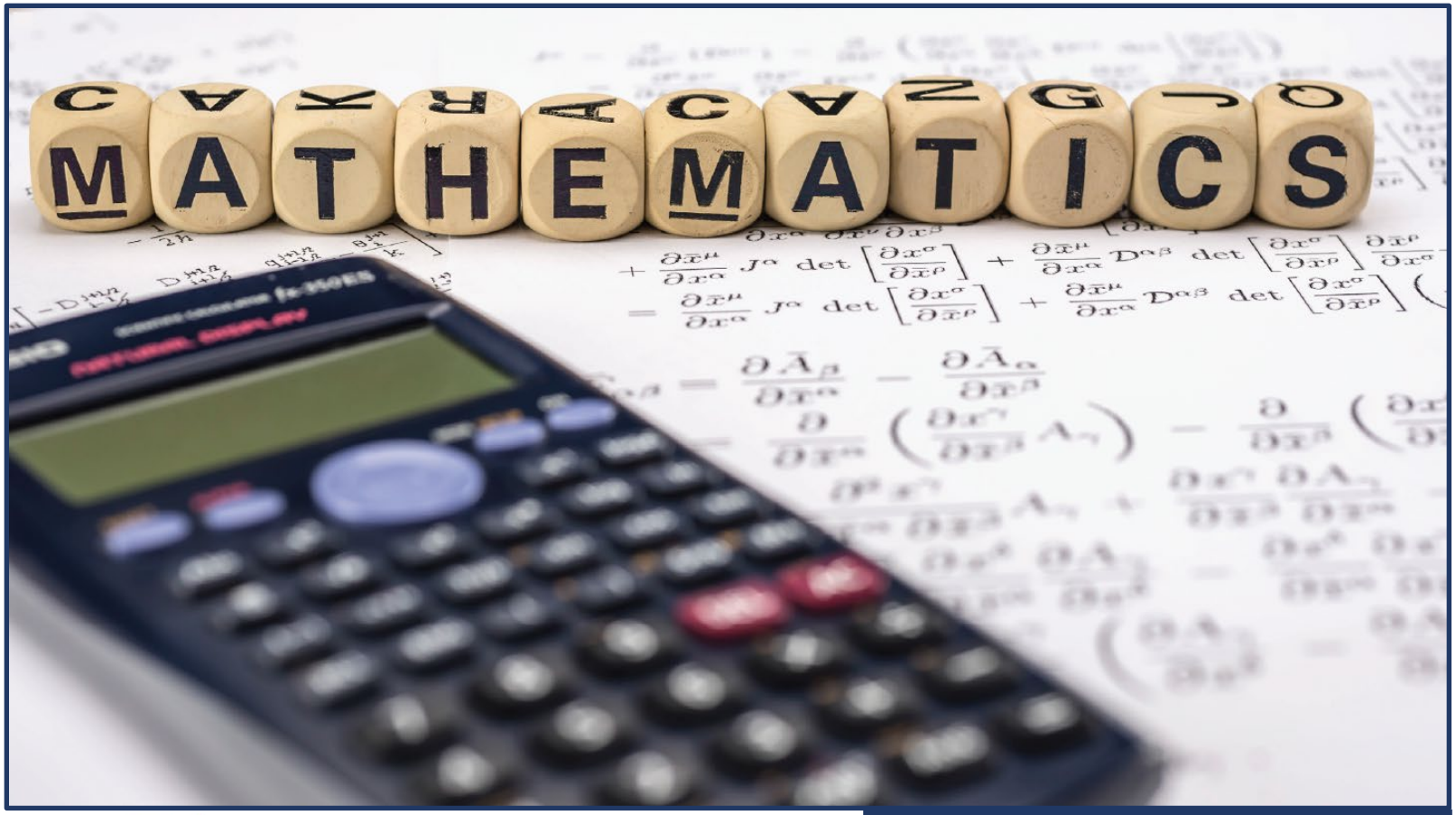
2019 Alabama Alternate Achievement Standards

Teaching and Learning Guide



Math

Ms. Crystal Richardson, Program Coordinator

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Introduction

The *Alabama Alternate Teaching and Learning Guides* were developed in conjunction with Alabama special education and content educators. The guides provide instructional guidance to teachers in instruction and promote learning for students with significant cognitive disabilities who qualify for the Alabama Alternate Assessment. The Alabama Alternate Achievement Standards listed in this document have been prioritized to cover the grade-level expectations of the Alabama State Standards and reflect a decreased depth and breadth of the content. The Alabama Alternate Teaching and Learning Guides will aid teachers during instruction by showing the alignment to the general education standards, outlining the achievement elements of the alternate achievement standards, providing key vocabulary, listing the progressions of the content toward mastery of the alternate achievement standards, and providing examples of how to apply the alternate achievement standards during instruction.

Purpose of the Alabama Alternate Achievement Standards

The Alabama Alternate Achievement Standards are for the instruction and assessment of students in the one-percent population, which is defined as students who

- have the most significant cognitive disabilities or multiple disabilities that affect intellectual functioning and adaptive behaviors and are unable to participate in the general assessment even with accommodations.

Purpose and Rigor of the Alabama Alternate Achievement Standards

Evidence from the Alabama Alternate Assessment shows that students with significant cognitive disabilities can and do learn. The data from this assessment over time also show that students in the one-percent population are ready for increased rigor and content. Furthermore, the Every Student Succeeds Act (ESSA) requires states to adopt challenging academic standards and assessments, which include alternate academic achievement standards for students with the most significant cognitive disabilities. States must ensure that alternate achievement standards

- are aligned with state standards.
- promote access to the general education curriculum.
- reflect professional judgment to the highest possible standards achievable by such students.
- are aligned to ensure the students meet the alternate standards and are on track to pursue postsecondary education or employment.

Purpose of the Teaching and Learning Guides

The Teaching and Learning Guides are designed to meet the following goals:

- To provide guidance for teaching the Alabama Alternate Achievement Standards that align to the Alabama State Standards.
- To provide content-based differentiation strategies, tools, and methodologies for instruction for students with significant cognitive disabilities that are content based.
- To provides teachers with a method to delineate the skills and knowledge needed for students with significant cognitive disabilities to master the alternate standards.
- To provide content-based examples to aid in instruction for students with significant cognitive disabilities.

How to Use the Guide

The Teaching and Learning Guides are organized by grade, strand, and objective in the text box at the top of the page. For each strand and objective, the following information is included to help guide instruction for students with significant cognitive disabilities:

- **General Education Standards** The General Education Standards listed in this document are the Alabama State standards that are most essential for students with significant disabilities.

- **Alabama Alternate Achievement Standards** The Alabama Alternate Achievement Standards in this document are aligned to the general education standards and reflect a reduction of the depth and breadth of these standards.
- **Achievement Elements** The Achievement Elements are statements that identify the essential skills needed for students to achieve the Alabama Alternate achievement standards.
- **Key Vocabulary** The Key Vocabulary section identifies the content language specific to the standard that is necessary for students to know to meet that standard.
- **Teaching and Learning Progressions** The Teaching and Learning Progressions sections map out the sequence of knowledge and skills necessary to achieve the Alabama Alternate Achievement Standards. Note: The teaching and learning progressions are not all-encompassing because students in this population may need instruction or skills that are not outlined in this guide.
- **Application of Alabama Alternate Achievement Standards** The Application of Alabama Alternate Achievement Standards sections provide teachers with some activities that are related to the Achievement Elements.
- **Levels of Standards** The Levels of Standards tables show examples of the skills students may be working toward at each level of achievement.

Connectivity of the Alabama Alternate Achievement Standards to Instruction and the General Education Standards

The Alabama Alternate Achievement Standards are aligned to the general education standards by design so students with cognitive disabilities can have access to age-appropriate content and the general education curriculum. Because the Alabama Alternate Achievement Standards reflect a less rigorous depth and breadth of the general education standards, the alternate standards work in concert with the general education curriculum. The information in the Teaching and Learning Progressions section in the guides can be used to show how students with significant cognitive disabilities are connected to and can access the general curriculum.

Students Who Are Pre-Symbolic Learners

All students can and do learn. The students who are considered pre-symbolic learners have the following characteristics:

- They gain attention through vocalizations, body movements, cries, changes in facial expressions, or changes in body position.
- They exhibit unclear or irregular responses to stimuli from others (e.g., smell, touch, speech, or vision).
- They have wants and needs that are interpreted by others based on their vocalizations, body movements, cries, changes in facial expressions, or changes in body position**.**

When teaching students who are pre-symbolic learners, teachers may want to include the following skills in their instruction to facilitate communication and learning:

- Communicate intentionally to continue or repeat an activity or action.
- Identify own interests.
- Seek out or request attention of others.
- Search for objects or people of interest.
- Direct attention to objects or people.
- Express a preference (e.g., likes or dislikes).
- Express an interest in something.
- Respond to something new or different.
- Express a like or dislike for a stimulus.
- Respond to a stimulus (e.g., follows a light with eyes, vocalizes when spoken to, shows interest in objects).

Using the Standards and the Teaching and Learning Guides to Write IEP Goals

The Alabama Alternate Achievement Standards inform teachers about what to teach but not *how* to teach. The IEP needs to reference the alternate achievement standards, and the goals need to address the specific skills in the standards that an individual student needs to be taught in order to meet the standards. Teachers may use the teaching and learning progressions as a guide for specific skills the student may need to access the alternate achievement standard.

Accommodations and Supports

The accommodations and supports listed in the table below are supported by the most current research and are available for assessments. Students with significant support needs may have additional accommodations and supports that are not listed here but are necessary to access instruction.

Accommodations and Supports

Accommodation	Description
American Sign Language	A teacher may use American Sign Language (ASL) or manually coded English as a presentation accommodation.
Assistive Technology	An assistive technology device is any item, piece of equipment, or product system, whether acquired commercially off the shelf, modified, or customized, that is used to present materials to a student.

Accommodation	Description
Braille	Braille, either contracted or uncontracted, may be by students who are blind or visually impaired.
Brailler/Braille Tools	For students who use braille, a brailler and other braille tools such as a raised number line, braille ruler, and braille clock may be used.
Breaks	Students in the one percent population may need frequent breaks to refocus or rest.
Color Overlays/High Contrast	Color transparencies may be placed over a paper-based assessment, and the color of computer background screens or text may be changed to provide the student with high contrast.
Computer, Speech to Text, or Another Device to Write	A computer or other device may be used to respond to a writing prompt.
Extended Time	Extended time is generally time and a half.
Flexible Scheduling	Flexible scheduling is imperative. A teacher is able to choose the time of day that is best for the student. Teachers may also stop and restart the test at any time based on the student's needs. Note: teachers may not administer any questions that had already been presented to or answered by the student.

Accommodation	Description
Highlighters	Highlighters may be used to color text in items or passages.
Individualized Instruction and Assessment	Instruction and assessment is administered in small groups or in a one-to-one setting.
Large Print	In general, large print is 18 pt. font.
Line Readers	Line readers may be used to isolate text.
Magnification Tool or Device	A magnifying glass or CCTV may be used if the student requires further enlargement of the student materials.
Masking	Masking involves blocking off content that is not of immediate need or that may be distracting to the student. Students are able to focus their attention on a specific part of a test item by masking.
Math Manipulative	Math manipulatives such as counters, a hundreds chart, and specialized rulers may be provided.
Physical Prompting	The teacher may assist a student with limited physical and/or mobility manipulate objects.
Picture Symbols	Picture symbols may be used for the writing prompt. An array of pictures on and off topic should be presented to the student.
Real Objects	Real objects may be substituted for pictures.

Accommodation	Description
Scratch Paper	Scratch paper to make notes, write computations, or record responses may be made available.
Scribe	A scribe may be used for students who have difficulty with writing or pointing. Student will dictate his or her responses to the teacher who records what is dictated by the student.

Grade: Kindergarten

Content Area: Mathematics
Strand: Counting and Cardinality
Objectives: Know number names and the count sequence.
Count to tell the number of objects.

General Education Standards

M.K.1 Count to 100 by ones and by tens.

M.K.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

M.K.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

M.K.5 Count to answer "how many" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Alternate Achievement Standards

M.AAS.K.1 Count to twenty by ones.

M.AAS.K.3 Distinguish numerals from other print (letters, symbols); recognize numerals 0 through 5 as written.

M.AAS.K.4 When counting objects, demonstrate one-to-one correspondence by saying the number names in standard order, pairing each object with one and only one number name, and each number name with one and only one object (limit numbers and objects to five).

M.AAS.K.5 Answer "how many" questions by counting objects arranged in a line and a rectangular array (limit objects to five).

Achievement Elements

Students will be able to count to ten by ones and recognize those written numerals as well as answer the question “how many” by counting objects (up to ten).

Key Vocabulary

count, number, how many

Teaching and Learning Progressions

- Given a set number of objects one through ten, answer the question “How many _____?”
- Pair the number of objects counted with “how many.”
- Understand that the last number name tells the number of objects counted.
- Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects.
- Pair a group of objects with a number representing the total number of objects in the group (up to ten objects).
- Count objects one-by-one, using only one number per object (up to ten objects).
- Recognize that numbers and numerals have meaning.
- Recognize numerals 0 (zero) through 10.
- Identify the difference between written numbers and other written things.
- Identify the difference between written numbers and objects.
- Rote count to ten.
- Rote count to five.
- Rote count to two.
- Communicate some number words.
- Communicate the number word *one*.
- Recognize *after*.
- Recognize *before*.

Application of Alternate Achievement Standards

- Provide two pictures of the same object(s), one labeled with the numeral of "how many" and one labeled with the name of the object. Compare the numerals/numbers with words.
- Demonstrate counting, using objects familiar to students (e.g., pencils, stickers, counting bears).
- Using items that students have (or are given), demonstrate answering the question "How many _____?"

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Count and answer "how many" for a number greater than ten.	Count and answer "how many" for a number up to ten.	Count up to ten objects.	Rote count up to three.

Grade: Kindergarten

Content Area: Mathematics
Strand: Counting and Cardinality
Objective: Compare numbers.

General Education Standards

M.K.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)

M.K.7 Compare two numbers between 1 and 10 presented as written numerals.

Alternate Achievement Standards

M.AAS.K.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (limit objects per group to five).

M.AAS.K.7 Compare two numbers between 1 and 5 presented as written numerals.

Achievement Elements

Students will be able to compare quantities up to ten, using greater/*more, less/fewer* and *equal/same*.

Key Vocabulary

greater/more, less/fewer, equal/same

Teaching and Learning Progressions

- When presented with two groups of objects (limited to up to ten objects in each group), compare the two groups, using *greater than, less than,* or *equal to.*
- When presented with two groups of objects (limited to up to five objects in each group), compare the two groups, using *greater than, less than,* or *equal to.*
- Recognize *less/fewer.*

- Recognize *greater/more.*
- Recognize *same/equal.*
- Count objects up to ten arranged in a line or an array.
- Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects.
- Rote count to ten.
- Rote count to five.
- Rote count to two.
- Communicate some number words.
- Communicate the number word *one*.
- Recognize *after*.
- Recognize *before*.

Application of Alternate Achievement Standards

- Using objects of student interest (e.g., beans, stickers, counting bears), demonstrate using two groups: more, less, equal.
- Using cards or items, have students match sets that are equal to each other.
- Demonstrate more, less, and equal in real-world situations (e.g., two hands/one nose, and two doors, and five windows in the room).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use comparative language to describe two sets of objects of up to ten objects.	Identify a set that is more, less, or the same as a given set (sets limited to no more than ten objects).	Identify a set that is equal to a given set (sets limited to five objects).	Count the number of objects in two independent sets (sets limited to five objects).

Grade: Kindergarten

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

General Education Standards

M.K.8 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problem. This applies wherever drawings are mentioned in the Standards.)

M.K.9 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

M.K.10 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

M.K.12 Fluently add and subtract within 5.

Alternate Achievement Standards

M.AAS.K.8 Represent addition as “add to/put together” and subtraction as “take from/take apart” with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, or verbal explanations (limited to five).

M.AAS.K.9 Solve addition and subtraction word problems, and add and subtract within 5, e.g., by using objects or drawings to represent the problem.

M.AAS.K.10 Demonstrate, using a model with objects, composing and decomposing numbers (limited to ten and less).

M.AAS.K.12 Represent addition and subtraction of 1 more or 1 less from a number 1 to 5.

Achievement Elements

Students will be able to understand addition and subtraction, using addition and subtraction of one.

Key Vocabulary

putting together, taking from/away, plus one, minus one

Teaching and Learning Progressions

- Subtract one from a set of objects (up to five objects).
- Add one to a set of objects (up to five objects).
- Given a group of objects (ten or less), divide the group into smaller groups in various ways.
- Given small groups of objects, create larger groups by combining the small groups.
- Take away objects from a large group to create two smaller groups.
- Put together two small groups of objects to create a larger group.
- Establish one-to-one correspondence between numbers and objects when given a picture, a drawing, or objects.
- Rote count to ten.

Application of Alternate Achievement Standards

- Using objects of student interest (e.g., stickers, counting bears), demonstrate one more and one less.
- Using objects that the students have (e.g., counting bears, chips, coins, crayons), demonstrate combining objects to make a larger set.
- Using a set of up to ten objects, have students take away items to demonstrate subtraction.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Given a verbal cue, demonstrate addition and subtraction of one from numbers one through ten.	Using objects, add and subtract one from a number one through five in response to a verbal cue.	Given a set of objects (ten or less in each), add or take away objects to make a smaller or larger set in response to a verbal cue.	Given two sets of objects (five or less in each), make a large set by combining.

Grade: Kindergarten

Content Area: Mathematics
Strand: Measurement and Data
Objective: Describe and compare measurable attributes.

General Education Standards

M.K.14 Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object.
M.K.15 Directly compare two objects, with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.

Alternate Achievement Standards

M.AAS.K.14 Describe common objects by attribute such as size (*big*/*small*), weight (*heavy*/*light*) and length (*long*/*short*).
M.AAS.K.15 Compare two objects with regard to size (*bigger*/*smaller*) and weight (*heavier*/*lighter*) and height (*taller*/*shorter*).

Achievement Elements

Students will be able to compare object attributes and communicate differences in size, weight, height, and length.

Key Vocabulary

bigger/smaller, heavier/lighter, longer or taller/shorter

Teaching and Learning Progressions

- Directly compare two objects with a measurable attribute in common and communicate the difference (*bigger/smaller, heavier/lighter, longer or taller/shorter*).
- Identify one of two objects given an attribute related to size, weight, length, and height (*bigger/smaller, heavier/lighter, longer or taller/shorter*).
- Classify common objects according to height (*tall/short*).
- Classify common objects according to length (*long/short*).

- Classify common objects according to weight (*heavy/light*).
- Classify common objects according to size (big/small).
- Communicate *big, small, heavy, light, long or tall,* and *short*.
- Recognize the attributes of common objects related to size, weight, and length.

Application of Alternate Achievement Standards

- Demonstrate attributes of height, length, weight, and size, using everyday objects in the classroom (e.g., a sheet of paper is light; a large book is heavy; a step stool is short; a cupboard is tall; a counting bear is small; an exercise ball is large).
- Have students identify an object with a certain attribute, using "I spy___." ("I spy something that is heavy.")
- Using objects or pictures, demonstrate the use of comparative language related to a given attribute.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Communicate the difference in two objects related to the same two attributes.	Communicate the difference in two objects related to one attribute.	Identify a difference in one attribute when given two objects.	Identify an object as big/small or heavy/light.

