

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

Impact of Meaningful Mathematical Discourse



Facilitate Meaningful Mathematical Discourse

Based on the WV Classroom Video

Video Clip

1. What teacher actions did you view that supported meaningful mathematical discourse? Cite evidence from the video to support your response.

2. Did the mathematical discourse within the lesson promote equity in the classroom? If yes, how? If no, what could the teacher have done to promote equity through mathematical discourse?

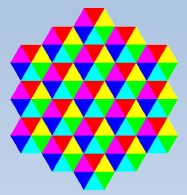
3. To what extent did the discourse facilitate student explanations or clarify their thinking?

A Closer Look

Tessellate This

WV Classroom Video Experience and the Effective Mathematics Teaching Practice:

Pose Purposeful Questions



Pose Purposeful Questions

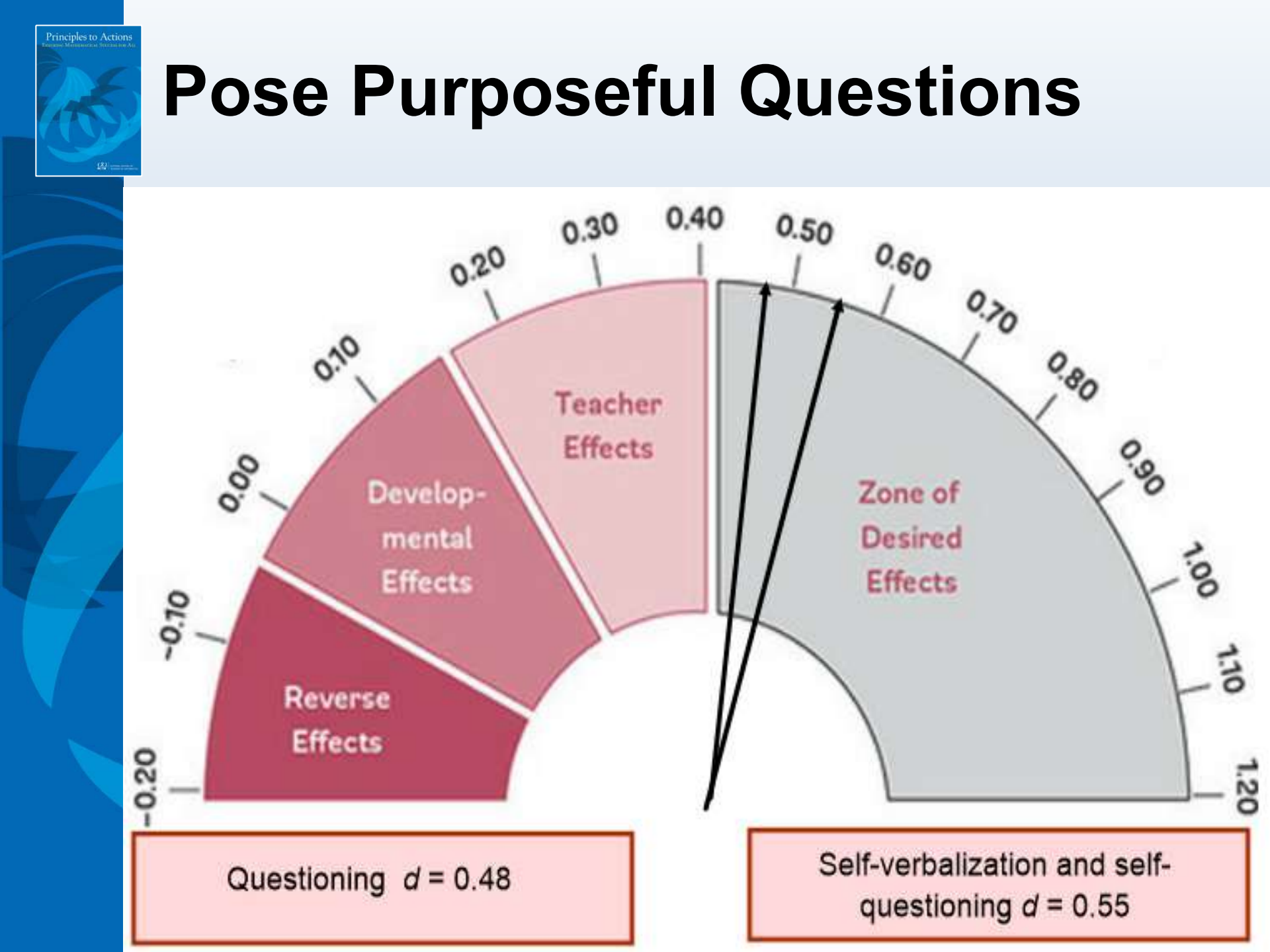
Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