Grade: Kindergarten

Content Area: Mathematics
Strand: Measurement and Data
Objective: Classify objects and count the number of objects in each category.

General Education Standard

M.K.16 Classify objects into given categories; count the number of objects in each category, and sort the categories by count. (Limit category counts to be less than or equal to 10.)

Alternate Achievement Standard

M.AAS.K.16 Explore a simple pictograph (limited to two categories and limit a combined quantity of 5 for both categories).

Achievement Elements

Students will be able to recognize a category and the number of objects in a category.

Key Vocabulary

names of common categories (e.g., pets, fruits, vegetables, cars, colors)

Teaching and Learning Progressions

- Identify items in a category.
- Sort items/pictures into common categories.
- Count objects up to ten.

Application of Alternate Achievement Standards

- Identify common categories in the classroom and school environment.
- Demonstrate sorting items/pictures into common categories.
- Create simple pictographs that represent student interests (e.g., pets, siblings, favorites).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify two categories and communicate the data in the categories on a simple pictograph.	Identify the category in a simple pictograph and communicate the data in that category.	Given a category, select items (from up to four choices) that fit into that category.	Count the number of items on a given pictograph.

Grade: Kindergarten

Content Area: Mathematics
Strand: Geometry
Objective: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

General Education Standards

M.K.17 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*.
M.K.18 Correctly name shapes regardless of their orientations or overall size.
M.K.19 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Alternate Achievement Standards

M.AAS.K.17 Match shapes of the same size and orientation; and describe the relative positions using *in front of* and *behind* (limited to circle, square, rectangle, and triangle).
M.AAS.K.18 Recognize a circle, square, rectangle, and triangle.
M.AAS.K.19 Match a real-life two-dimensional or three-dimensional object with a drawing of the object.

Achievement Elements

Students will be able to identify basic shapes (circle, square, rectangle, triangle), become aware of shapes in the real world, and be able to identify the relative position of objects (in front of, behind).

Key Vocabulary

front, behind, circle, square, rectangle, triangle

Teaching and Learning Progressions

- Communicate the relative position of two shapes in front of/behind.
- Know *in front of* and *behind* as they relate to objects.

- When given objects, manipulate them relative to position.
- Match shapes of the same size and orientation.
- Discriminate shapes: circle, square, rectangle, and triangle.
- Identify same and different when given three objects/pictures, two of which are the same.
- Indicate same and different when given two objects/pictures.
- Sort shapes given circle, square, rectangle, and triangle.
- Sort shapes given three shapes (limited to a total of six items).
- Sort shapes given two shapes (limited to a total of six items).
- Match objects with a drawing of the object.
- Recognize shape as an attribute of a two- or three-dimensional object/drawing.

Application of Alternate Achievement Standards

- Identify shapes in the real world of the classroom and school, using "shape hunts" or "I spy," etc.
- Using hands-on shapes (e.g., cards/cut outs/blocks) identify circle, square, rectangle, triangle.
- Demonstrate various sizes and orientations of the same shape using manipulatives.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Discriminate between objects in front of, behind, beside.	Identify the position of two objects when they are in front of and behind one another.	Place one object behind or in front of another object given a verbal cue.	Manipulate two objects during play.
Recognize that a shape stays the same if its orientation changes.	Match shapes of the same size and orientation.	Match shapes of the same size.	Match objects.

Grade: Kindergarten

Content Area: Mathematics
Strand: Geometry
Objective: Analyze, compare, create, and compose shapes.

General Education Standard

M.K.21 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Alternate Achievement Standard

M.AAS.K.21 Match a shape to common objects (real or picture; limited to circle, square, rectangle, and triangle).

Achievement Elements

Students will be able to recognize that shapes can be found in the world around them.

Key Vocabulary

circle, square, rectangle, triangle, shape

Teaching and Learning Progressions

- Match a shape to common objects.
- Recognize shape as an attribute of objects and objects in pictures or drawings.
- Match like objects, pictures, or line drawings.

Application of the Alternate Achievement Standards

- Pair circles, squares, rectangles, triangles to objects and/or pictures of objects in the real world.
- Develop games to help students search for and recognize shapes that match real-world objects or pictures of objects.
- Demonstrate how real-world objects are the same and different from two-dimensional pictures and shapes.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe the shape represented in a given real-world object.	Match a real-world object to a given shape (circle, square, rectangle, triangle).	Identify the shapes: circle, square, rectangle, triangle.	Match like objects, pictures, or line drawings.

Grade: 1

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Represent and solve problems involving addition and subtraction.

General Education Standards

M.1.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

M.1.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Alternate Achievement Standards

M.AAS.1.1 Represent addition as "add to/put together" and subtraction as "take from/take apart" with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, or verbal explanations (limited to 15).

M.AAS.1.2 Solve problems with two given sets using "putting together" with objects and drawings (each set limited to ten).

Achievement Elements

Students will be able to add and subtract within twenty and solve simple word/real-world problems within ten.

Key Vocabulary

addition/add, subtraction/subtract, common vocabulary

Teaching and Learning Progressions

- Using models, solve word/real-world problems with two given sets (e.g., objects, drawings); using *putting together,* add within ten.
- Using models, solve word/real-world problems with two given sets (e.g., objects, drawings); using *putting together,* add within five.

- Represent addition and subtraction with objects, pictures, fingers, or sounds within twenty.
- Understand addition as *putting together* and subtraction as *taking from*.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to twenty.

Application of the Alternate Achievement Standards

- Using a set of up to ten objects, have students take away items to demonstrate subtraction and add objects to demonstrate addition.
- Demonstrate a real-world problem (within ten) and ways to solve it. (e.g., "There are four colored pencils. You take two colored pencils to finish your art project. How many colored pencils are left?")
- Introduce using a representation (e.g., sticks) to represent objects/numbers in math problems.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use models to solve problems with two given sets (e.g., objects, drawings) using putting together to add within twenty.	Use models to solve problems with two given sets (e.g., objects, drawings) using putting together to add within ten.	Add and subtract within twenty using counting tools.	Count objects up to twenty. Take away and add to a group of objects.

Grade: 1

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Add and subtract within 20.

General Standards

M.1.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

M.1.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.

Alternate Achievement Standards

M.AAS.1.5 Represent addition and subtraction of 1 more or 1 less from a number 1 to 15.

M.AAS.1.6 Add and subtract numbers 1 to 15 using objects, pictures, and fingers.

Achievement Elements

Students will be able to add and subtract within twenty using manipulatives.

Key Vocabulary

taking away, putting together, add, subtract, count

Teaching and Learning Progressions

- Add and subtract numbers within twenty using objects, pictures, and fingers.
- Pair *taking away* with subtraction.
- Take a smaller set out of a larger set.
- Pair *putting together* with adding.

- Combine two sets to make a larger set up to twenty.
- Count items in a set up to twenty.
- Establish one-to-one correspondence between numbers and objects.
- Using counting, find one less than a number 2 through 20.
- Using counting, find one more than a number 1 through 20.
- Rote count to twenty.
- Rote count to ten.

Application of the Alternate Achievement Standards

- Using objects of student interest (e.g., stickers, counting bears), demonstrate addition and subtraction up to twenty.
- Use repetitive movement (e.g., steps, jumps, arm swings, finger taps) with pairs of students to practice addition up to twenty.
- Introduce using a representation (e.g., sticks) to represent objects/numbers in math problems.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Add and subtract within twenty without models or manipulatives.	Add and subtract within twenty using models or manipulatives.	Demonstrate addition and subtraction within ten using manipulatives.	Count objects up to ten.

Grade: 1

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Extend the counting sequence.

General Education Standard

M.1.9 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Alternate Achievement Standard

M.AAS.1.9 Count to 30 by ones. Distinguish numerals from other print (letters, symbols); recognize numerals 0 through 15 as written. When given a numeral 0 to 15, represent the numeral with objects.

Achievement Elements

Students will be able to write numerals representing a given number of objects up to ten.

Key Vocabulary

write, number, count

Teaching and Learning Progressions

- Write the numerals 1 through 10 in response to a given number of objects or objects in a drawing/picture.
- Write the numerals 1 through 10.
- Match a given written numeral with the number of objects in a given set up to ten.
- Understand that numerals represent numbers.
- Count objects up to twenty.
- Rote count to twenty.
- Distinguish between numerals and letters.

Application of the Alternate Achievement Standards

- Using various writing utensils (e.g., markers, sidewalk chalk), demonstrate writing numbers that represent real-world groups of objects (e.g., three chairs, four pencils, five children).
- Using sets of objects familiar to students, demonstrate counting out a number of objects in response to a written numeral within 10.
- Using sets of objects (within ten) and numeral cards, demonstrate matching the numeral with the set.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Write numerals representing a given number of objects beyond ten.	Write the numerals representing a given number of objects within ten.	Match written numerals to a given set of objects within ten.	Make purposeful marks in response to a given number of objects within three.

Grade: 1

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Understand place value.

General Education Standards

M.1.10 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones, called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

M.1.11 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Alternate Achievement Standards

M.AAS.1.10 Recognize and create sets of ten (limit to three sets).

M.AAS.1.11 Compare two sets of items using greater than, less than or same (sets limited to 1 to 19 items; each set differs by less than 4).

a. Identify whether the number of objects in one group is greater than, less than, or same/equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to 15 objects.)
b. Compare two numbers between 1 and 15 presented as written numerals.

Achievement Elements

Students will be able to compare two sets of items using *equal to, greater than, less than*.
Students will be able to group items into sets of ten up to three groups.

Key Vocabulary

greater than/more, less than/fewer/less, equal/same

Teaching and Learning Progressions

- Compare two sets of items using greater than, less than, or same/equal.
- Count the number of items in each of two given sets.
- Understand the concepts of more, less, greater than, less than, or same.
- Separate three sets of ten from a larger group of objects.
- Separate two sets of ten from a larger group of objects.
- Separate a set of ten from a larger group of objects (up to thirty).
- Separate a set of five from a larger group of objects (up to twenty).
- Count the number of items in a given set.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to thirty.
- Rote count to twenty.
- Rote count to ten.

Application of the Alternate Achievement Standards

- Using objects familiar to students, demonstrate counting out groups of two, three, four, five, and ten from a larger group of objects. (e.g., "Show me five counting bears. Now, show me another five counting bears.")
- Demonstrate equal sets using objects and pictures of objects.
- Using pairs of students and a number of objects they have individually selected, demonstrate comparing quantities using greater than, less than, and equal to.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Compare quantities less than twenty in the real world using greater than, less than, or equal to.	Compare two groups of items using *greater than, less than, or equal to* when given two groups of less than twenty items that differ by up to four.	Appropriately use the terms *more* and *less* to compare two groups of less than ten items that differ by at least six.	Match equal sets of objects.
Form groups of ten and some left over from a given set of objects in response to a verbal cue.	Separate three groups of ten from a larger group of objects in response to a verbal cue.	Count out ten objects from a group of fifteen objects in response to a verbal cue.	Count out two objects from a group of five objects in response to a verbal cue.

Grade: 1

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Use place value understanding and properties of operations to add and subtract.

General Education Standard

M.1.12 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Alternate Achievement Standard

M.AAS.1.12 Compose and decompose numbers from 1 to 15 into one ten and ones using objects, drawings, or pictures.

Achievement Elements

Students will be able to compose and decompose numbers up to fifteen into a variety of sets.

Key Vocabulary

take apart, put together, ten plus___

Teaching and Learning Progressions

- Describe an original given set of objects as a group of ten plus the remaining number of objects in numbers one through fifteen.
- Count the number of objects remaining after a set of ten objects is counted out from a given set of objects.
- From a given set of objects, count out a set of ten objects.
- Given a set of objects, create two sets of objects using all of the objects up to fourteen.
- Describe a larger set in terms of combining two smaller sets.

- Given two smaller sets of objects (one of which is a set of ten), combine the sets and determine the total number of objects up to fifteen.
- Given two smaller sets of objects, combine the sets and determine the total number of objects up to fifteen.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to ten.

Application of the Alternate Achievement Standards

- Using objects familiar to students (e.g., counting bears, blocks, chips), demonstrate separating a group of objects between ten and twenty into a group of ten with some left over.
- Demonstrate counting out ten objects into a group and then adding a smaller group of objects (one through nine) and counting the number in the combined group.
- Using small groups of objects, demonstrate counting the number of objects in each of two small groups and then combining them and counting the number of objects in the combined group.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use objects, drawings, or pictures to compose and decompose numbers (up to thirty) into a group(s) of ten and some/no ones.	Use objects, drawings, or pictures to compose and decompose numbers (ten through twenty) into a group(s) of ten and some/no ones.	Use objects, drawings, or pictures to compose and decompose some numbers ten and under.	Combine two sets of numbers (each under five) to make a larger number.

Grade: 1

Content Area: Mathematics
Strand: Measurement and Data
Objective: Measure lengths indirectly and by iterating length units.

General Education Standard

M.1.15 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Alternate Achievement Standard

M.AAS.1.15 Compare lengths of objects (real or pictures) in terms of *longer/shorter* and *taller/shorter*.

Achievement Elements

Students will be able to compare length using longer/shorter and taller/shorter.

Key Vocabulary

long, longer, short, shorter, tall, taller

Teaching and Learning Progressions

- Directly compare two objects (real or pictures) with a measurable attribute in length and communicate the difference (longer/shorter, taller/shorter).
- Use comparative language (longer/shorter, taller/shorter) for the attributes of objects related to length.
- Communicate *long, tall,* and *short*.
- Recognize the length attributes of objects (long/short, tall/short).
- Recognize length as the measurement of something from end to end.

Application of the Alternate Achievement Standards

- Demonstrate comparisons of length using physical movement, such as walking to two locations in the classroom or school.
- Using common household or school materials (e.g., string, connected paper clips) of a given length, demonstrate making examples that are longer or shorter.
- Demonstrate comparisons of height using classroom objects.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Compare more than two lengths in real life or pictures using tall/taller/tallest, short/shorter/ shortest, or long/longer/longest.	Compare two lengths in real life or in pictures using taller/shorter, longer/shorter.	Given a picture of two objects, determine which object is taller or longer.	Identify an object as tall or short.

Grade: 1

Content Area: Mathematics
Strand: Measurement and Data
Objective: Tell and write time.

General Education Standard

M.1.17 Tell and write time in hours and half-hours using analog and digital clocks.

Alternate Achievement Standard

M.AAS.1.17 Demonstrate an understanding of the concept of time using words such as yesterday, today, tomorrow, morning, afternoon, day, and night; identify activities that come before, next, and after on a daily schedule using a clock limited to time in hours.

Achievement Elements

Students will be able to understand relative time.

Key Vocabulary

yesterday, today, tomorrow, morning, afternoon, day, night, before, next, after

Teaching and Learning Progressions

- Identify activities on a daily schedule that come before, next, or after other activities.
- Know *before, next,* and *after.*
- Use a daily schedule that contains times (in hours) and activities (in pictures).
- Tell time in hours on an analog clock.
- Demonstrate an understanding of yesterday, today, tomorrow, morning, afternoon, day, and night.
- Recognize *yesterday, today,* and *tomorrow.*

- Recognize *morning, afternoon,* and *evening/night.*
- Recognize *day* and *night.*
- Understand the concept of time.

Application of the Alternate Achievement Standards

- Use daily circle time to discuss topics that relate to comprehending *yesterday, today, and tomorrow*.
- Use visual daily schedules (classroom or individual) to pair activities with times (to the hour) and demonstrate events before, next, or after.
- Demonstrate times of day (morning, afternoon, evening/night) using pictures and/or pairing with daily routine events.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Locate yesterday, today, and tomorrow on a calendar.	Identify events that take place during times of day and days of reference.	Follow a daily schedule indicating an activity that happens next.	Recognize a visual daily schedule.

Grade: 1

Content Area: Mathematics
Strand: Measurement and Data
Objective: Represent and interpret data.

General Education Standard

M.1.18 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Alternate Achievement Standard

M.AAS.1.18 Sort objects or pictures into common categories (e.g., shapes, pets, fruits; limited to two categories and a combined total of 15 objects/pictures for the categories).

Achievement Elements

Students will be able to categorize objects.

Key Vocabulary

common category names (e.g., shapes, colors, pets, fruits, vegetables, transportation)

Teaching and Learning Progressions

- Given a set of objects or pictures, sort them into common categories.
- Given a choice of two categories, identify the correct category for a given object.
- Given an object or a picture of an object, place it in a given category.
- Understand *same* and *different*.
- Know common category names such as pets, fruits, and cars.

Application of the Alternate Achievement Standards

- Demonstrate sorting into categories using two given categories and objects or cards that can be sorted into those categories.
- Demonstrate labeling objects by category using objects in the school or classroom (e.g., apples for snack, apples are fruit).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Sort items (up to fifteen) into three common categories.	Sort items (up to fifteen) into two common categories.	Sort items (up to eight) into two basic categories.	Sort items (up to four) by color.

Grade: 1

Content Area: Mathematics
Strand: Geometry
Objective: Reason with shapes and their attributes.

General Education Standards

M.1.19 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

M.1.20 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")

M.1.21 Partition circles and rectangles into two and four equal shares; describe the shares using the words *halves, fourths,* and *quarters*; and use the phrases *half of, fourth of*, and *quarter o*f. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Alternate Achievement Standards

M.AAS.1.19 Determine similarities and differences among shapes (limited to circle, square, rectangle, and triangle).

M.AAS.1.20 Sort shapes of the same size and orientation (limited to circle, square, rectangle, and triangle).

M.AAS.1.21 Put together two equal size pieces to make a shape that relates to a whole (limited to circle, square, rectangle, and triangle).

Achievement Elements

Students will be able to determine relative size and position of objects.

Key Vocabulary

small, smaller, smallest, big/large, bigger/larger, biggest/largest, on, off, same, different

Teaching and Learning Progressions

- Given two halves of a circle or rectangle, place them to form a whole.
- Recognize that circles and rectangles can be cut in half.
- Sort shapes into categories given multiple shapes, some of which are the same (circle, square, rectangle, and triangle)
- Sort shapes into categories given three shapes that are different.
- Sort shapes into categories given two shapes that are different.
- Identify the relative position of objects in the real world.
- Demonstrate on, off, in, and out using real objects.
- Know *on, off, in*, and *out*.
- Sequence three objects by size.
- Compare two objects according to size.
- Identify an object by its size.
- Know *small/smaller, big/large, bigger/larger*, and *biggest/largest*.
- Know *same* and *different*.

Application of the Alternate Achievement Standards

- Demonstrate relative position for this standard using classroom or school items in daily life (e.g., "The coats are hanging on the hooks; the backpacks are in the cubbies.").
- Using construction paper circles and rectangles, demonstrate cutting them in half and reconstructing the wholes.
- Demonstrate sorting shapes using wooden shapes such as picture cards.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use sequencing language to describe a sequence of three objects.	Sequence three objects by size.	Compare two objects by size.	Identify an object as small or large.
Identify the location of objects (in, out, off, on, beside).	Identify the location of objects (in, out, off, on).	Place objects in and out, and off and on, in response to a verbal cue.	Identify the position of on and off.
Sort shapes of different sizes and orientations (circle, square, triangle, rectangle).	Sort shapes of the same size and orientation (circle, square, triangle, rectangle).	Select a circle and a square from three different shapes.	When given a visual cue, identify a circle and a square.

Grade: 2

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Represent and solve problems involving addition and subtraction.

General Education Standard

M.2.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Alternate Achievement Standard

M.AAS.2.1 Represent addition and subtraction by using objects, pictures, fingers, or sounds (within a quantity of 30).

Achievement Elements

Students will be able to add and subtract, using manipulatives, within fifty.

Key Vocabulary

add, subtract

Teaching and Learning Progressions

- Add and subtract numbers within fifty using number representations (e.g., objects, slash marks, ten frames).
- Subtract groups of tens to make 40, 30, 20, and 10.
- Add groups of tens to make 20, 30, 40, and 50.
- Represent numbers between 10 and 50 as groups of ten with additional ones.
- Use groups of ten to represent 10, 20, 30, 40, and 50.
- Count items in a set up to fifty.

- Establish one-to-one correspondence between numbers and objects or other representations (e.g., slash marks, counting sticks).
- Rote count to fifty.
- Rote count to twenty.

Application of the Alternate Achievement Standards

- Have students count common objects in the classroom environment, such as people and supplies.
- Use songs and nursery rhymes counting by tens.
- Use manipulatives to demonstrate addition and subtraction.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Add and subtract within fifty using manipulatives with regrouping.	Add and subtract within fifty using manipulatives (no regrouping).	Add and subtract within twenty using manipulatives (no regrouping).	Add and subtract within ten using manipulatives (no regrouping).

Grade: 2

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Work with equal groups of objects to gain foundations for multiplication.

General Education Standards

M.2.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

M.2.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Alternate Achievement Standards

M.AAS.2.3 Separate even numbers of objects into two groups (limited to twenty total objects).

M.AAS.2.4 Find the total number of objects in two equal groups (limit of twenty total objects).

Achievement Elements

Students will be able to develop skills for multiplication and division.

Key Vocabulary

equal

Teaching and Learning Progressions

- Given two equal sets of objects (up to ten in each set), count the total number of objects.
- Given two sets of objects up to ten, count the total number of objects.
- Given sets containing even numbers of objects up to twenty objects, separate the objects into two equal sets.
- Given sets of six, eight, and ten objects, separate the objects into two equal sets.
- Understand *equal*.
- Given a set of four objects, separate the objects into two sets of two objects.

- Given a set of two objects, separate the objects into two sets of one object.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to twenty.
- Rote count to ten.

Application of the Alternate Achievement Standards

- Demonstrate equal groups of objects using objects familiar to students.
- Use real-world classroom and school situations to demonstrate dividing objects into equal groups (e.g., dividing supplies between students, dividing students for a physical education activity).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify and divide up to twenty objects into two equal groups.	Divide up to twenty objects into two equal groups.	Divide up to ten objects into two equal groups.	Divide four objects into two groups of two.
Find the total number of objects made by combining three equal groups of up to ten objects each.	Find the total number of objects made by combining two equal groups of up to ten objects each.	Determine the total number of objects made by combining two equal groups of up to five objects each.	Determine the total number of objects made by combining two equal groups of one or two objects in each.

Grade: 2

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Understand place value.

General Education Standards

M.2.5 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens, called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

M.2.6 Count within 1000; skip-count by 5s, 10s, and 100s.

M.2.7 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

M.2.8 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.

Alternate Achievement Standards

M.AAS.2.5 Recognize and represent numbers up to 30 with sets of tens and ones (objects, columns, arrays).

M.AAS.2.6 Count consistently to 50 by ones. When given a numeral 0 to 25, name the next number in a three-item sequence.

M.AAS.2.7 Recognize numerals 0 through 30 as written. When given a numeral 0 to 30, represent the numeral with objects. Match a numeral 1 to 30 to a quantity.

M.AAS.2.8 Compare sets of objects and numbers using appropriate vocabulary (greater than, less than, equal to; limited to thirty objects in a group).

Achievement Elements

Students will be able to read and write numerals up to 30 and understand their application.

Students will be able to begin sequencing numbers and continue comparing numbers using appropriate vocabulary.

Key Vocabulary

greater than, less than, equal to, next number

Teaching and Learning Progressions

- Compare numerals up to 20 using *equal to, greater than,* and *less than.*
- Match numerals 11–20 to quantities.
- Match numerals 1–10 to quantities.
- Identify numerals 11–20.
- Identify numerals 1–10.
- Compare a given set of objects with another set of objects, using *equal to, greater than,* and *less than.*
- Compare a set of objects numbering between ten and twenty with a set of objects containing less than ten objects, using *greater than* and *less than.*
- Compare given sets of objects (up to ten objects in each group) as equal to or not equal to each other.
- Pair *more* with *greater than, less* with *less than,* and *same* with *equal to*.
- Understand *more, less,* and *same.*
- Match numerals 11–20 to quantities.
- Match numerals 1–10 to quantities.
- Identify numerals 21–30.
- Identify numerals 11–20.
- Identify numerals 1–10.
- Given three numbers in sequence, name the next number up to 19.
- Using a number line, name the number after a given number up to 19.
- Using a number line, locate a number after a given number up to 19.
- Understand *before* and *after* on a number line.
- Count to thirty using a number line.
- Using manipulatives and counting, recognize and represent the numbers 21 through 29 as two sets of ten and some ones.
- Using manipulatives and counting, recognize and represent the number 20 as two sets of ten.
- Using manipulatives and counting, recognize and represent the numbers 11 through 19 as a set of ten and some ones.
- Using manipulatives and counting, recognize and represent the number 10 as a place value unit.

- Using manipulatives and counting, recognize and represent the numbers 1 through 9.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to thirty.

Application of the Alternate Achievement Standards

- Use real-world opportunities to demonstrate the recognition and use of numbers up to 30 (e.g., counting, recognizing numerals).
- Demonstrate beginning sequencing, using a number line to locate a number and the next number.
- Using a number line, demonstrate greater than and less than.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify, count, recognize numerals, and use numbers up to 30.	Count, recognize numerals, and use numbers up to 30.	Count and recognize numerals up to 20.	Count and recognize numerals up to 10.
Sequence up to five numbers and compare numbers, using appropriate language and symbols.	Sequence up to three numbers and compare numbers, using appropriate language.	Using a number line, tell which number comes after a given number.	Count forward from a given number (not one) within ten.

Grade: 2

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Use place value understanding and properties of operations to add and subtract.

General Education Standards

M.2.9 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

M.2.10 Add up to four two-digit numbers using strategies based on place value and properties of operations.

Alternate Achievement Standards

M.AAS.2.9 Identify the meaning of the + sign (add, plus, put together) and the – sign (subtract, take away, take from) and the = sign (equal, the same as); compose and decompose numbers up to 30 using objects, pictures, drawings, or numbers.

M.AAS.2.10 Add and subtract numbers 0 to 30 using objects, pictures, and numbers.

Achievement Elements

Students will be able to use numbers up to 30 to add, subtract, and compare numbers.

Key Vocabulary

add, plus, combine, subtract, separate, take away, equal, same as

Teaching and Learning Progressions

- Add and subtract numbers 0–30 using numbers.
- Recognize numbers 0–30 as representing quantities.
- Add and subtract numbers 0–30 using pictures.
- Add and subtract numbers 0–30 using objects.
- Know mathematical symbols for add and subtract.
- Given a set of objects, find the total number of objects when a given set is removed.

- Given a set of objects, find the total number of objects when another set is combined with the original set.
- Given two sets of objects (less than ten objects each), count the total number of objects.
- Count objects to thirty.
- Establish one-to-one correspondence between numbers and objects.
- Identify the = sign as *equal*.
- Pair *same* and *equal*.
- Know *same* when comparing numbers of objects.
- Recognize cue words for *minus* (subtract, take away, separate).
- Identify the – sign as *minus*.
- Recognize cue words for *plus* (add, plus, combine).
- Identify the + sign as *plus*.
- Using manipulatives and counting, recognize and represent the number 30 as three sets of ten.
- Using manipulatives and counting, recognize and represent the numbers 21 through 29 as two sets of ten and some ones.
- Using manipulatives and counting, recognize and represent the number 20 as two sets of ten.
- Using manipulatives and counting, recognize and represent the numbers 11 through 19 as a set of ten and some ones.
- Using manipulatives and counting, recognize and represent the number 10 as a place value unit.
- Using manipulatives and counting, recognize and represent the numbers 1 through 9.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to thirty.

Application of the Alternate Achievement Standards

- Pair number and symbol cards with objects, using verbal cues, to demonstrate simple addition and subtraction expressions.
- Demonstrate representing numbers up to 30 as sets of ten and some ones using base ten blocks or other manipulatives.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Add and subtract within 30 from number expressions with symbols and regrouping.	Add and subtract within 30 from a simple number sentence using numerals and symbols.	Add and subtract within twenty using manipulatives.	Add and subtract within ten using manipulatives.

Grade: 2

Content Area: Mathematics
Strand: Measurement and Data
Objective: Measure and estimate lengths in standard units.

General Education Standards

M.2.14 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

M.2.16 Estimate lengths using units of inches, feet, centimeters, and meters.

Alternate Achievement Standards

M.AAS.2.14 Identify standard tools associated with measurement (clock, ruler, scale, measuring cup); measure the lengths of objects using nonstandard units (e.g., hands, paper clips).

M.AAS.2.16 Order three objects by length (long/longer/longest; short/shorter/shortest).

Achievement Elements

Students will be able to measure and compare lengths of objects using nonstandard units and recognize standard measurement tools.

Key Vocabulary

clock, ruler, scale, measuring cup, longer, longest, shorter, shortest

Teaching and Learning Progressions

- Compare three items using comparative language (long, longer, longest; or short, shorter, shortest).
- Compare two items using comparative language (e.g., longer, shorter).
- Identify an item as long or short.
- Know long/short, longer/longest, and shorter/shortest.
- Recognize length as the measurement of something from end to end.
- Identify a clock as the standard tool for measuring time.

- Recognize time as measurements (e.g., in minutes, hours, days, weeks).
- Identify a measuring cup as a standard tool for measuring volume.
- Recognize volume as a measure of how much space something takes up.
- Identify a scale as the standard tool for measuring weight.
- Recognize weight as the measure of how heavy something is.
- Identify a ruler as the standard tool for measuring length.
- Measure the lengths of objects using nonstandard units (e.g., hands, paperclips).
- Recognize length as the measurement of something from end to end.

Application of the Alternate Achievement Standards

- Using either a digital or analog clock, demonstrate time of day for activities during the classroom/school day.
- Relate calendar activities to the passing of time.
- Demonstrate measuring a line and an item with a straight line (e.g., paper edge) with a ruler.
- Use a set of measuring cups to compare quantities (more, less) or volume.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify a clock (digital, analog), a ruler, a scale, a yardstick, measuring cups and spoons as tools for measurement.	Identify a clock, a ruler, a scale, and a measuring cup as tools for measurement.	Identify a clock and a ruler as tools for measuring time and length.	Identify a clock as a tool for measuring time.
Use a ruler to make a measurement of less than 12 inches.	Make and compare nonstandard measurements of length.	Make nonstandard measurements of length.	Identify a line as long or short.

Grade: 2

Content Area: Mathematics
Strand: Measurement and Data
Objective: Relate addition and subtraction to length.

General Education Standards

M.2.18 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

M.2.19 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2 . . ., and represent whole-number sums and differences within 100 on a number line diagram.

Alternate Achievement Standards

M.AAS.2.18 Increase or decrease length by adding or subtracting nonstandard unit(s).

M.AAS.2.19 Represent whole-number sums within 20 using a number line.

Achievement Elements

Students will be able to use and arithmetically manipulate standard and nonstandard units of length.

Key Vocabulary

increase, add, decrease, subtract

Teaching and Learning Progressions

- Add units of length using a number line.
- Understand counting on from a given number.
- Locate whole numbers on a number line scale.
- Identify a number line (0–20).
- Identify inches and feet as units of measure for length.
- Recognize length as the measurement of something from end to end.

- Using manipulatives, decrease the length by subtracting nonstandard units.
- Know *decrease, make shorter,* and *subtract.*
- Using manipulatives, increase the length by adding nonstandard units.
- Know *increase, make longer,* and *add.*
- Measure length using nonstandard units.

Application of the Alternate Achievement Standards

- Demonstrate increasing or decreasing lengths in nonstandard units by adding more units or taking off units (e.g., taking off or adding paper clips to a string of paper clips, cutting off or taping on units of paper to a length of paper).
- Using a ruler and common classroom items, demonstrate how to measure inches (e.g., length and width of a book, desk, or chair seat).
- Gather standard measures of common classroom items in a chart and compare them using a number line and appropriate terms.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use a ruler and a yardstick to measure and compare lengths.	Use standard and nonstandard measuring tools and compare measurements using a number line.	Use a nonstandard measuring tool to add more to a length.	Use a nonstandard measuring tool to measure length.

Grade: 2

Content Area: Mathematics
Strand: Measurement and Data
Objective: Work with time and money.

General Education Standards

M.2.20 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.21 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

Alternate Achievement Standards

M.AAS.2.20 Identify the time that matches a routine activity using a digital or analog clock (limited to hour).
M.AAS.2.21 Demonstrate knowledge that money has value; identify coins (penny, nickel, dime).

Achievement Elements

Students will be able to use an analog or a digital clock to identify a routine activity.
Students will be able to identify and know the values of basic coins.

Key Vocabulary

clock, penny, nickel, dime

Teaching and Learning Progressions

- Identify *penny, nickel,* and *dime*.
- Recognize *penny, nickel,* and *dime*.
- Differentiate coins from other objects.
- Know that money is used to purchase goods and services.
- Identify the time on an analog and a digital clock of a routine activity, limited to hour.

- Use a daily schedule of routine activities.
- Tell time to the hour on an analog and a digital clock.
- Identify the display of hours and minutes on a digital clock.
- Identify the hour and minute hands on an analog clock.
- Identify analog and digital clocks as instruments for measuring time in hours and minutes.

Application of the Alternate Achievement Standards

- Set up a simple store to demonstrate purchasing with pennies, nickels, and dimes.
- Designate a timekeeper to notify the class when it is time for the next activity.
- Have students match digital and analog clock times.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify a penny, a nickel, a dime, and a quarter, and know the value of each.	Identify a penny, a nickel, and a dime, and know the value of each.	Identify a penny, a nickel, and a dime.	Identify coins as money.
Use an analog or a digital clock to determine the time for a routine daily activity to the half hour.	Use an analog or a digital clock to determine the time for a routine daily activity to the hour.	Identify an analog or a digital clock as a tool for measuring time.	Identify a schedule of daily activities.

Grade: 2

Content Area: Mathematics
Strand: Measurement and Data
Objective: Represent and interpret data.

General Education Standard

M.2.23 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Alternate Achievement Standard

M.AAS.2.23 Use a pictograph, limited to 2 categories, to answer more/less, most/least, or equal to questions (limited to two categories and a combined total of no more than 30 objects/pictures shown for the 2 categories).

Achievement Elements

Students will be able to interpret a simple pictograph.

Key Vocabulary

pictograph, more, less, most, least

Teaching and Learning Progressions

- Answer a more/less, most/least question using a pictograph.
- Recognize the elements of a pictograph.
  - Main title
  - Descriptive label that includes the variable or quantity that changes (limited to three)
  - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.
- Know *more/less* and *most/least.*

Application of the Alternate Achievement Standards

- Demonstrate how data is entered on a pictograph using student-gathered data (e.g., various fruit stickers are applied to a pictograph to indicate each student's preference for apples, oranges, or strawberries).
- Demonstrate determining more or less using counting and length of the data column/row.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Answer a more/less, most/least, and how much more/less question using data on a pictograph.	Answer a more/less and most/least question using data on a pictograph.	Identify the data in one category on a pictograph.	Count pictures or drawings of objects in horizontal or vertical orientation.

Grade: 2

Content Area: Mathematics
Strand: Geometry
Objective: Reason with shapes and their attributes.

General Education Standards

M.2.24 Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

M.2.26 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds, half of, a third of,* etc.; and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Alternate Achievement Standards

M.AAS.2.24 Identify two-dimensional shapes (limited to circle, square, rectangle, and triangle).

M.AAS.2.26 Identify half as being two equal parts of a shape (limited to circle, square, rectangle, and triangle).

Achievement Elements

Students will be able to identify basic shapes and understand that half is one of two equal parts of a shape.

Key Vocabulary

circle, square, triangle, rectangle

Teaching and Learning Progressions

- Identify half of a circle, square, or rectangle.
- Know that two halves are equal parts.
- Identify the two equal parts of a constructed shape as halves.
- Using manipulatives, construct a shape from two halves.

- Recognize *circle, square,* and *rectangle* as shapes.
- Recognize shape as a characteristic of a drawing.

Application of the Alternate Achievement Standards

- Using manipulatives, demonstrate forming circles, squares, rectangles, and triangles out of various materials (e.g., paper, play dough,).
- Using student-made models of shapes, demonstrate half (e.g., by cutting; by placing a pencil or string).
- Demonstrate selecting two halves of one shape from a group of halves of shapes and reconstructing the whole.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify and separate common shapes (circle, square, rectangle, triangle) into halves and reconstruct the wholes.	Separate common shapes (circle, square, rectangle, triangle) into halves and reconstruct the wholes.	Identify common shapes (circle, square, rectangle, triangle).	Identify a circle and a square.

Grade: 3

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Represent and solve problems involving multiplication and division.

General Education Standards

M.3.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

M.3.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

Alternate Achievement Standards

M.AAS.3.1 Find the sum of equal groups of objects using repeated addition (sums up to 30).

M.AAS.3.2 Divide a group of items into smaller equal groups (limit given group to fifteen items or less; limit equal groups to two, three, four, five, ten).

Achievement Elements

Students will be able to add using repeated addition.
Students will be able to divide numbers.

Key Vocabulary

sum, less than, equal, same, count by, more than, group

Teaching and Learning Progressions

- Given sets containing even numbers of objects (up to twenty objects), separate the objects into two equal sets.
- Given sets of six and eight objects, separate the objects into two equal sets.
- Given a set of four objects, separate the objects into two sets of two objects.
- Understand *equal*.

- Establish one-to-one correspondence between numbers and objects.
- Sum the number of objects in equal sets using repeated addition.
- Know *repeated addition*.
- Sum the number of objects in two equal sets by counting.
- Count the number of objects in a given set.
- Establish one-to-one correspondence between numbers and objects.
- Rote count to twenty.
- Rote count to ten.

Application of the Alternate Achievement Standards

- Practice counting by counting objects.
- Pair number and symbol cards with objects, using verbal cues, to demonstrate simple addition expressions.
- Demonstrate representing numbers up to 30 as sets of ten and some ones using base ten blocks or other manipulatives.
- Practice the concept of division by equally sharing a candy bar or pizza.
- Use skip counting for efficiency when sorting into groups.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the unknown in an addition or subtraction equation and add and subtract single-digit numbers.	Demonstrate the concept of addition (limited to single-digit numbers from 1–30).	Identify the functions of addition (limited to single-digit numbers from 1–30).	Recognize the addition sign.
Divide up to a two-digit dividend by a one-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.	Demonstrate the concept of division (limited to fifteen items or fewer and equal groups of two, three, four, five, and ten).	Identify the functions of division (limited to fifteen items or fewer and equal groups of two, three, four, five, and ten).	Recognize the division sign.

Grade: 3

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Multiply and divide within 100.