Pose Purposeful Questions

Effective Questions should:

- Reveal students' current understandings
- Encourage students to explain, elaborate or clarify their thinking
- Make the mathematics more visible and accessible for students

Teachers' questions are crucial *in helping students make connections and learn important mathematics concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.*


Principles to Actions
Pose Purposeful Questions
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0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
Reverse Effects
Develop-mental Effects
Teacher Effects
Zone of Desired Effects
Questioning d = 0.48
Self-verbalization and self-questioning d = 0.55

Five Types of Questions

Question Type	Purpose
Gathering Information	Ask students to recall facts, definitions, or procedures.
Probing thinking	Ask students to explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or completion of a task.
Making the mathematics visible	Ask students to discuss mathematical structures and make connections among mathematical ideas and relationships.
Encouraging reflection and justification	Reveal deeper insight into student reasoning and actions, including asking students to argue for the validity of their work.
Engaging with the reasoning of others	Help students to develop an understanding of each other's solution paths and thinking, and lead to the co-construction of mathematical ideas.

Pose Purposeful Questions

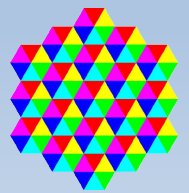
Based on the WV Classroom Video:

Video Clip

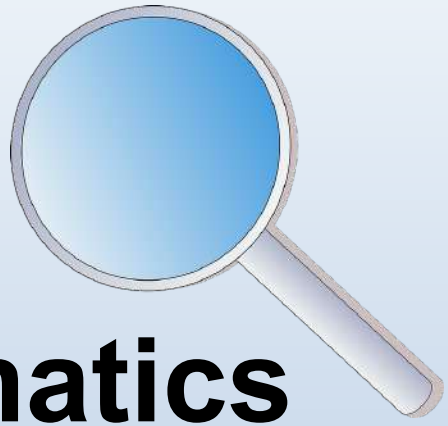
1. What did you notice about the questions the teacher asked?

2. What purposes did the questions appear to serve?

3. Were all students' ideas and questions heard, valued and pursued? Cite evidence from the video to support your response.



A Closer Look



Effective Mathematics Teaching Practice:

Build Procedural Fluency from Conceptual Understanding

Build Procedural Fluency from Conceptual Understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

A rush to fluency undermines students' confidence and interest in mathematics and is considered a cause of mathematics anxiety.

(Ashcraft 2002; Ramirez Gunderson, Levine, & Beilock, 2013)

Build Procedural Fluency from Conceptual Understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding
- Over time (months, years), result in known facts and generalized methods for solving problems
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

Build Procedural Fluency from Conceptual Understanding

To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect.

Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

Build Procedural Fluency from Conceptual Understanding

What are teachers doing?	What are students doing?
• Providing students with opportunities to use their own reasoning strategies and methods for solving problems. • Asking students to discuss and explain why the procedures that they are using work to solve particular problems. • Connecting student-generated strategies and methods to more efficient procedures as appropriate. • Using visual models to support students' understanding of general methods. • Providing students with opportunities for distributed practice of procedures.	• Making sure that they understand and can explain the mathematical basis for the procedures that they are using. • Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. • Determining whether specific approaches generalize to a broad class of problems. • Striving to use procedures appropriately and efficiently.

Build Procedural Fluency from Conceptual Understanding

1. What procedural fluency skills are essential to completing a task?

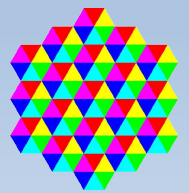
2. What teacher actions do teachers use to building procedural fluency from conceptual understanding?

A Closer Look

Tessellate This

WV Classroom Video Experience and the Effective Mathematics Teaching Practice:

Support Productive Struggle in Learning Mathematics



Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

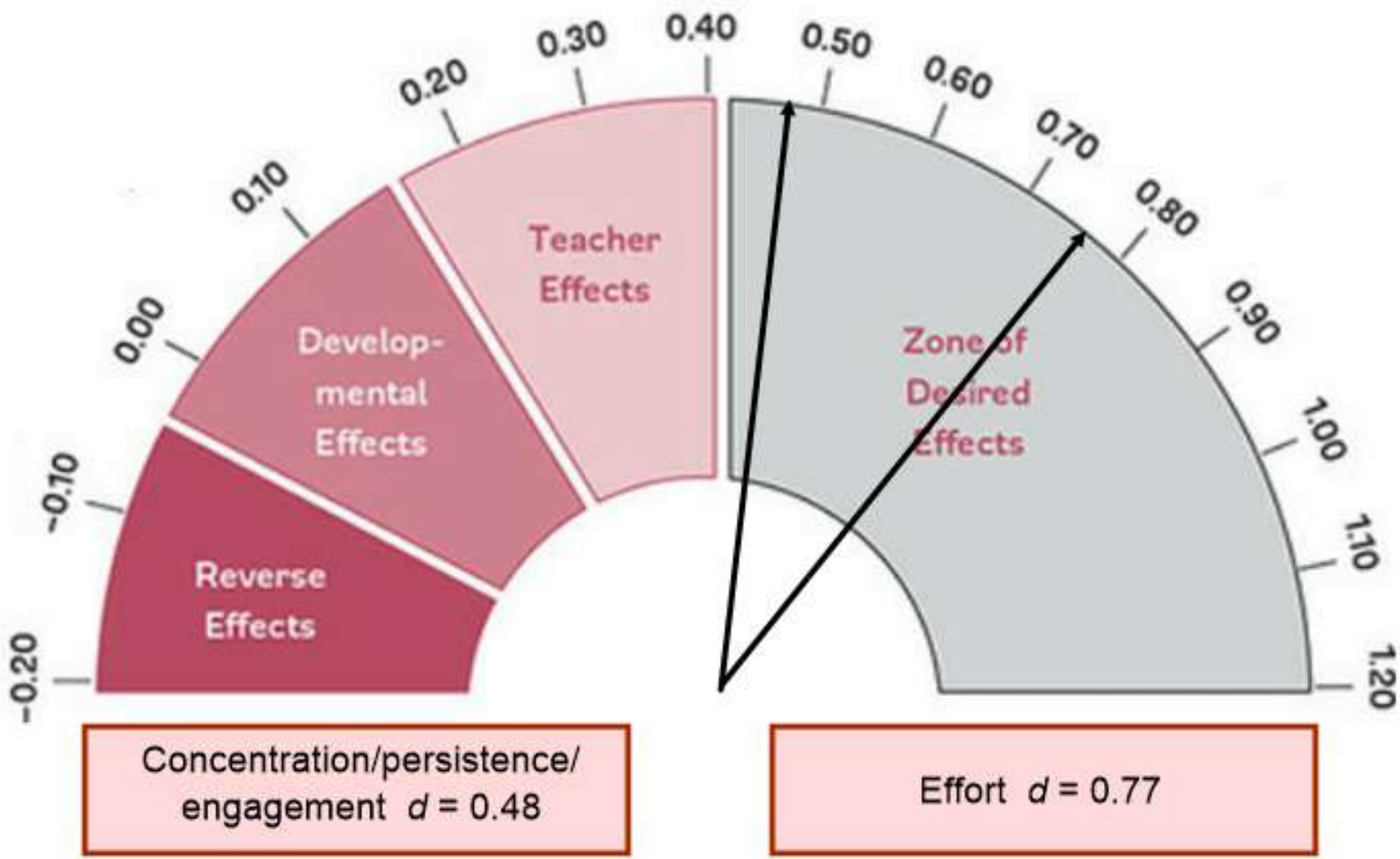
Hiebert & Grouws, 2007, pp. 387-388

Support Productive Struggle in Learning Mathematics

Productive Struggle entails:

- Students individually and collectively grappling with mathematical ideas and relationships
- Teachers and students understanding that frustration may occur, but perseverance is important
- Communicating about thinking to make it possible for students to help one another make progress on the task

Support Productive Struggle in Learning Mathematics



Support Productive Struggle in Learning Mathematics

Support productive struggle in learning mathematics Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

Support Productive Struggle in Learning Mathematics

Based on the WV Classroom Video:

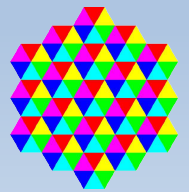
Video Clip

1. How did the teacher support productive struggle among the students – individually and collectively?

1. Did the teacher restrain from "taking over" the thinking of the students?

   If YES, cite evidence from the video that the restrain occurred.

   If NO, cite when it occurred in the video and suggest what the teacher could have done to help the students persevere.