General Education Standard

M.3.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Alternate Achievement Standard

M.AAS.3.7 Multiply and divide one-digit numbers using repeated addition or repeated subtraction where the products are within twenty and the factors are one, two, three, four, five, or ten using multiplication and division tools.

Achievement Elements

Students will be able to multiply and divide one-digit whole numbers (product within 20 and factors of 2, 3, 4, 5, and 10).

Key Vocabulary

multiply, divide, product, factor, inverse

Teaching and Learning Progressions

- Using multiplication/division tools, multiply and divide one-digit numbers where the products are within 20 and the factors are 1, 2, 3, 4, 5, or 10.
- Know how to use multiplication/division tools.
- Recognize division as the inverse operation of multiplication.
- Recognize factor pairs for the multiplication facts up to 20.
- Use multiplication tools to multiply up to 20.

Application of the Alternate Achievement Standards

- Use real-world opportunities to demonstrate the recognition and use of multiplication and division, within the limits of the Achievement Elements:
- Sharing: “Do you have enough for everyone?”
- Planning a party: "Do you have enough chairs and food for everyone?”
- Asking: “Do you have enough money to buy _____?”
- Cooking: “How do you double the recipe? How do you divide the recipe?”

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Fluently multiply and divide within 100 using a variety of strategies.	Multiply and divide one-digit numbers using 1, 2, 3, 4, 5, and 10.	Demonstrate the concept of multiplication.	Recognize the multiplication sign.
Demonstrate the concept of division using a variety of strategies.	Demonstrate the concept of division using manipulatives to divide into equal groups.	Recognize multiplication factors and products.	Recognize the division sign.

Grade: 3

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

General Education Standards

M.3.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify an order (Order of Operations).)

M.3.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Alternate Achievement Standards

M.AAS.3.8 Solve one-step real-world problems using addition or subtraction without regrouping.

M.AAS.3.9 Extend numeric and non-numeric patterns of two terms.

Achievement Elements

Students will be able to solve real-world addition or subtraction problems.
Students will be able to extend patterns of two terms.

Key Vocabulary

add, subtract, pattern, solve, identify

Teaching and Learning Progressions

- Extend two-term numeric and nonnumeric patterns.
- Recognize two-term numeric and nonnumeric patterns.

- Identify terms in a pattern as different.
- Discriminate between drawings, letters, and numbers.
- Solve a one-step, real-world problem using addition or subtraction.
- Select the appropriate method of computation (addition/subtraction) needed to find a solution.
- Select subtraction as the method of computation when given real-world problems involving subtraction.
- Recognize cue words for subtraction in real-world problems.
- Select addition as the method of computation when given real-world problems involving addition.
- Recognize cue words for addition in real-world problems.
- Identify the information (numbers) in a one-step real-world problem that is needed to find the solution.
- Subtract without regrouping.
- Add without regrouping.

Application of the Alternate Achievement Standards

- How much money can one earn during the summer for doing jobs such as pulling weeds?
- Use the pattern in a garden to continue planting.
- Continue the pattern of numbers, objects, and manipulatives to complete a sequence or next term in a pattern.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Given a real-world situation, solve an addition or subtraction problem.	Given a real-world situation, solve an addition or subtraction problem without regrouping.	Given a real-world situation, recognize the concepts of addition and subtraction.	Recognize whether a real-world situation requires addition or subtraction.
Given the rule, determine the next two terms in a pattern.	Given the rule, determine the next term in a picture pattern or sequence.	Identify symbolic, repeating, and pictorial patterns.	Recognize patterns.

Grade: 3

Content Area: Mathematics
Strand: Numbers and Operations In Base Ten
Objective: Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

General Education Standards

M.3.10 Use place value understanding to round whole numbers to the nearest 10 or 100.
M.3.11 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
M.3.12 Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Alternate Achievement Standards

M.AAS.3.10 Use decade numbers (0, 10, 20, 30) as benchmarks to demonstrate understanding of place value for rounding numbers 0 to 34 using rounding tools (charts, number lines).
M.AAS.3.11 Add and subtract one- and two-digit numbers up to 30 (no regrouping).
M.AAS.3.12 Relate groups of ten to multiplying by ten up to 100, using objects, skip counting by tens.

Achievement Elements

Students will be able to recognize place value for rounding numbers 0 to 34.
Students will be able to add and subtract one- and two-digit numbers up to 30.
Students will be able to use objects to multiply by 10 up to 100.

Key Vocabulary

set, object, add, subtract, digit, place value, tens

Teaching and Learning Progressions

- Using sets of ten objects, skip count to one hundred.
- Using sets of ten objects, count to one hundred.
- Count out groups of 10 objects.
- Rote count to one hundred.
- Add and subtract two-digit whole numbers without regrouping.
- Using numbers, add and subtract one-digit numbers without regrouping.
- Using numbers, add and subtract one-digit numbers.
- Using manipulatives, subtract a one-digit number from a two-digit number without regrouping.
- Using manipulatives, add two-digit numbers without regrouping.
- Using manipulatives, compose two-digit numbers as a unit/some number of units of ten and some ones (up to 30).
- Using manipulatives, represent one-digit numbers.
- Round one- and two-digit numbers up to 34 to the nearest 10 using a number line.
- Recognize that 0, 1, 2, 3, and 4 are closest to the lower decade number and 5, 6, 7, 8, and 9 are closer to the higher decade number.
- Recognize that the number 5 and a number composed of some number of tens and five is halfway between each decade.
- Understand rounding as finding the decade number closest to a given number.
- Know halfway, middle, closer, nearest, about.
- Identify decade numbers (including 0) on a chart or number line.
- Recognize two-digit numbers as some number of tens and some ones.
- Understand place value of ones and tens and their relationships.

Application of the Alternate Achievement Standards

- Estimate how much money is required to purchase something.
- Plan a party, using counting (addition) and sharing (cutting) to determine, e.g., number of balloons, cookies, or cakes.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Round whole numbers from 0–100 to the nearest 10.	Round whole numbers from 0–30 to the nearest 10.	Understand the point when a number should be rounded up (limited to numbers from 1–10).	Recognize a unit in the place value system (limited to ones and tens place values).
Solve an addition or subtraction problem.	Solve an addition or subtraction problem without regrouping.	Recognize the concept of addition and subtraction.	Recognize addition or subtraction.
Skip count by tens to 100.	Relate groups of ten to multiply by 10 up to 100, and using objects, skip count by tens.	Group objects of ten and skip count to fifty.	Group objects of ten.

Grade: 3

Content Area: Mathematics
Strand: Numbers and Operations–Fractions
Objective: Develop understanding of fractions as numbers.

General Education Standards

M.3.13 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand *a* fraction *a*/*b* as the quantity formed by *a* parts and size 1/*b*.

M.3.14 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

M.3.15 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Alternate Achievement Standards

M.AAS.3.13 Use models to represent unit fractions as parts of a whole (halves and fourths).
M.AAS.3.14 Use a number line to represent halves and fourths.
M.AAS.3.15 Compare fractions.
M.AAS.3.15a Use models to identify two equivalent fractions (limit to fourths and halves).

M.AAS.3.15b Recognize two equivalent fractions (limit to fourths and halves).
M.AAS.3.15c Use models of fourths and halves to make a whole.

Achievement Elements

Students will be able to use models and number lines to represent fractions as parts of a whole (halves and fourths).
Students will be able to compare, identify, use models, and recognize equivalent fractions (quarters and halves).

Key Vocabulary

whole, half, quarter, fourth, equal, part, fraction, number line, like, compare, model, unit

Teaching and Learning Progressions

- Given models of wholes divided into two or four equal parts, identify two like fractions.
- Given fractional pieces (halves and fourths), construct a whole.
- Using a number line marked in fourths, identify whole numbers, halves, and fourths.
- Using a number line marked in fourths, demonstrate that each half is made up of two fourths.
- Using a number line marked in fourths, demonstrate that each whole number increment is made up of four equal parts.
- Using a number line marked in halves, identify whole numbers and halves.
- Using a number line marked in halves, demonstrate that each whole number increment is made up of two equal parts.
- Using a number line marked in whole number increments, identify whole numbers.
- Given a model of a whole divided into two or four equal parts, recognize ½ and ¼.
- Understand that a unit fraction 1/*b* is one part of a whole divided into equal pieces of size *b*.

Application of the Alternate Achievement Standards

- Use pizza or cracker squares to practice fractions.
- Demonstrate breaking or cutting the cracker or pizza into equal parts such as halves or fourths. Demonstrate the denominator and numerator; and the total number of parts of the cracker or slices of pizza (denominator) and the number of parts remaining, eaten, or shared (numerator).
- Color or cut paper plates into fractions.
- Compare groups of boys or girls to the whole group (e.g., at recess, at lunch).

Levels of Standards

Level 4: Exceed Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts and size 1/*b*.	Use models to represent unit fractions as parts of a whole (halves and fourths).	Identify a half and a fourth.	Identify a half.
Understand and represent a fraction as a number line.	Use a number line to represent halves and fourths.	Identify fraction models that are divided into the same number of parts.	Identify a fraction from a list of numbers (e.g., two whole numbers and one fraction).
Compare fractions by reasoning about their sizes.	Use models to identify like fractions and make a whole (halves and fourths).	Identify two fractions.	Identify shapes divided into equal parts.
Explain why fractions are equivalent by using models.	Recognize two equivalent fractions (halves and fourths).	Identify a half and a fourth	Identify a fraction.

Grade: 3

Content Area: Mathematics
Strand: Measurement and Data
Objective: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

General Education Standards

M.3.16 Tell and write time to the nearest minute, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

M.3.17 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of "times as much").)

Alternate Achievement Standards

M.AAS.3.16 Tell time to the nearest half hour on a clock**.**

M.AAS.3.17 Identify the appropriate measurement tool to measure liquid; identify the appropriate standard unit of measurement (grams, kilograms, and liters).

Achievement Elements

Students will be able to solve measurement problems and estimate time intervals, liquid values, and masses of objects.

Key Vocabulary

unit, cup, pint, quart, liter, measure, time, minute, hour, analog, digital

Teaching and Learning Progressions

- Identify cup, pint, quart, and liter as units of measurements for liquids.
- Identify a measuring cup as a standard tool to measure liquids.

- Measure liquids using nonstandard tools.
- Identify tools used to measure liquids (e.g., cups, jugs, containers).
- Recognize when measuring liquids is used in the real world.
- Tell time to the half hour on an analog and a digital clock.
- Tell time to the hour on an analog and a digital clock.
- Identify the display of hours and minutes on a digital clock.
- Identify the hour and minute hands on an analog clock.
- Identify analog and digital clocks as instruments for measuring time in hours and minutes.

Application of the Alternate Achievement Standards

- Practice reading a digital and/or an analog clock or watch by incorporating activities throughout the day that require the students to read the clock and compare the time to a class schedule that shows what is happening at different times in the day.
- Measure liquids using measuring cups or weigh objects using a scale to help students understand volume and units of measure.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Tell and write time to the hour and half hour on both types of clocks (analog, digital).	Tell time to the nearest half hour on a clock.	Match the time to an activity from a list.	Identify a clock as a tool to measure time.
Measure and estimate liquid volumes and masses of objects, using standard units.	Identify the appropriate standard unit of measurement for liquid volume and mass.	Identify a cup as a standard tool to measure liquids.	Identify a cup.

Grade: 3

Content Area: Mathematics
Strand: Measurement and Data
Objective: Represent and interpret data.

General Education Standards

M.3.18 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

M.3.19 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Alternate Achievement Standards

M.AAS.3.18 Use a pictograph or bar graph to answer questions about data (limit to three categories).

M.AAS.3.19 Measure lengths of objects using standard tools (rulers, yardsticks, meter sticks). Limit to whole numbers.

Achievement Elements

Students will be able to use a pictograph or bar graph to answer questions about data.
Students will be able to measure lengths of objects using standard tools.

Key Vocabulary

length, graph, ruler, measure, tool, yardstick, meter stick, data

Teaching and Learning Progressions

- Measure lengths of objects using standard tools.
- Identify ruler, yardstick, and meter stick as standard tools for measuring length.
- Measure the lengths of objects using nonstandard units (e.g., hands, paperclips).
- Recognize length as the measurement of something from end to end.
- Answer questions about the data on a pictograph or bar graph.

- Recognize elements of a bar graph.
  - Title
  - Labels that describe the data shown (side and bottom)
  - Scales that show the units used (numbers)
  - Bars that show the measure of the data number (limited to three)
- Recognize the elements of a pictograph.
  - Main title
  - Descriptive label that includes the variable or quantity that changes (limited to three)
  - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.

Application of the Alternate Achievement Standards

- Use bar graphs from newspapers and magazine articles to demonstrate how to interpret the graph data.
- Students can measure objects using an inch ruler; for example, the students can measure the length and width of a picture to see whether it can fit in a certain frame size.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Draw a scaled picture and bar graph to represent a data set with several categories.	Use a pictograph or bar graph to answer questions about data (limit to three categories).	Identify information from a bar graph.	Identify a bar graph.
Measure lengths using rulers marked with halves and fourths of an inch. Show the data on a line plot.	Measure lengths of objects using standard tools (rulers, yardsticks, meter sticks).	Identify inches as a measure of length.	Identify a ruler.

Grade: 3

Content Area: Mathematics
Strand: Measurement and Data
Objective: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

General Education Standards

M.3.20 Recognize area as an attribute of plane figures, and understand concepts of area measurement.

a. A square with side length 1 unit called "a unit square," is said to have "one square unit" of area and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

M.3.22 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Alternate Achievement Standards

M.AAS.3.20 Identify a model that demonstrates area and/or recognize one square unit of area as a "unit square" to use when measuring area.

M.AAS.3.22 Find the area of a rectangle with side lengths of no more than one, two, three, four, or five.

Achievement Elements

Students will be able to identify a model that demonstrates area.
Students will be able to find the area of a rectangle (side lengths of no more than one, two, three, four, and five units).

Key Vocabulary

length, area, model, rectangle, grid, squares, space, flat, dimension, unit

Teaching and Learning Progressions

- Use counting to determine the area of a rectangle on a grid.
- Identify the squares on a grid as square units.
- Recognize square units as the measure of area.
- Know that area is the amount of space inside the boundary of a flat (two-dimensional) object.
- Count the number of units for the length of a given line on a grid.
- Understand grid lines as marking unit measure.
- Recognize models that demonstrate area.

Application of the Alternate Achievement Standards

- Use classroom objects to measure lengths.
- Represent classroom objects using counting squares to measure area.
- Create a garden: plan the size and shape and then measure the area.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Recognize area as an attribute of plane figures and understand concepts of area measurement.	Identify a model that demonstrates area.	Recognize that a tile can be used to measure area.	Recognize a tile.
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems and present whole-number products as rectangular areas in mathematical reasoning.	Find the area of a rectangle with side lengths of no more than one, two, three, four, or five units.	Recognize a rectangle.	Recognize side lengths.

Grade: 3

Content Area: Mathematics
Strand: Measurement and Data
Objective: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

General Education Standard

M.3.23 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Alternate Achievement Standard

M.AAS.3.23 Find the perimeter of a rectangle with lengths limited to one to ten units.

Achievement Elements

Students will be able to find the perimeter of a rectangle with lengths limited to one to ten units.

Key Vocabulary

rectangle, length, unit, perimeter

Teaching and Learning Progressions

- Use counting to determine the perimeter of a rectangle on a grid.
- Recognize units as the measure of perimeter.
- Know that perimeter is the continuous line forming the boundary of a closed geometric figure.
- Count the number of units for the length of a given line on a grid.
- Understand grid lines as marking unit measure.
- Add whole numbers.

Application of the Alternate Achievement Standards

- Use classroom objects to measure sides.
- Use a ruler to measure the perimeter of sheets of paper.
- Use a ruler to measure the sides of tangrams.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve real-world problems involving the perimeter of polygons when given the side lengths.	Find the perimeter of a rectangle (length of one to ten units).	Recognize a rectangle.	Recognize side lengths.

Grade: 3

Content Area: Mathematics
Strand: Geometry
Objective: Reason with shapes and their attributes.

General Education Standards

M.3.24 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Alternate Achievement Standards

M.AAS.3.24 Identify two-dimensional shapes by their attributes (circle, square, rectangle, triangle).

Achievement Elements

Students will be able to identify two-dimensional shapes by their attributes.

Key Vocabulary

two-dimensional, length, side, triangle, rectangle, square, circle

Teaching and Learning Progressions

- Identify a square as having four sides that are all the same length.
- Identify a rectangle as having four sides.
- Identify a triangle as having three sides.
- Recognize that triangles, squares, and rectangles are closed figures and have straight sides.
- Identify a circle when given the attribute of being round.

- Recognize that a circle is an enclosed figure with a boundary that is an equal distance from a center point.
- Recognize manipulatives and drawings of a triangle, a rectangle, a square, and a circle.
- Differentiate shapes from other drawings.

Application of the Alternate Achievement Standards

- Use classroom objects to draw different shapes.
- Build shapes with building blocks.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category.	Identify two-dimensional shapes by their attributes (triangle, rectangle, square, circle).	Identify a two-dimensional shape.	Recognize a rectangle.

Grade: 4

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Use the four operations with whole numbers to solve problems.

General Education Standard

M.4.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Alternate Achievement Standard

M.AAS.4.3 Solve one-step real-world problems using addition, multiplication, or subtraction (within forty; no regrouping); select the appropriate method of computation (limited to addition or subtraction) when problem solving.

Achievement Elements

Students will be able to solve word problems using addition, subtraction, and multiplication, within forty, no regrouping.

Key Vocabulary

addition, subtraction, multiplication

Teaching and Learning Progressions

- Solve a one-step, real-world problem using addition, subtraction, or multiplication.
- Select the appropriate method of computation (addition/subtraction/multiplication) needed to find a solution.
- Select multiplication as the method of computation when given real-world problems involving multiplication.
- Recognize cue words for multiplication in real-world problems.
- Understand multiplication as repetitive addition.
- Select subtraction as the method of computation when given real-world problems involving subtraction.

- Recognize cue words for subtraction in real-world problems.
- Select addition as the method of computation when given real-world problems involving addition.
- Recognize cue words for addition in real-world problems.
- Identify the information (numbers) needed to find the solution to a one-step, real-world problem.
- Multiply with products of forty and less.
- Subtract without regrouping.
- Add without regrouping.

Application of the Alternate Achievement Standards

- Use objects familiar to students to create word problems that demonstrate addition, subtraction, and multiplication within forty.
- Show students addition, subtraction, and multiplication facts they know within forty, using objects and written form.
- Connect facts by solving word problems.

Levels of Standards

Level 4: Exceed Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve one-step, real-world problems using multiplication (0–10) or division (2–5, no remainders).	Solve one-step addition, subtraction, and multiplication word problems.	Determine the unknown in an addition or a subtraction equation.	Demonstrate the concepts of addition and subtraction.

Grade: 4

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Gain familiarity with factors and multiples.

General Education Standards

M.4.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a whole number in the range 1–100 is prime or composite.

Alternate Achievement Standards

M.AAS.4.4 Arrange, match, and/or recognize factor pairs limited to ones, twos, threes, fours, fives, and tens to their products using models or tools.

Achievement Elements

Students will be able to recognize symbol and number patterns related to factors.

Key Vocabulary

factor

Teaching and Learning Progressions

- Given multiples of 2, 3, 4, 5, and 10, match the product and the factor pairs.
- Given a multiple of 2, 3, 4, 5, and 10, identify the factor pair.
- Recognize factor pairs for the multiplication facts of ones, twos, threes, fours, fives, and tens.
- Use multiplication tools to multiply up to 10.

Application of the Alternate Achievement Standards

- Use a rug or other familiar object with a pattern and have the student identify the pattern.
- Using objects familiar to students, create real-world word problems that demonstrate multiplication facts for numbers 1– 5 and 10.
- Using a multiplication fact students know, connect the multiplication fact to a real-world problem.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify factor pairs for multiples of numbers 1–10.	Identify factor pairs for multiples of 1, 2, 3, 4, 5, and 10 using a multiplication tool.	Skip count by 2, 5, and 10.	Skip count by 2.

Grade: 4

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Generalize place value understanding for multi-digit whole numbers.

General Education Standards

M.4.6 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

M.4.7 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

M.4.8 Use place value understanding to round multi-digit whole numbers to any place.

Alternate Achievement Standards

M.AAS.4.6 Compose and decompose numbers from 11 to 50 into a number of tens and a number of ones.

M.AAS.4.7 Compare the value of two numbers up to 100 and read a whole number up to 100.

M.AAS.4.8 Round a whole number from 1 to 99 to the nearest ten.

Achievement Elements

Students will be able to compare and round numbers (to the nearest 10) up to 100.

Key Vocabulary

rounding

Teaching and Learning Progressions

- Round whole numbers from 1 to 99 to the nearest 10.
- Using manipulatives, understand that a number from 95 through 99 rounded to the nearest 10 is 100.
- Round two-digit numbers from 19 to 89 to the nearest 10.
- Round one- and two-digit numbers up to 50 to the nearest 10, using a number line.

- Understand place value of ones, tens, and hundreds.
- Using place value, compare the value of two numbers up to 100.
- Understand place value of ones, tens, and hundreds.
- Name numbers 1–100.
- Write numbers 30–50 as a number of tens plus a number of ones.
- Using manipulatives, compose and decompose two-digit numbers 30–50 as some number of tens and some ones.
- Write numbers 11–29 as a number of tens plus a number of ones.
- Using manipulatives, compose and decompose two-digit numbers 11–29 as a/some number of ten(s) and some ones.

Application of the Alternate Achievement Standards

- Read and recognize the rounding of numbers using bingo game activities.
- Demonstrate rounding of numbers with familiar objects arranged in arrays.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Round whole numbers 1–199 to the nearest 10 or 100.	Round whole numbers 1–99 to the nearest 10.	Identify when a number can be rounded up to the nearest 10.	Recognize a place value unit (ones, tens).

Grade: 4

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Use place value understanding and properties of operations to perform multi-digit arithmetic.

General Education Standards

M.4.9 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

M.4.10 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

M.4.11 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Alternate Achievement Standards

M.AAS.4.9 Add and subtract one- and two-digit numbers with regrouping.

M.AAS.4.10 Multiply a two-digit number by a one-digit number with no regrouping using a calculator.

M.AAS.4.11 Divide a two-digit number by a one-digit number with no remainder using a calculator.

Achievement Elements

Students will be able to add and subtract two-digit numbers with regrouping.
Students will be able to multiply two-digit numbers by one-digit numbers without regrouping.
Students will be able to divide a two-digit number by a one-digit number without regrouping.

Key Vocabulary

addition, subtraction, multiplication, division

Teaching and Learning Progressions

- Divide two-digit numbers by one-digit numbers without remainders.
- Divide one-digit numbers by one-digit numbers without remainders.
- Understand division as separating a number into equal parts.
- Multiply one-digit numbers.
- Subtract with and without regrouping.
- Multiply a two-digit number by a one-digit number without regrouping.
- Multiply a one-digit number by a one-digit number.
- Understand multiplication as repeated addition.
- Add without regrouping.
- Add and subtract two-digit whole numbers with regrouping.
- Using manipulatives, subtract a one-digit number from a two-digit number with regrouping that involves borrowing a unit of ten.
- Using manipulatives, add two-digit numbers with regrouping so that the answer is composed of a unit/some number of units of ten and some ones.
- Using manipulatives, compose two-digit numbers as a unit/some number of units of ten and some ones.

Application of the Alternate Achievement Standards

- Use arrays to visually demonstrate division.
- Practice menu math to buy two entrees.
- Plan a pizza party (e.g., count toppings to demonstrate addition/multiplication, cut/share to demonstrate division).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Add and subtract a two-digit number and a three-digit number.	Add and subtract two-digit numbers with regrouping.	Add and subtract two-digit numbers without regrouping.	Recognize addition and subtraction symbols.
Multiply a two-digit number by a one-digit number with regrouping.	Multiply and divide two-digit numbers by one-digit numbers without regrouping.	Multiply a two-digit number by a one-digit number without regrouping, multiplicand within twenty.	Multiply one-digit numbers using a multiplication tool.

Grade: 4

Content Area: Mathematics
Strand: Numbers and Operations-Fractions
Objective: Extend understanding of fraction equivalence and ordering.

General Education Standard

M.4.13 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols for >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Alternate Achievement Standard

M.AAS.4.13 Using models, identify a fraction that is greater than, less than, or equal to a given fraction (limited to halves, thirds, and fourths).

Achievement Elements

Students will be able to identify fractions and use models to compare fractions.

Key Vocabulary

fraction

Teaching and Learning Progressions

- Using comparative language (greater than, less than, equal to), compare fractional parts constructed from unit fractions.
- Using models, construct parts of a whole made of two or more fraction parts.
- Using comparative language (greater than, less than, equal to), compare one-half, one-third, and one-fourth.
- Using models, compare one-half, one-third, and one-fourth.
- Identify the written labels for one-half, one-third, and one-fourth.

- Given a model of a whole divided into two, three, or four equal parts, recognize one-half, one-third, and one-fourth.
- Understand that a unit fraction 1/*b* is one part of a whole divided into equal pieces of size *b*.
- Know *greater than, less than,* and *equal to* in relation to size.

Application of the Alternate Achievement Standards

- Use fraction models of various shapes (circles, strips, squares, rectangles) to demonstrate fractional parts (e.g., by drawing, cutting paper strips).
- Demonstrate how to compare fractional parts using models and overlays.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Place fractional parts in order from smallest to largest using models.	Compare fractional parts to a given fractional part (limited to halves, thirds, and fourths) using models.	Identify fractional parts (halves, thirds, fourths) of a whole using models.	Identify a fractional part of a whole using models.

Grade: 4

Content Area: Mathematics
Strand: Numbers and Operations–Fractions
Objective: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

General Education Standards

M.4.15 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$.
b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Alternate Achievement Standard

M.AAS.4.15 Multiply a one-digit whole number by a unit fraction (limited to whole numbers to 1 to 5 and fractions of halves, fourths, and thirds).

Achievement Elements

Students will be able to multiply a whole number (one-digit) by halves, fourths, and thirds.

Key Vocabulary

fraction, multiply, numerator, denominator, “of” (as multiplication)

Teaching and Learning Progressions

- Multiply a one-digit whole number by a unit fraction (½, ⅓, ¼).
- Pair *repeated addition* with *multiply by*, using models, given a unit fraction and a given whole number (1 to 5).

- Calculate the repeated addition of the same unit fraction (up to five times) including ½, ⅓, and ¼.
- Using models, show repeated addition of the same unit fractions (up to five times) including ½, ⅓, and ¼.
- Understand multiplication as repeated addition.

Application of the Alternate Achievement Standards

- Demonstrate multiplication as repeated addition using fraction manipulatives.
- Plan a pizza party (e.g., given a number of children, if each child receives one piece of pizza, how many pieces are needed all together?).
- Using addition of fractions of various colors of paper, plan and construct streamers for the classroom and desks.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Multiply numbers 1–10 by halves, thirds, and fourths.	Multiply numbers 1–5 by unit fractions ½, ⅓, and ¼.	Use repeated addition to find the multiples of ½, ⅓, and ¼.	Pair repeated addition of whole numbers with multiplication.

Grade: 4

Content Area: Mathematics
Strand: Numbers and Operations–Fractions
Objective: Understand decimal notation for fractions, and compare decimal fractions.

General Education Standards

M.4.17 Use decimal notation for fractions with denominators 10 or 100.

Alternate Achievement Standards

M.AAS.4.17 Use decimal notation for a fraction with a denominator of 10.

Achievement Elements

Students will be able to use decimal notation for fractions with a denominator of 10.

Key Vocabulary

fraction, decimal point, denominator, numerator

Teaching and Learning Progressions

- Use decimal notation for a given fraction with a denominator of 10.
- Pair the 10 in the denominator of a fraction with tenths.
- Recognize decimal notation for the tenths place.
- Recognize fractions with denominators of 10.

Application of the Alternate Achievement Standards

- Use decimal/fraction tiles to demonstrate equivalency between a fraction and its decimal representation.
- Demonstrate the structure of writing decimals with a place-value chart or other place-value manipulatives.
- Use decimal notation to show the relationship between dimes and a one-dollar bill.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Write a fraction from a decimal notation with the denominator of 10.	Write a fraction with a denominator of 10 as a decimal.	Identify the decimal point and the tenths place on a place-value chart.	Identify a decimal point on a place-value chart.

Grade: 4

Content Area: Mathematics
Strand: Measurement and Data
Objective: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

General Education Standards

M.4.19 Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

M.4.20 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

M.4.21 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

Alternate Achievement Standards

M.AAS.4.19 Identify the smaller measurement unit that comprises a larger unit within a measurement system (inches/feet, minutes/hours, feet/yards).

M.AAS.4.20 Tell time to the half-hour; identify the hour before or after a given time; measure weight using standard units; recognize the value of coins in cents.

M.AAS.4.21 Given a drawing of a square or rectangle on a grid, determine the area or perimeter (sum or product limited to 40).

Achievement Elements

Students will be able to develop the background skills for problem solving with measurement in length, time, weight, money, and perimeter.

Key Vocabulary

feet, inch, yard, minute, half hour, hour, quarter, dime, nickel, penny, perimeter, area, square unit

Teaching and Learning Progressions

- Given an expression, determine the area of a square or rectangle on a grid.
- Use counting to determine the area of a square or rectangle on a grid.
- Recognize square units as the measure of area.
- Know that area is the amount of space inside the boundary of a flat (two-dimensional) object.
- Given an expression, determine the perimeter of a square or rectangle on a grid.
- Use counting to determine the perimeter of a square or rectangle on a grid.
- Recognize units as the measure of perimeter.
- Know that perimeter is the continuous line forming the boundary of a closed geometric figure.
- Count the number of units for the length of a given line on a grid.
- Understand grid lines as marking unit measure.
- Add and multiply whole numbers.
- Identify the hour before or after a given time.
- Tell time to the half hour on an analog and a digital clock.
- Tell time to the hour on an analog and a digital clock.
- Identify the display of hours and minutes on a digital clock.
- Identify the hour and minute hands on an analog clock.
- Identify an analog and a digital clock as instruments for measuring time in hours and minutes.
- Use a scale to determine the weight of common objects.
- Identify pounds, ounces, and kilograms as units of measure of the weight of objects.
- Identify a scale as an instrument for measuring the weight of objects.
- Identify the value of coins including penny, nickel, dime, and quarter.
- Identify a penny, a nickel, a dime, and a quarter as coins with monetary value.
- Identify the smaller measurement units that comprise a larger unit within a measurement system (inches/feet, minutes/hours, feet/yards).
- Understand that 60 minutes (smaller unit) comprise an hour (larger unit).
- Compare minutes and hours.
- Using the appropriate tool, identify the passage of time, in hours, up to 8 hours.
- Using the appropriate tool, measure the passage of time, in minutes, up to 60 minutes.
- Identify common measurements of time: minutes, hours.
- Identify *time* as a measure of *how long*.
- Identify the smaller measurement unit in the comparisons of smaller units of length.

- Understand that larger units are composed of smaller units of length (inches/feet, feet/yards).
- Compare feet and yards.
- Compare inches and feet.
- Using the appropriate tool, measure the length of common objects in a smaller unit (e.g., objects up to 24 inches in inches, objects up to 36 inches in inches and feet).
- Using the appropriate tool, measure the length of common objects in their appropriate measures (e.g., objects up to 12 in inches, objects up to 3 feet in feet).
- Identify common measurements of length: inches, feet, yards.
- Identify length as the measure of something from end to end.

Application of the Alternate Achievement Standards

- Use classroom objects to measure length and width.
- Use classroom objects and counting squares to determine perimeter and area.
- Using advertisements and sales flyers, have students select the appropriate coins needed to purchase items under one dollar.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use mixed measurements in length, weight, and time.	Determine whether measurements within one system are larger or smaller (inch, feet, yard, pound, ounce, minute, hour).	Distinguish between systems of measurement.	Compare two measurements within the same unit.
Tell time to the quarter hour.	Tell time to the half hour and identify the hour before or after the given time.	Tell time to the hour.	Recognize that a clock is the measurement tool for time.
Determine the value of a collection of coins less than one dollar.	Know the value of quarter, dime, and nickel.	Identify a quarter, dime, and nickel.	Identify a quarter.
Find area and perimeter given side lengths.	Find area and perimeter by counting squares.	Identify side lengths on a rectangle and square.	Identify a square and rectangle.

Grade: 4

Content Area: Mathematics
Strand: Measurement and Data
Objective: Represent and interpret data.

General Education Standard

M.4.22 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Alternate Achievement Standard

M.AAS.4.22 Interpret data on a pictograph or bar graph to solve a problem.

Achievement Elements

Students will be able to use data on a pictograph or bar graph to answer questions.

Key Vocabulary

pictograph, bar graph, data, bars, title, labels, scale, key

Teaching and Learning Progressions

- Answer questions about the data on a pictograph or bar graph.
- Recognize elements of a bar graph.
  - Title
  - Labels that describe the data shown (side and bottom)
  - Scales that show the units used (numbers)
  - Bars that show the measure of the data number

- Recognize the elements of a pictograph.
  - Main title
  - Descriptive label that includes the variable or quantity that changes
  - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.

Application of the Alternate Achievement Standards

- Create pictographs and bar graphs using student-generated data (e.g., class preferences for pets, colors, snacks, cars).
- Demonstrate the use of data by asking questions about pictographs and bar graphs and modeling by interpreting the data.
- Explore pictographs and bar graphs with students using media such as magazines and newspapers.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Gather data and construct a bar graph or pictograph.	Use a bar graph or pictograph to answer questions.	Identify what a given bar graph or pictograph is about.	Discriminate between a pictograph and a bar graph.

Grade: 4

Content Area: Mathematics
Strand: Measurement and Data
Objective: Geometric measurement: understand concepts of angle and measure angles.

General Education Standards

M.4.23 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles.
b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

Alternate Achievement Standards

M.AAS.4.23 Identify an angle in a given shape (square, rectangle, triangle).

Achievement Elements

Students will be able to identify angles in a square, rectangle, and triangle.

Key Vocabulary

square, rectangle, triangle, angle

Teaching and Learning Progressions

- Identify angles in a square, rectangle, and triangle.
- Using models, identify that the sides of an enclosed figure are part of the rays forming an angle.
- Identify an angle as being formed when two rays intersect at a point.

Application of the Alternate Achievement Standards

- Using models and drawings, identify free-standing angles.
- Identify angles in the classroom (e.g., corners of paper, book, cupboard, door).
- Use squares, rectangles, and triangles of different sizes and in different orientations when identifying angles in a shape.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Measure angles in squares, rectangles, and triangles.	Identify angles in squares, rectangles, and triangles.	Identify sides of squares, rectangles, and triangles.	Identify a square, rectangle, and triangle.

Grade: 4

Content Area: Mathematics
Strand: Geometry
Objective: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

General Education Standards

M.4.26 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

M.4.28 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Alternate Achievement Standards

M.AAS.4.26 Recognize angles, parallel and perpendicular lines, and intersecting lines.

M.AAS.4.28 Given a drawing of a shape with a line drawn across the shape, identify if it is divided symmetrically.

Achievement Elements

Students will be able to recognize angles; parallel, perpendicular, and intersecting lines; and lines of symmetry.

Key Vocabulary

angle, parallel, perpendicular, intersecting, symmetry

Teaching and Learning Progressions

- Identify perpendicular lines as two lines that intersect at one point and form right angles.
- Identify a right angle.
- Recognize that angles can have many different arc measurements.
- Identify two rays that have one point in common as an angle.
- Identify two lines that do not intersect and would not intersect if extended as parallel lines.
- Identify two lines that cross (or have a point in common) as intersecting lines.

- Identify a line segment as straight.
- Given a drawing of a regular shape and a line drawn through the shape, identify the line of symmetry.
- Recognize that a line of symmetry partitions a shape into two equal areas.
- Fold a regular shape in half and identify the fold as a line of symmetry.
- Fold a variety of shapes “in half” and identify which folds result in two identical “halves.”

Application of the Alternate Achievement Standards

- Create parallel, perpendicular, and intersecting lines and lines of symmetry by folding square and rectangular papers.
- Create lines of symmetry by folding paper cutouts of other shapes.
- Incorporate art projects with drawings made only of angles (different measures, different side lengths).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify parallel, perpendicular, and intersecting lines in two-dimensional figures.	Recognize parallel, perpendicular, and intersecting lines and lines of symmetry.	Recognize parallel and intersecting lines.	Differentiate lines and other geometric shapes.

Grade: 5

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Write and interpret numerical expressions.

General Education Standards

M.5.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Alternate Achievement Standards

M.AAS.5.2 Select the expression that represents a given calculation (include parentheses).

Achievement Elements

Students will be able to recognize the mathematical expression that represents a given calculation.

Key Vocabulary

expression, add, subtract, parentheses, times/multiply, divide

Teaching and Learning Progressions

- Recognize the expression that represents a given calculation, including parentheses.
- Know order of operations when parentheses are present.
- Understand the mathematical notation for multiplying a number times a quantity that is the result of addition in a mathematical expression (e.g., 3(2+7)).
- Understand parentheses in mathematical expressions.
- Recognize the expressions representing addition and subtraction calculations.
- Add and subtract numbers in series from left to right.
- Add numbers in series from left to right.
- Add, subtract, and multiply whole numbers.

Application of the Alternate Achievement Standards

- Using number and symbol cards, create various basic expressions.
- Model simple real-world situations with basic expressions with and without parentheses.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Interpret a given mathematical expression with parentheses.	Match a mathematical expression with a given expression (addition, subtraction, multiplication, or division, with/without parentheses).	Match a simple mathematical expression with an addition or subtraction calculation.	Represent numbers and operations (addition, subtraction) with written numerals and symbols.

Grade: 5

Content Area: Mathematics
Strand: Operations and Algebraic Thinking
Objective: Analyze patterns and relationships.

General Education Standard

M.5.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Alternate Achievement Standard

M.AAS.5.3 Extend an addition or subtraction number pattern given one rule and the starting point of less than 10.

Achievement Elements

Students will be able to recognize and extend addition and subtraction number patterns.

Key Vocabulary

pattern, extend

Teaching and Learning Progressions

- Extend an addition or subtraction number pattern when given the rule and a starting point of less than 10.
- Given a number pattern of at least three items, determine the addition or subtraction pattern rule.
- Fluently add and subtract whole numbers.

Application of the Alternate Achievement Standards

- Demonstrate a variety of patterns from a simple shape pattern to a simple number/object pattern to simple addition or subtraction patterns.