A Closer Look

Effective Mathematics Teaching Practice:

Elicit and Use Evidence of Student Thinking

Elicit and Use Evidence of Student Thinking

Effective teaching of mathematics **uses evidence of student thinking** to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

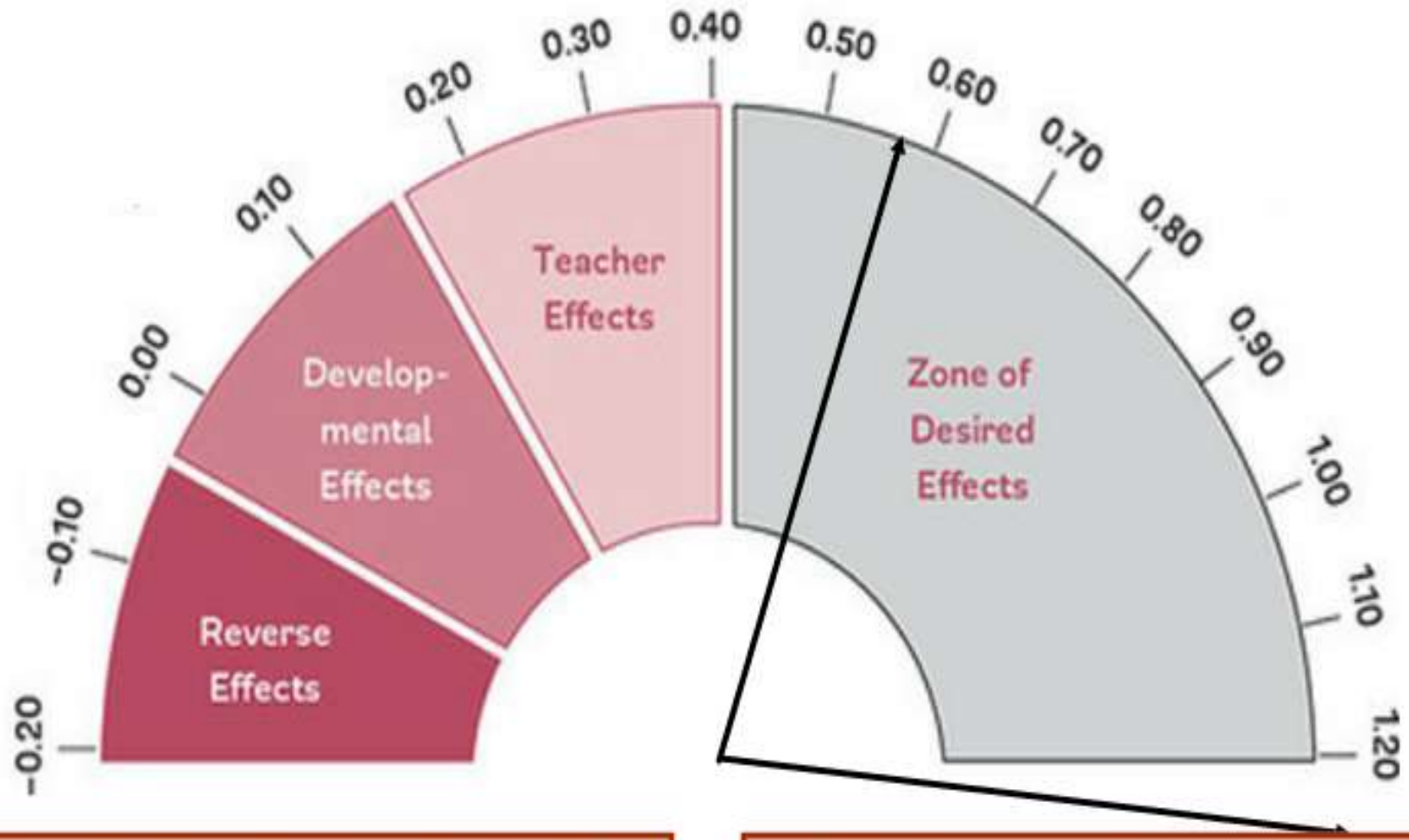
Elicit and Use Evidence of Student Thinking

Evidence should:

- Provide a window into students' thinking
- Help the teacher determine the extent to which students are reaching the math learning goals
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

Elicit and Use Evidence of Student Thinking



Providing formative evaluation $d = 0.58$

Cognitive task analysis $d = 1.29$

Elicit and Use Evidence of Student Thinking

1. Identify specific places during the lesson in which a teacher could elicited evidence of student learning.

2. Discuss how a teacher might use that evidence to adjust instruction to support and extend student learning.

Going Forward

As you reflect on the Effective Mathematics Teaching Practices, identify 1-2 Practices that will strengthen your own instruction.

Working with a partner, develop a list of actions to begin the next steps of your journey toward ensuring mathematical success for all your students.

Tessellate This:
A WV Classroom Video Experience

School: Barboursville Middle School
Barboursville, WV

Teacher: Felicia Backus

Class: Grade 8

Curriculum: *Illustrative Mathematics*

Size: 25 students