- Have students demonstrate understanding of the pattern +1 (e.g., “Here is one student; let’s add one more. How many students are there? Let’s add one more. How many students are there now?”)

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Recognize a given numerical pattern that begins at between 10 and 25, and implement the rule to extend the numerical pattern by three items.	Recognize a given numerical pattern that begins at less than 10, and implement the rule to extend the numerical pattern by three items.	Recognize a numerical pattern that begins at 5 or less.	Recognize a symbol pattern.

Grade: 5

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Understand the place value system.

General Education Standards

M.5.6 Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (^1/_{10}) + 9 \times (^1/_{100}) + 2 \times (^1/_{1000})$.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

M.5.7 Use place value understanding to round decimals to any place.

Alternate Achievement Standards

M.AAS.5.6 Compare numbers, including decimals up to hundredths.

M.AAS.5.7 Round three-digit whole numbers from 100 to 949 to the nearest 10 or 100 and round decimals to the nearest hundredths using dollars and cents.

Achievement Elements

Students will be able to round three-digit whole numbers.
Students will be able to compare numbers, including decimals up to hundredths.

Key Vocabulary

decimal, tenths, hundredths, rounding

Teaching and Learning Progressions

- Round three-digit numbers to the nearest 10.
- Round three-digit numbers to the nearest 100.

- Using manipulatives, understand that a number from 95 through 99 rounded to the nearest 10 is 100.
- Round two-digit numbers from 19 to 89 to the nearest 10.
- Round one- and two-digit numbers up to 50 to the nearest 10, using a number line.
- Understand place value of ones, tens, and hundreds and their relationships.
- Using place value, compare whole numbers, decimals less than 1, and decimals greater than 1.
- Using models, compare whole numbers and numbers containing decimals in terms of greater than and less than.
- Using models, compare decimals in terms of greater than and less than.
- Using models, compare whole numbers in terms of greater than and less than.
- Understand place value to tenths and hundredths.
- Using models, determine the relationship between ones, tenths, and hundredths.
- Understand place value of ones, tens, and hundreds.
- Using models, determine the relationship between ones, tens, and hundreds.

Application of the Alternate Achievement Standards

- Use advertisements and websites to demonstrate how to compare decimals.
- Use number lines and place value charts to round numbers.
- Use coin values to demonstrate how to compare decimals.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Round numbers to any place.	Round numbers from 100 to 949 to the nearest 10 or 100.	Round numbers from 5 to 49 to the nearest 10.	Using a number line (whole numbers only), tell to which 10 an indicated number is closer.
Use comparative symbols to communicate comparisons of numbers with decimals up to thousandths.	Compare numbers, including decimals up to the hundredths.	Read numbers to the tenths place.	Recognize decimals to the tenths place.

Grade: 5

Content Area: Mathematics
Strand: Numbers and Operations in Base Ten
Objective: Perform operations with multi-digit whole numbers and with decimals to hundredths.

General Education Standards

M.5.8 Fluently multiply multi-digit whole numbers using the standard algorithm.

M.5.9 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

M.5.10 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used.

Alternate Achievement Standards

M.AAS.5.8 Multiply a two-digit number by a one-digit number with regrouping using a calculator.

M.AAS.5.9 Divide a three-digit number by a one-digit number with no remainder using a calculator.

M.AAS.5.10 Add and subtract two-digit numbers with regrouping (include numbers with decimals of tenths) using a calculator.

Achievement Elements

Students will be able to add and subtract two-digit numbers, including decimals (tenths).
Students will be able to multiply and divide two- and three-digit numbers by one-digit numbers.

Key Vocabulary

add, subtract, multiply, divide, decimal, regroup

Teaching and Learning Progressions

- Add and subtract two-digit numbers with regrouping, using numbers to the tenths.
- Understand place value to the tenths place.
- Using manipulatives, subtract a one-digit number from a two-digit number with regrouping that involves borrowing a unit of ten.
- Using manipulatives, add two-digit numbers with regrouping so that the answer is composed of a unit/some number of units of ten and some ones.
- Using manipulatives, compose two-digit numbers as a unit/some number of units of ten and some ones.
- Divide three-digit numbers by one-digit numbers without remainders.
- Divide two-digit numbers by one-digit numbers without remainders.
- Divide one-digit numbers by one-digit numbers without remainders.
- Multiply one-digit numbers.
- Subtract with and without regrouping.
- Multiply a two-digit number by a one-digit number with regrouping.
- Multiply a two-digit number by a one-digit number without regrouping.
- Multiply a one-digit number by a one-digit number.
- Add with and without regrouping.

Application of the Alternate Achievement Standards

- Use arrays with preferred objects to demonstrate multiplication.
- Sort classroom supplies for activities to demonstrate division.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Multiply a two-digit number by a two-digit number.	Multiply a two-digit number by a one-digit number with regrouping.	Multiply a two-digit number by a one-digit number without regrouping.	Multiply a one-digit number by a one-digit number.
Divide a three-digit number by a two-digit number with no remainder.	Divide a three-digit number by a one-digit number with no remainder.	Divide a two-digit number by a one-digit number with no remainder.	Separate a given group of objects into two or three groups with no objects left over.
Add and subtract numbers with decimals up to hundredths.	Add and subtract numbers, with regrouping, up to tenths.	Add and subtract whole numbers within 20 with regrouping.	Add and subtract whole numbers within 10 without regrouping.

Grade: 5

Content Area: Mathematics
Strand: Numbers and Operations–Fractions
Objective: Use equivalent fractions as a strategy to add and subtract fractions.

General Education Standards

M.5.11 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

M.5.12 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally, and assess the reasonableness of answers.

Alternate Achievement Standards

M.AAS.5.11 Use a model to add and subtract fractional parts with like denominators.

M.AAS.5.12 Solve word problems involving addition and subtraction of fractions with like denominators.

Achievement Elements

Students will be able to solve problems using fractions with like denominators.

Key Vocabulary

fraction, denominator, numerator

Teaching and Learning Progressions

- Solve word problems involving addition and subtraction of fractions with like denominators.
- Add and subtract fractions with the same denominator.
- Decompose a given fraction into a sum of fractions with the same denominator in one or more ways.
- Understand addition and subtraction of fractions as joining and separating parts of the same whole.
- Using models, add and subtract fractions with the same denominator.

- Using models, decompose a given fraction into a sum of fractions with the same denominator in one or more ways.
- Using models, join unit fractions together to create larger parts of the whole.
- Understand addition and subtraction of fractions as joining and separating parts of the same whole.

Application of the Alternate Achievement Standards

- Use a measuring cup and a given measuring spoon (e.g., ½ teaspoon, ¾ teaspoon) to demonstrate addition of like fractions.
- Use concrete models (e.g., apple slices in halves and fourths).
- Double recipes.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Create equivalent fractions with unlike denominators.	Add, subtract, and solve problems using fractions with like denominators.	Add and subtract fractions with like denominators, using models/manipulatives.	Add and subtract thirds and fourths using manipulatives.

Grade: 5

Content Area: Mathematics
Strand: Number and Operations–Fractions
Objective: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

General Education Standards

M.5.14 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(^{a}/_{b})$ x q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x $q \div b$.
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

M.5.16 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

M.5.17 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade.)

a. Interpret division of a unit fraction by a nonzero whole number, and compute such quotients.
b. Interpret division of a whole number by a unit fraction, and compute such quotients.
c. Solve real-world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

Alternate Achievement Standards

M.AAS.5.14 Find the product of unit fractions (with denominators of 2, 3, 4, 5, 10).

M.AAS.5.16 Use a model to solve one-step real-world problems involving multiplying a whole number by a unit fraction.

M.AAS.5.17 Divide a whole number by a unit fraction.

Achievement Elements

Students will be able to multiply unit fractions.
Students will be able to multiply and divide a whole number by a given fraction.

Key Vocabulary

whole number, fraction, multiply, divide

Teaching and Learning Progressions

- Using models, divide a whole number by a given fraction.
- Understand division of a whole number by a fraction as finding out how many of the given part make up the whole.
- Understand division as separating into parts of a given size (divisor).
- Use a model to solve a one-step real-world problem involving multiplying a whole number by a unit fraction.
- Pair *repeated addition* with *multiply by* using models given a unit fraction and a given whole numbers (1 to 5).
- Calculate the repeated addition of the same unit fraction (up to five times) including ½, ⅓, ¼.
- Using models, show repeated addition of the same unit fractions (up to five times) including ½, ⅓, ¼.
- Understand multiplication as repeated addition.
- Know the algorithm for multiplication of fractions.
- Identify the denominator and numerator of a fraction.
- Know that the product is the result of multiplication.
- Fluently multiply by 2, 3, 4, 5, and 10.

Application of the Alternate Achievement Standards

- Demonstrate multiplication of fractions by using repetitive addition of manipulative fraction parts and corresponding wholes.
- Demonstrate real-world problems involving the multiplication of a whole number by a fraction.
- Using manipulative fraction parts and a whole, demonstrate dividing a whole by a fraction (e.g., “How many of these parts are in a whole?”)

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve real-world problems that require multiplication and division of fractions and whole numbers.	Multiply and divide a whole number by a fraction.	Using models, multiply unit fractions.	Recognize repeated addition of unit fractions as multiplication.

Grade: 5

Content Area: Mathematics
Strand: Measurement and Data
Objective: Convert like measurement units within a given measurement system.

General Education Standards

M.5.18 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multistep, real-world problems.

Alternate Achievement Standards

M.AAS.5.18 Given a smaller unit of measurement, determine how many smaller units it would take to make the larger unit.

Achievement Elements

Students will be able to tell time to the quarter hour and determine elapsed clock and calendar time.
Students will be able to use standard units to measure weight and length.
Students will be able to determine the value of a group of coins up to one dollar.

Key Vocabulary

inch, foot, yard, pound, ounce, quarter, nickel, dime, penny, minute, hour, quarter hour, elapsed time

Teaching and Learning Progressions

- Determine elapsed time for a given real-world event using a calendar.
- Identify the beginning and ending days for a given real-world event.
- Understand elapsed time, in days, as the number of days included in an event covering multiple days.
- Locate days of the week on a calendar.
- Recognize a calendar as an instrument for measuring time in days and weeks.
- Using an appropriate clock, determine elapsed time for a real-world event.
- Identify beginning and ending time for a real-world event on an analog and a digital clock.

- Understand elapsed time as the difference between a beginning time and an ending time.
- Tell time to the quarter hour on an analog and a digital clock.
- Tell time to the half hour on an analog and a digital clock.
- Tell time to the hour on an analog and a digital clock.
- Identify the display of hours and minutes on a digital clock.
- Identify the hour and minute hands on an analog clock.
- Identify an analog and a digital clock as instruments for measuring time in hours and minutes.
- Identify pounds, ounces, and kilograms as units of measure of the weight of objects.
- Identify a scale as an instrument for measuring the weight of objects.
- Identify inches, feet, yards, centimeters, and meters as units of measure of the length of objects.
- Identify a ruler and a yardstick/meter stick as instruments for measuring the length of objects.
- Determine the value of a group of coins (up to one dollar).
- Identify the value of coins including penny, nickel, dime, and quarter.
- Identify penny, nickel, dime, and quarter as coins with monetary value.

Application of the Alternate Achievement Standards

- Measure lengths, widths, and distances throughout the classroom and school.
- Use the school or classroom schedule to calculate elapsed time in clock time and days (vacations).
- Have students take coins out of a grab bag and calculate the value of the coins taken (up to one dollar).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use standard units to measure and weigh objects in units and subunits.	Use standard units to measure and weigh objects.	Use a ruler to measure an object in inches. Use a scale to measure an object in ounces.	Use nonstandard tools to measure length.
Calculate the value of coins and bills up to five dollars.	Calculate the value of a group of coins up to one dollar.	Calculate the value of groups of coins up to twenty-nine pennies, ten nickels, and five dimes.	Identify and know the value of a penny, a nickel, and a dime.
Determine elapsed clock time (to the nearest quarter hour) and calendar time (days and weeks) and tell time to the minute.	Determine elapsed clock time (to the nearest hour and half hour) and calendar time, and tell time to the quarter hour.	Tell time to the half hour and determine elapsed time to the nearest hour on a class or a school schedule.	Tell time to the hour and use a personal daily schedule.

Grade: 5

Content Area: Mathematics
Strand: Measurement and Data
Objective: Represent and interpret data.

General Education Standard

M.5.19 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

Alternate Achievement Standard

M.AAS.5.19 Interpret data on a bar graph, pictograph, or line plot to display a data set of measurements in fractions of a unit (limited to halves).

Achievement Elements

Students will be able to interpret data on a bar graph, pictograph, or line plot.

Key Vocabulary

data, pictograph, bar graph, line plot

Teaching and Learning Progressions

- Interpret data shown on graphic representations.
- Determine data shown on graphic representations.
- Recognize elements of a line plot.
  - Title
  - Key/Legend that tells what each dot represents
  - *x*-axis that runs horizontally (flat) and has pictures/labels that represent different time or names of things being compared
  - *y*-axis that runs vertically (up and down) and typically, has numbers for the amount of things being measured

- Recognize elements of a bar graph.
  - Title
  - Labels that describe the data shown (side and bottom)
  - Scales that show the units used (numbers)
  - Bars that show the measure of the data number
- Recognize the elements of a pictograph.
  - Main title
  - Descriptive label that includes the variable or quantity that changes
  - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.

Application of the Alternate Achievement Standards

- Use student-collected data to construct pictographs, bar graphs, line plots.
- Demonstrate construction of graphic representations.
- Model determining and interpreting data from graphic representations found in various forms of media.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Draw conclusions to answer questions and make predictions using data from a bar graph or line plot.	Interpret data from a pictograph, bar graph, and line plot (numerical data to halves).	Answer questions about data in a pictograph and bar graph (whole number numerical data).	Identify the topic of a pictograph or bar graph.

Grade: 5

Content Area: Mathematics
Strand: Measurement and Data
Objective: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

General Education Standard

M.5.21 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.

Alternate Achievement Standards

M.AAS.5.21 Determine the volume of a three-dimensional figure by counting unit cubes (where at least the width or height or depth is 1).

Achievement Elements

Students will be able to determine volume by counting unit cubes.

Key Vocabulary

volume, cube, cubic unit

Teaching and Learning Progressions

- Determine the volume of a three-dimensional figure using unit cubes.
- Know that a solid figure that can be packed without gaps or overlaps using *n* unit cubes has a volume of *n* cubic units.
- Recognize cubic units as the measure of volume.
- Recognize that a cube with side lengths of one unit is called a *unit cube*.
- Know that volume is the amount of space that an object occupies or that is enclosed within a container.

Application of the Alternate Achievement Standards

- Use measuring cups or beakers (filling and dumping) to introduce the concept of volume.
- Have students build models made out of cubes in different configurations and determine the volume of each structure.
- Transfer knowledge of three-dimensional structures to two-dimensional drawings of the same structure (“hidden” cubes).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the volume of a regular rectangular prism given the length, width, and height of the prism.	Determine the volume of a regular rectangular prism by counting unit cubes.	Identify that volume is the amount of space occupied by or contained in an object.	Differentiate between a three- and a two-dimensional object.

Grade: 5

Content Area: Mathematics
Strand: Geometry
Objective: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

General Education Standards

M.5.23 Use a pair of perpendicular number lines, called axes, to define a coordinate system with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

M.5.24 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Alternate Achievement Standards

M.AAS.5.23 Identify quadrant 1 and the origin on a coordinate system grid.

M.AAS.5.24 Identify the coordinate values of a point with whole number coordinates in quadrant 1 (*x* and *y* values limited to 5 or less).

Achievement Elements

Students will be able to identify points on a coordinate plane in quadrant 1.

Key Vocabulary

coordinate plane, coordinates, ordered pair, grid, point

Teaching and Learning Progressions

- Locate/identify points on the coordinate grid given the *x*- and *y*-values of the ordered pair.
- Understand that a point is located on the coordinate plane by using an ordered pair of numbers where the first number indicates how many units to move from the origin to the right on the *x*-axis and the second number indicates how many units to move from that point vertically/up.

- Count unit spaces on the *x*- and *y*-axis.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize quadrant 1 of a coordinate grid.
- Identify quadrant 1 and the origin of a coordinate grid.
- Know that the axes that border quadrant 1 are scaled with positive numbers.
- Know that the point of intersection of the axes is called the origin.
- Know that the axes are scaled from the point of intersection with positive numbers from the point of intersection to the right and from the point of intersection horizontally up.
- Understand that a coordinate plane is divided into four spaces/quadrants by coordinate axes.
- Understand that two lines perpendicular to each other form a coordinate grid on the coordinate plane.

Application of the Alternate Achievement Standards

- Use simple maps to determine routes by moving vertically and horizontally on a grid.
- Have students roll a die to make a horizontal move and then vertical move on a grid.
- Make a grid on the floor with painter's tape and have students move to ordered pair locations.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify the parts of a coordinate system.	Locate given points on a coordinate plane in quadrant 1.	Identify a number on a vertical or horizontal number line.	Recognize a point on a number line.

Grade: 5

Content Area: Mathematics
Strand: Geometry
Objective: Classify two-dimensional figures into categories based on their properties.

General Education Standards

M.5.25 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

Alternate Achievement Standards

M.AAS.5.25 Classify two-dimensional figures and identify the attributes (angles, number of sides, corners) they have in common.

Achievement Elements

Students will be able to identify similar attributes of two-dimensional figures and classify them as circles, squares, rectangles, triangles, pentagons, and hexagons.

Key Vocabulary

circle, square, rectangle, triangle, pentagon, hexagon

Teaching and Learning Progressions

- When given a geometric figure, classify it as a circle, square, rectangle, triangle, pentagon, or hexagon.
- Recognize attributes of a geometric figure (sides, angles for figures with straight sides and boundary, radius, diameter for a circle).
- Recognize a geometric figure on and not on a coordinate grid.

Application of the Alternate Achievement Standards

- Using shape manipulatives, have students identify attributes of two-dimensional figures.
- Using transparency overlays, have students discriminate differences in two-dimensional figures.
- Given an attribute, have students select figures that have that attribute from a group of figures.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe classes of two-dimensional figures when given the class.	Classify two-dimensional figures using their attributes.	Identify square, rectangle, triangle, circle, and pentagon.	Identify square, rectangle, triangle, and circle.

Grade: 6

Content Area: Mathematics
Strand: Ratios and Proportional Relationships
Objective: Understand ratio concepts and use ratio reasoning to solve problems.

General Education Standards

M.6.1 Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities.

M.6.2 Understand the concept of a unit rate $^{a}/_{b}$ associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship.

M.6.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $^{30}/_{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities**.**

Alternate Achievement Standards

M.AAS.6.1 Select a ratio to match a given statement and representation

M.AAS.6.2 Recognize rate vocabulary in a real-world situation (e.g., miles per hour, dollars per pound).

M.AAS.6.3 Solve simple real-world problems using ratio/rate reasoning.

M.AAS.6.3a Answer simple questions about a table of equivalent ratios with whole-number measurements.

M.AAS.6.3b Calculate unit-rate problems, including those involving unit pricing.

M.AAS.6.3c Identify a percentage equivalent to a fraction (e.g., 1/2, 1/4, 1).

M.AAS.6.3d Identify the decimal equivalent of a percentage (limited to 10%, 20%, 25%, 40%, and 50%).

Achievement Elements

Students will be able to show a simple ratio relationship between two quantities.
Students will be able to recognize rate vocabulary.
Students will be able to solve real-world problems.
Students will be able to interpret a table with whole number ratios.
Students will be able to calculate unit-rate/pricing problems.
Students will be able to identify a fraction equivalent to a percentage.
Students will be able to convert percentages to decimals.

Key Vocabulary

rate, table, ratio, unit rate, fraction, equivalent, pricing, percentage, decimal, divide,

Teaching and Learning Progressions

- Interpret a table displaying ratios.
- Use ratio language to describe a real-world relationship.
- Understand the concept of a ratio as comparing two quantities.
- Calculate simple unit rates.
- Use rate language to describe a real-world situation.
- Understand the concept of a unit rate associated with a ratio with a second element of 1.
- Identify rates in the real world.
- Recognize rate as a measure of one quantity against another quantity.
- Recognize the fraction equivalent and decimal notation for common percentages.
- Write a common percent fraction as a decimal.
- Write a common percent as a fraction with 100 in the denominator.
- Recognize common percentages.
- Understand that *percent* means per one hundred.
- Use rate language to describe a real-world situation.
- Understand the concept of a unit rate associated with a ratio with a second element of 1.
- Identify rates in the real world.
- Recognize rate as a measure of one quantity against another quantity.
- Show a simple ratio relationship between two real-world quantities.

Application of the Alternate Achievement Standards

- Read newspaper and magazine articles with tables and identify the rate of change in the table.
- Teach the concept of ratios with pictures of objects compared to other pictures of objects.
- Practice parts to a whole with circle graphs.
- Practice grocery store math with dollars per pounds or cents per ounce.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Understand the concept of a ratio as the relationship between two quantities, including part-to-part and part-to-whole relationships.	Understand the concept of a ratio as the relationship between two quantities (limit to part-to-whole relationships).	Identify a ratio that matches a context.	Identify a ratio.
Understand the concept of a unit rate associated with a ratio and use rate language in context.	Recognize rate vocabulary in a real-world situation (e.g., miles per hour, dollars per pound).	Identify rate vocabulary in a real-world situation.	Identify rate vocabulary.
Use ratio and rate to solve real-world and mathematical problems.	Use ratios to solve real-world problems.	Identify real-world problems with ratios.	Identify a ratio.
Make tables of equivalent ratios relating quantities and whole-number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane.	Interpret a table displaying ratios with whole number measurements.	Identify a table displaying ratios with whole number measurements.	Identify a ratio.

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve unit rate problems, including those involving unit pricing and constant speed.	Calculate unit rate problems, including those involving unit pricing.	Identify a whole number when presented in monetary form. Identify a decimal when presented in monetary form.	Identify whole and part.
Find the percentage of a quantity as a rate per 100. Use ratio reasoning to convert measurement units.	Identify a fraction equivalent to a percentage and convert percentages to decimals.	Identify same and different fractions or decimal numbers from pictorial representations.	Identify a whole number or a fraction.

Grade: 6

Content Area: Mathematics
Strand: The Number System
Objective: Apply and extend previous understandings of multiplication and division to divide by fractions.

General Education Standard

M.6.4 Interpret and compute quotients of fractions, and solve word problems involving division of fractions, e.g., by using visual fraction models and equations to represent the problem.

Alternate Achievement Standard

M.AAS.6.4 Divide fractions using visual fraction models.

Achievement Elements

Students will be able to divide ½ by 2, 4, and 5.

Key Vocabulary

fraction, half, part, whole, divide, equal, part

Teaching and Learning Progressions

- Using models, a number line, or an algorithm, divide ½ by 2, 4, and 5.
- Using a number line, divide ½ by 2.
- Represent fractions on a number line.
- Understand a fraction as a number on a number line.
- Understand that pieces are part of a whole.
- Using models, divide ½ into two equal parts (divide by 2).
- Understand division as separating a quantity into parts.
- Understand the fraction ½ as the quantity formed by one part when a whole is partitioned into two equal parts.

Application of the Alternate Achievement Standards

- Use arrays or models for division and multiplying and factors.
- Determine the price of produce based on weight.
- Divide a single piece of fruit (e.g., apple, watermelon, cantaloupe) in one-half and then by 2 or more to compare numbers.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Interpret and compute quotients of fractions and solve word problems involving division of fractions (e.g., represent the problem).	Divide ½ by 2, 3, 4, or 5.	Apply the relationship between multiplication and division.	Demonstrate the concept of division.

Grade: 6

Content Area: Mathematics
Strand: The Number System
Objective: Compute fluently with multi-digit numbers and find common factors and multiples.

General Education Standards

M.6.5 Fluently divide multi-digit numbers using the standard algorithm using a calculator.

M.6.6 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation using a calculator.

Alternate Achievement Standards

M.AAS.6.5 Divide a multi-digit whole number by a single-digit with a remainder. Divide a multi-digit whole number by a two-digit number with no remainders.

M.AAS.6.6 Calculate addition, subtraction, and multiplication problems involving solving two-digit addition, subtraction, and multiplication problems with decimals up to hundredths (e.g., money problems).

Achievement Elements

Students will be able to compute fluently with multi-digit numbers with decimals up to hundredths (e.g., money problems).

Key Vocabulary

multiplication, add, subtract, multiply, divide, single digit, two-digit, dollar, cent, hundredths, equal

Teaching and Learning Progressions

- Use multiplication to solve money problems involving two-digit dollars and then cents (.01–.99). The multiplier is limited to a single digit.
- Use multiplication to solve money problems involving single-digit dollars and then cents (.01–.99). The multiplier limited to a single digit.
- Add and subtract money problems involving two-digit dollars and then cents (.01–.99).

- Add and subtract money problems involving single-digit dollars and then cents (.01–.99).
- Add, subtract, and multiply two-digit numbers, including tenths place decimals. The multiplier is limited to one digit.
- Solve addition, subtraction, and multiplication problems using one-digit numbers.
- Using a division algorithm, divide numbers up to 99 by a one-digit number with or without remainders.
- Using a division algorithm, divide numbers up to 20 by a one-digit number with or without remainders.
- Using models, divide numbers up to 20 by a one-digit number with or without remainders.
- Understand division as the action of separating something into parts.
- Using models, decompose numbers into equal groups with leftovers.
- Using models, decompose even numbers into equal groups.

Application of the Alternate Achievement Standards

- Buy items in the school store to practice adding prices of items and subtracting the amount of money a student has to spend and what the student wants to buy.
- Practice banking, making hypothetical deposits and withdrawals.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Fluently divide multi-digit numbers using the standard algorithm using a calculator.	Divide a two-digit number by a one-digit number with a remainder using a calculator.	Apply the relationship between multiplication and division.	Demonstrate the concept of division.
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation using a calculator.	Calculate addition, subtraction, and multiplication problems that involve solving two-digit addition, subtraction, and multiplication problems with decimals up to hundredths (e.g., money problems) using a calculator.	Identify a whole number when presented in monetary form. Identify a decimal when presented in monetary form.	Identify whole and part.

Grade: 6

Content Area: Mathematics
Strand: The Number System
Objective: Apply and extend previous understanding of numbers to the system of rational numbers.

General Education Standards

M.6.8 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts explaining the meaning of 0 in each situation.

M.6.9 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, for example - (-3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

M.6.11 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Alternate Achievement Standards

M.AAS.6.8 Identify positive and negative numbers in real-world situations (e.g., using visual representations related to credits/debits, temperatures above/below zero).

M.AAS.6.9a Identify positive and negative numbers on a number line.

M.AAS.6.9b Locate or plot positive and negative numbers on a number line.

M.AAS.6.9c Find given points between -10 and 10 on both axes of a coordinate plane.

M.AAS.6.11 Graph or identify points in all four quadrants of the coordinate plane, given a coordinate plane on graph paper between -10 and 10.

Achievement Elements

Students will be able to identify positive and negative numbers in real-world situations and on a number line.
Students will be able to plot/identify a point in quadrant 1 from 0 to 5.

Key Vocabulary

plot, point, coordinate grid, *x*- and *y*-values, pair, *x*-axis, *y*-axis, number line, positive, negative, zero

Teaching and Learning Progressions

- Plot points on the coordinate grid given the *x*- and *y*-values of the ordered pair.
- Locate/identify points on the coordinate grid given the *x*- and *y*- values of the ordered pair.
- Understand that a point is located on the coordinate plane by using an ordered pair of numbers, where the first number indicates how many units to move from the origin to the right on the *x*-axis and the second number indicates how many units to move from that point vertically/up.
- Count unit spaces on the *x*- and *y*-axes.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal,* and *vertical*.
- Recognize the axes, origin, and quadrant 1 of a coordinate grid.
- Recognize a coordinate grid.
- Given a number line from -20 to +20, identify positive numbers, 0, and negative numbers.
- Compare a number line that shows positive numbers with a number line that shows both positive and negative numbers.
- Recognize a number line.
- Identify positive and negative numbers to represent quantities in real-world contexts (e.g., temperature, elevation, credits/debits).
- Understand that positive and negative numbers are used to identify quantities having opposite directions or values.
- Given a number line from -20 to +20, identify positive numbers, 0, and negative numbers.

Application of the Alternate Achievement Standards

- Play *Battleship* on a coordinate plane.
- Use maps during a classroom or school scavenger hunt.
- Use a thermometer to find the temperature inside and outside the school.
- Use an elevator or stairs to understand *positive, negative,* and *zero.*

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use concepts of equality and inequality to write and to explain real-world and mathematical situations.	Identify positive and negative numbers in real-world situations (e.g., using visual representations related to credits/debits, temperatures above/below zero).	Given two points on a number line, identify a statement using less than (<), greater than (>), and equal to (=) to describe their locations on the number line.	Identify whether two statements are equal (=) or not equal (≠).
Identify the absolute value of a number as the distance from zero on a number line.	Identify positive and negative numbers on a number line.	Given two points on a number line, identify a statement using less than (<), greater than (>), and equal to (=) to describe their locations on the number line.	Identify whether two statements are equal (=) or not equal (≠).
Plot integers on number lines and ordered pairs on the coordinate grid.	Plot/identify a point in quadrant 1 from 0 to 5.	Plot a number on a vertical or horizontal number line.	Identify a number on a vertical or horizontal number line.

Grade: 6

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Apply and extend previous understandings of arithmetic to algebraic expressions.

General Education Standards

M.6.13 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers.
b. Identify parts of an expression using mathematical terms (*sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Alternate Achievement Standards

M.AAS.6.13 Describe a given mathematical or real-world problem with an expression including one unknown.
M.AAS.6.13a Evaluate expressions at specific values of their variables (e.g., $m + x = ?$, where $x = 3$ and $m = 2$).
M.AAS.6.13b Identify parts of an expression using mathematical terms (e.g., *sum, product, difference, quotient*).

Achievement Elements

Students will be able to convert repeated multiplication into exponents (e.g., $2 \times 2 \times 2 = 2^3$).
Students will be able to describe a given mathematical or real-world problem with an expression including one unknown.
Students will be able to evaluate an expression at specific values of the variables.
Students will be able to identify parts of an expression using mathematical terms.

Key Vocabulary

variable, value, expression, exponent, unknown, equal, integer, exponential notation

Teaching and Learning Progressions

- Substitute values for variables and calculate the value of a given expression.
- Substitute given numbers for given variables in an expression.
- Identify the parts of an expression using mathematical terms.
- Create simple addition, subtraction, and multiplication expressions to represent a real-world problem.
- Create simple addition, subtraction, and multiplication expressions to represent a given model.
- Use variables to represent numbers.
- Know that a variable(s) can represent an unknown number(s).
- Know *equal* and *variable*.
- Add, subtract, multiply, and divide integers.
- Match 2 x 2, 3 x 3, 4 x 4, and 5 x 5 to their exponential notations.
- Recognize the exponential notation for the numbers 2, 3, 4, and 5 squared.
- Understand that an exponent indicates how many times to use that number in a multiplication.

Application of the Alternate Achievement Standards

- Play a game using addition, subtraction, multiplication, and division to get to a given number; use whole numbers only.
- Use an outside activity such as running to determine distance ran in a time or time to run a distance, and then graph the results.
- Play hopscotch to practice using the Order of Operations.
- Play hiring students for jobs and determining pay.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Understand exponents to simplify numerical expressions that include integer exponents.	Understand exponents to simplify numerical expressions.	Expand exponents into repeated multiplication.	Match repeated multiplication to its exponential form.
Write, read, and evaluate expressions in which letters stand for numbers.	Describe a given mathematical or real-world problem with an expression that includes one unknown.	Identify an algebraic expression in a real-world problem.	Recognize that an unknown number can be represented by a variable in a real-world problem.
Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.	Evaluate expressions at specific values of their variables (e.g., $m + x = ?$, where $x = 3$ and $m = 2$).	Identify the relationship between two variables.	Recognize that an unknown value can be represented by a variable.
Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Identify parts of an expression using mathematical terms (e.g., sum, product, difference, quotient).	Given an equation, identify the first step of Order of Operations.	Given an equation, identify the symbols for the four functions (+, –, x, ÷).

Grade: 6

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Reason about and solve one-variable equations and inequalities.

General Education Standards

M.6.17 Use variables to represent numbers, and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.

M.6.18 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

Alternate Achievement Standards

M.AAS.6.17 Match a phrase to the corresponding one-step one-variable expression (e.g., "a number plus 3" matches "$x + 3$").

M.AAS.6.18 Solve real-world, single-step linear equations involving positive rational numbers.

Achievement Elements

Students will be able to match a phrase to a corresponding one-step, one-variable expression.
Students will be able to solve an addition or subtraction word problem with one step and one variable.

Key Vocabulary

one step, one variable, substitution, equation, true, unknown, equal, integer

Teaching and Learning Progressions

- Solve an addition or subtraction word problem with one step and one variable.
- Use substitution to determine whether a given number in a given set makes an equation true.
- Use variables to represent numbers.
- Know that a variable can represent an unknown number.
- Know *equal* and *variable*.

- Add and subtract integers.
- Match a phrase to the corresponding one-step, one-variable expression.
- Pair the verbal expression of a one-step, one-variable model to its written expression.
- Verbally express simple addition, subtraction, and multiplication expressions that represent a given model.
- Create simple addition, subtraction, and multiplication expressions to represent a given model.
- Use variables to represent numbers.
- Know *equal* and *variable*.
- Add, subtract, and multiply integers.

Application of the Alternate Achievement Standards

- Read articles with graphs and interpret graphs.
- Identify linear equations that represent real-world situations. For example, Peter is paid $5 for walking one dog, $10 for walking two dogs, and $15 for walking three dogs. Which line shows how much Peter gets paid for walking dogs?
- Use a clothes dryer to understand that a longer drying time creates drier clothes.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use variables to represent numbers, and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.	Match a phrase to the corresponding one-step, one-variable expression (e.g., "a number plus 3" matches "$x + 3$").	Identify an algebraic expression.	Distinguish between a number and a letter.
Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.	Solve an addition or subtraction word problem with one step and one variable.	Identify the relationship between two variables.	Recognize that an unknown value can be represented by a variable.

Grade: 6

Content Area: Mathematics
Strand: Geometry
Objective: Solve real-world and mathematical problems involving area, surface area, and volume.

General Education Standards

M.6.21 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

M.6.22 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

M.6.24 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Alternate Achievement Standards

M.AAS.6.21 Calculate problems about perimeter of squares, triangles, rectangles, and other polygons with sides up to ten units; calculate problems involving finding the area of rectangles and squares with sides up to ten units.

M.AAS.6.22 Solve simple problems about volume using unit cubes.

M.AAS.6.24 Identify a three-dimensional shape (cube, cone, cylinder) and match it to its nets.

Achievement Elements

Students will be able to calculate problems about perimeter of polygons with sides up to ten units; and find the area of rectangles and squares with sides up to ten units.
Students will be able to solve problems about volume using unit cubes.
Students will be able to identify a three-dimensional shape and match it to its net.

Key Vocabulary

net, cube, three-dimensional, two-dimensional, polygon, unit cube, solid, length, volume, perimeter, figure, side

Teaching and Learning Progressions

- Match the net of a cube, cone, and cylinder to the three-dimensional figure it represents.
- Understand a net as a pattern that can be cut and folded to make a model of a solid shape.
- Identify a three-dimensional shape: cube, cone, cylinder.
- Solve simple problems about volume using unit cubes.
- Use comparative language to compare the volumes of various solid figures.
- Know that a solid figure that can be packed without gaps or overlaps using *n* unit cubes has a volume of *n* cubic units.
- Recognize cubic units as the measure of volume.
- Recognize a cube with side lengths of one unit is called a *unit cube*.
- Know that volume is the amount of space that an object occupies or that is enclosed within a container.
- Solve simple problems about perimeters and areas of given figures.
- Use comparative language to compare the perimeters and areas of given figures.
- Know that a plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
- Recognize square units as the measure of area.
- Know that a square with a side length of one unit on all sides is called a *unit square* and has an area of *one square unit*.
- Know that area is the amount of space inside the boundary of a flat (two-dimensional) object.
- Apply the perimeter formulas for squares, triangles, rectangles, and other regular polygons.
- Know that *perimeter* is the continuous line forming the boundary of a closed geometric figure.

Application of the Alternate Achievement Standards

- Have students determine where a rug can fit in a room by calculating area.
- Have students determine the amount of fencing needed for a vegetable garden by calculating perimeter.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Calculate problems about the perimeter of squares, triangles, rectangles, and other polygons with sides up to ten units. Calculate problems involving finding the area of rectangles and squares with sides up to ten units.	Calculate the perimeter by adding all the side lengths. Calculate the area by counting the square units.	Recognize measurable attributes.
Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism.	Solve simple problems about volume using unit cubes.	Use unit cubes to measure.	Recognize unit cubes.
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Identify a three-dimensional shape (cube, cone, cylinder) and match it to its net.	Identify a three-dimensional shape.	Point to a three-dimensional net.

Grade: 6

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Develop understanding of statistical variability.

General Education Standard

M.6.25 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

Alternate Achievement Standard

M.AAS.6.25 Interpret a simple graph representing statistical data.

Achievement Elements

Students will be able to answer simple number questions about data on a given graph.

Key Vocabulary

graph, data

Teaching and Learning Progressions

- Answer questions related to the total number of data points, how many are in each category, and how many more or less are in one category than another.
- Recognize the parts of a simple graph (title, scale, key).

Application of the Alternate Achievement Standards

- Demonstrate pictographs, line plots, histograms, and the information that is available on the graphs (e.g., total number of data points, how many in a given category, if and how many more or less are in one category).
- Create pictographs, line plots, and histograms from classroom data.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Make a simple prediction using data on a given graph.	Answer simple number questions about data on a given graph.	Locate given information on a graph.	Identify the data, as a group, on a graph.

Grade: 6

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Summarize and describe distribution.

General Education Standards

M.6.28 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

M.6.29 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Alternate Achievement Standards

M.AAS.6.28 Interpret numerical data on a dot plot or histogram.

M.AAS.6.29 Using a data display, describe the data trend (increasing/going up, decreasing/going down).

Achievement Elements

Students will be able to interpret numerical data on a dot plot or histogram and describe the data trend.

Key Vocabulary

dot plot, histogram, trend, increasing, describe, decreasing

Teaching and Learning Progressions

- Describe the data trend for a given data display.
- Recognize that as you move left to right on a data display, values from top to bottom will indicate a decrease.
- Recognize that you move left to right on a data display, values from bottom to top will indicate an increase.
- Display numerical data in plots or displays such as dot plots, histograms, or line plots.
- Recognize that scale values on a data display increase from left to right and bottom to top.
- Understand a data set as information collected about a given topic or question.
- Answer questions related to the total number of data points, how many are in each category, and how many more or less are in one category than another.
- Display collected numerical data in a dot plot or histogram.
- Recognize the parts of a dot plot or histogram (e.g., title, scale, key).

Application of the Alternate Achievement Standards

- Have students put numerical data into a dot plot or histogram manually or using computer simulations.
- Have students identify the numbers of objects in given groups and create a bar/stack representing that data using blocks.
- Determine or plan classroom situations where students can demonstrate increasing and decreasing.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Compare two data plots or histograms on the same topic and describe the difference in data trends.	Interpret numerical data on a dot plot or histogram and describe the data trend.	Using a dot plot or histogram, answer a more/less question and answer categorical questions (how many in a given category).	Identify a dot plot and a histogram and identify data in a given category.

Grade: 7

Content Area: Mathematics
Strand: Ratios and Proportional Relationships
Objective: Analyze proportional relationships and use them to solve real-world and mathematical problems.

General Education Standards

M.7.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

M.7.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

M.7.3 Use proportional relationships to solve multistep ratio and percent problems.

Alternate Achievement Standards

M.AAS.7.1 Calculate a unit rate (numbers limited to whole numbers under 100).

M.AAS.7.2 Use a ratio to model or describe a real-world relationship (ratio or rate).

M.AAS.7.3 Calculate 10%, 20%, 25%, 50% of a number up to 100, to identify a proportional relationship using a calculator.

Achievement Elements

Students will be able to recognize proportional relationships in real-world settings, including ratios and percentages.

Key Vocabulary

proportional, ratio, percentage, unit rate

Teaching and Learning Progressions

- Calculate 10%, 20%, 25%, and 50% of a number up to 100.
- Know that “of” in calculating a percent means to multiply.
- Understand that percent means per one hundred.
- Use rate language to describe a real-world situation.
- Understand the concept of a unit rate associated with a ratio.
- Use ratio language to describe a real-world relationship.
- Understand the concept of a ratio.
- Calculate simple unit rates.
- Use rate language to describe a real-world situation.
- Understand the concept of a unit rate associated with a ratio with a second element of 1.
- Identify rates in the real world.
- Recognize rate as a measure of one quantity against another quantity.

Application of the Alternate Achievement Standards

- Use classroom data collection to discuss ratios and proportions (e.g., number of girls to the total number of students, number of boys to the total number of students).
- Use sets of chips in two colors to create ratios and proportions.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use percentages to solve a real-world problem using a calculator.	Calculate 10%, 20%, 25%, and 50% of numbers up to 100 using a calculator.	Recognize 25% and 50% of numbers up to 50 using a calculator.	Understand part and whole relationships.
Solve real-world and mathematical problems involving unit rate.	Calculate a unit rate.	Identify a unit rate in a real-world situation.	Pair per hour and per pound with a real-world situation.
Given a graph or table, determine whether two quantities are in a proportional relationship.	Use a ratio to model a real-world relationship.	Identify a proportional relationship in a real-world situation.	Identify equivalent relationships in real-world situations.

Grade: 7

Content Area: Mathematics
Strand: The Number System
Objective: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

General Education Standards

M.7.4 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0.
b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

M.7.5 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

M.7.6 Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Alternate Achievement Standards

M.AAS.7.4 Given a number line divided by increments of 1/4, visual representations, or manipulatives, add and subtract fractions with like and unlike denominators of 2 and 4.
M.AAS.7.5a Multiply proper fractions to include 1/2, 1/4, 1/8, and 1/10.
M.AAS.7.5b Simplify proper fractions.
M.AAS.7.5c Solve division problems with divisors up to five and also with a divisor of 10 without remainders.
M.AAS.7.6 Solve real-world and mathematical problems involving addition, subtraction, and multiplication with rational numbers (fractions to include 1/2, 1/4, 1/8, and 1/10).

Achievement Elements

Students will be able to apply addition, subtraction, and multiplication to solve real-world and mathematical problems of rational numbers.
Students will be able to simplify fractions.

Key Vocabulary

solve, fraction, symbols for addition, subtraction, and multiplication; equivalent, simplify, divisor, numerator, denominator

Teaching and Learning Progressions

- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
- Recognize multiplication of fractions as repeated addition.
- Decompose a given fraction into a sum of fractions with the same denominator in one or more ways.
- Understand addition and subtraction of fractions as joining and separating parts of the same whole.
- Understand a fraction a/b is a sum of fractions $1/b$.
- Simplify proper fractions by recognizing equivalent fractions.
- Recognize and generate simple equivalent fractions.
- Understand two fractions as equivalent if they are the same size or the same point on a number line.
- Using models, show why two fractions are equivalent.
- Compare fractions by reasoning about their size.
- Recognize multiplication of fractions as repeated addition.
- Decompose a given fraction into a sum of fractions with the same denominator in one or more ways.

- Understand addition and subtraction of fractions as joining and separating parts of the same whole.
- Understand a fraction *a*/*b* is a sum of fractions 1/*b*.
- Add and subtraction fractions with like and unlike denominators (limited to 2 and 4).
- Understand two fractions as equivalent if they are the same size or the same point on a number line ($_{2/4 = 1/2}$).
- Decompose a given fraction into a sum of fractions with the same denominator in one or more ways ($_{1/4 + 1/4 = 2/4}$).
- Understand addition and subtraction of fractions as joining and separating parts of the same whole.
- Represent fractions on a number line.
- Understand a fraction as a number on a number line.
- Understand a fraction 1/*b* as the quantity formed by 1 part of a whole when the whole is partitioned into *b* equal parts.

Application of the Alternate Achievement Standards

- Use fraction manipulatives to create equivalent fractions and introduce simplifying fractions.
- Use measuring cups and daily tasks such as watering plants to calculate total water needed.
- Plan a party involving multiple individual servings of whole units (e.g., cake, pizza).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Convert improper fractions to mixed numbers using a calculator.	Simplify fractions using a calculator.	Create equivalent fractions.	Recognize equivalent fractions.
Multiply fractions using a calculator.	Multiply fractions (limited to one half, one fourth, one eighth, and one tenth) using a calculator.	Recognize repeated addition as multiplication using a calculator.	Add up to three unit fractions.
Solve two-step real world and mathematical problems using addition, subtraction, and multiplication of rational numbers using a calculator.	Solve one-step real-world and mathematical problems using addition, subtraction, and multiplication of rational numbers (fractions limited to	Solve one-step real-world and mathematical problems using addition and subtraction of rational numbers (fractions limited to one half and one	Solve one-step real-world problems using addition with whole numbers and fractions limited to one fourth.

	one half, one fourth, one eighth, and one tenth) using a calculator.	fourth) using a calculator.	

Grade: 7

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

General Education Standards

M.7.9 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.

M.7.10 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem.

Alternate Achievement Standards

M.AAS.7.9 Solve addition and subtraction of positive and negative numbers in real-world situations (e.g., credits and debits, temperatures, elevations) using a calculator.

M.AAS.7.10 Solve one-step addition, subtraction, or multiplication problems with one variable using a calculator.

Achievement Elements

Students will be able to apply knowledge of addition and subtraction with positive and negative numbers to real-world situations (e.g., credits and debits, temperatures, elevations).
Students will be able to solve one-step addition, subtraction, or multiplication problems with one variable.

Key Vocabulary

addition, subtraction, multiplication, positive number, negative number, credit, debit, temperature, elevation

Teaching and Learning Progressions

- Solve one-step addition, subtraction, or multiplication problems with one variable.
- Use substitution to determine whether a given number in a given set makes an equation true.
- Use variables to represent numbers.
- Know that a variable can represent an unknown number.
- Know *equal* and *variable*.
- Add, subtract, and multiply.
- Solve addition and subtraction problems based on real-world problems using positive and negative numbers.
- Apply and extend previous understanding of addition and subtraction to add and subtract positive and negatives numbers on a horizontal or vertical number line.
- Explain the meaning of zero.
- Use positive and negative numbers to represent quantities in real-world contexts (e.g., temperature, elevation, credits/debits).
- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values.
- Given a number line from -20 to +20, identify positive numbers, 0, and negative numbers.

Application of the Alternate Achievement Standards

- Have students record temperatures from an area that has positive and negative daytime or nighttime temperatures.
- Have students move up and down on stairs or in an elevator to demonstrate positive and negative.
- Use classroom situations to model equations with one variable.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve a mathematical problem using positive and negative numbers.	Solve real-world problems related to credits and debits, changes in temperature, and changes in elevation.	Use positive and negative numbers to represent quantities in real-world contexts.	Identify 0 and positive and negative numbers on a number line from -20 to +20.
Solve real-world problems with one variable, including positive and negative numbers.	Solve one-step problems with one variable (addition, subtraction, multiplication).	Given a real-world situation, create a number sentence using a variable.	Identify a variable in a mathematical expression.

Grade: 7

Content Area: Mathematics
Strand: Geometry
Objective: Draw, construct, and describe geometrical figures and describe the relationships between them.

General Education Standards

M.7.11 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

M.7.13 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Alternate Achievement Standards

M.AAS.7.11 Given a geometric figure, recognize a similar scaled figure with the same orientation.

M.AAS.7.13 Match a two-dimensional shape with a three-dimensional shape that shares an attribute (rectangle with a rectangular prism, square with a cube, circle with a sphere).

Achievement Elements

Students will be able to recognize similar figures.
Students will be able to match two-dimensional shapes with three-dimensional shapes that share a common attribute.

Key Vocabulary

similar figures, attribute, two-dimensional, three-dimensional, rectangular prism, cube, sphere, side, angle, radius, diameter, face, orientation

Teaching and Learning Progressions

- Given a two-dimensional shape, identify a three-dimensional shape that shares an attribute.
- Identify that a sphere is made up of circles.
- Recognize that a sphere is made up of circles.
- Identify any face of a rectangular prism as a square or rectangle.

- Recognize that rectangular prisms have faces that are rectangular (including squares).
- Identify that any face of a cube is a square.
- Recognize that cubes have faces that are shaped like squares.
- Recognize a three-dimensional object: rectangular prism, cube, and sphere.
- Recognize a two-dimensional drawing of a rectangle, square, and circle.
- Given a geometric figure, recognize a similarly scaled figure with the same orientation.
- When given the same geometric figures in various sizes but the same orientation, identify them as the same figure.
- Identify geometric figures with the same attributes and in the same orientation as the same figure.
- Recognize attributes of geometric figures: sides; angles for figures with straight sides; and boundary, radius, and diameter for a circle).
- Recognize a geometric figure on and not on a coordinate grid.

Application of the Alternate Achievement Standards

- Use real objects, drawings of those objects, and line drawings of the basic shapes that share attributes with the objects to explore those attributes and recognize them in other objects.
- Compare real objects of various sizes (e.g., balls, cubes, chairs) to demonstrate characteristics that are the same and different.
- Find examples of the same shape in various sizes and orientations in the classroom and other environments.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Create a scaled drawing of a given shape.	Given a geometric figure, recognize similar shapes.	Identify shapes of different sizes as the same shape.	Sort shapes into categories.
Describe the attributes of three-dimensional figures.	Match two-dimensional and three-dimensional shapes that share an attribute.	Identify the faces of a rectangular prism as a square or rectangle.	Identify shapes.

Grade: 7

Content Area: Mathematics
Strand: Geometry
Objective: Solve real-world mathematical problems involving angle measure, area, surface area, and volume.

General Education Standards

M.7.14 Know the formulas for the area and circumference of a circle, and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

M.7.15 Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

M.7.16 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Alternate Achievement Standards

M.AAS.7.14 Identify the radius, diameter, and circumference of a circle.

M.AAS.7.15 Categorize angles as acute, obtuse, or right (freestanding or within a triangle).

M.AAS.7.16 Solve real-world and mathematical problems involving volumes of cubes or rectangular prisms.

Achievement Elements

Students will be able to identify attributes of a circle: radius, diameter, and circumference.
Students will be able to categorize angles as acute, obtuse, or right.
Students will be able to solve real-world and mathematical problems involving volumes of cubes and rectangular prisms.

Key Vocabulary

radius, diameter, circumference, acute, obtuse, right, volume

Teaching and Learning Progressions

- Use problem-solving skills to solve real-world problems involving volumes of cubes or rectangular prisms.
- Calculate the volume of cubes and rectangular prisms in the real world.

- Recognize cubes and rectangular prisms in the real world.
- Know that volume is calculated by multiplying the length times the width times the height of a cube or given rectangular prism.
- Use manipulatives to determine the volume of given cubes or rectangular prisms.
- Recognize cubic units as the measure of volume.
- Know that volume is the amount of space that an object occupies or that is enclosed within a container.
- Given an angle within a triangle, identify the angle as *right*, *obtuse*, or *acute*.
- Given a triangle, identify the three angles of the triangle.
- Recognize a triangle as an enclosed figure with three angles.
- Given various angles, identify angles smaller than 90 degrees as *acute*.
- Given various angles, identify angles greater than 90 degrees as *obtuse*.
- Identify a right angle as an angle with a 90-degree arc or one formed by two rays that are perpendicular to each other.
- Recognize the measure of an angle as the length of the arc inside the angle.
- Locate the arc of an angle.
- Identify an angle as a figure formed by two rays (lines) that share a common endpoint.
- Recognize *more/greater, less/fewer,* and *same/equal*.
- Given a circle, identify the radius, diameter, and circumference of the circle.
- Know that the circumference of a circle is the line that forms the outer boundary of the drawn circle.
- Know that the diameter of a circle is the line segment drawn from boundary to boundary of a circle and passing through the center of the circle.
- Know that the radius of a circle is the line segment drawn from the center of the circle to the outer boundary of the circle.
- Locate the center of a circle.
- Identify the drawing of a circle on and not on a coordinate grid.

Application of the Alternate Achievement Standards

- Have students create circles using various mathematical attributes, including the radius and diameter.
- Have students use body movements to create acute, right, and obtuse angles.
- Have students measure real objects in the classroom (e.g., shoeboxes, tissue boxes) and calculate the volume of each object.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Calculate the diameter of a circle given the radius.	Identify the radius, diameter, and circumference of a given circle.	Identify the diameter of a circle.	Identify a circle.
Measure the arc of given angles free-standing or in a triangle.	Categorize given angles as acute, right, or obtuse.	Draw angles with a variety of arc measurements.	Identify an angle.
Solve a real-world two-step problem involving the volume of a rectangular prism.	Calculate the volume of cubes and other rectangular prisms.	Determine the volume of a cube or rectangular prism using unit cubes.	Recognize that volume is the amount of “stuff” a rectangular prism will hold.

Grade: 7

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Use random sampling to draw inferences about a population.

General Education Standards

M.7.17 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Alternate Achievement Standards

M.AAS.7.17 Given a statistical display (line graph, dot plot, histogram), in everyday language, identify what the display measures.

Achievement Elements

Students will be able to identify what information in a line graph, dot plot, and histogram is measuring.

Key Vocabulary

line graph, dot plot, histogram, measure, topic, key, legend, title

Teaching and Learning Progressions

- Recognize elements of a line graph.
  - Title
  - Key/Legend that tells what each line represents
  - Data/Line that represents the information the graph contains
  - *x*-axis that runs horizontally (flat) and has numbers that represent different time periods or names of things being compared
  - *y*-axis that runs vertically (up and down) and typically has numbers for the amount of stuff being measured

- Recognize elements of a histogram.
    - Title
    - Labels that describe the data shown (side and bottom)
    - Scales that show the units used (numbers)
    - Bars that show the measure of the data number
- Recognize the elements of a pictograph.
    - Main title
    - Descriptive label that includes the variable or quantity that changes
    - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.

Application of the Alternate Achievement Standards

- Construct various displays of classroom-gathered data, including a line graph, dot plot, and histogram.
- Gather and discuss different data displays and describe what they measure.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Interpret data on a given line graph, dot plot, or histogram.	Identify what a given line graph, dot plot, or histogram is measuring.	Identify a line graph, dot plot, and histogram.	Identify a pictograph, bar graph, and line graph.

Grade: 7

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Draw informal comparative inferences about two populations.

General Education Standard

M.7.19 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Alternate Achievement Standard

M.AAS.7.19 Compare two sets of data within a single data display such as a pictograph or bar graph.

Achievement Elements

Students will be able to compare data displays within a pictograph or bar graph to answer questions such as “how much more” and “how much less.”

Key Vocabulary

pictograph, bar graph, more, less

Teaching and Learning Progressions

- Compare two sets of data on a single data display (pictograph or bar graph).
- Determine the arithmetic operations needed to determine “how much more” and “how much less.”
- Know “how much more” and “how much less.”
- Know *more* and *less*.
- Recognize elements of a bar graph.
    - Title
    - Labels that describe the data shown (side and bottom)
    - Scales that show the units used (numbers)
    - Bars that show the measure of the data number

- Recognize the elements of a pictograph.
    - Main title
    - Descriptive label that includes the variable or quantity that changes
    - Key that gives the symbol and shows what the symbol represents
- Recognize data as a representation of real events.

Application of the Alternate Achievement Standards

- Create a bar graph using classroom data (e.g., shoe size) and demonstrate calculating more/less between categories.
- Have students represent data with real objects and compare quantities to determine more/less of the groups.
- Use paper strips that can be removed and overlapped to represent quantities on a bar graph to determine more/less.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Compare and interpret points on a line graph.	Determine more/less and answer “how much more” and “how much less” questions using a pictograph or bar graph.	Answer “how much more” and “how much less” questions when given two sets of objects.	Answer more/less questions when given two sets of objects.

Grade: 7

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Investigate change processes and develop, use, and evaluate probability models.

General Education Standards

M.7.21 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

M.7.22 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

M.7.23 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Alternate Achievement Standards

M.AAS.7.21 Describe the probability of events occurring as possible or impossible.

M.AAS.7.22 Given a data set that represents a series of events, identify most likely event.

M.AAS.7.23 Model an event with two outcomes by flipping a coin.

Achievement Elements

Students will be able to determine whether an event is possible, likely, or impossible.
Students will be able to identify the most likely event given data about a series of events.

Key Vocabulary

probability, possible, likely, impossible, most likely, outcomes, flip, probable

Teaching and Learning Progressions

- Model an event with two outcomes by flipping a coin.
- Pair flipping a coin with an event that has two and only two possible outcomes.
- Identify events that have two and only two possible outcomes.
- Know the meaning of *flip*.
- Given a series of events, compare the data to determine which is most likely.
- Given three probability events, compare the data to determine which is most likely.
- Given two probability events, compare the data to determine which is most likely.
- Given a probability event, understand what the data convey.
- Understand data in relation to the event they represent.
- Understand that data represent real events.
- Understand probabilities of greater than 0 as possible.
- Differentiate *possible* and *probable*.
- Understand a probability of 1 as the *only outcome*.
- Understand impossibility as a probability of 0.
- Express probability as a fraction using probability data such as coin tosses or die tosses.
- Experience various probability events using coins or number cubes.
- Understand the probability of an event as the number of times an outcome gives the event divided by the total number of possible outcomes.
- Understand an outcome of a chance event in relation to the event it represents.
- Understand that outcome data represent real-world events.

Application of the Alternate Achievement Standards

- With students, identify a real-world situation that has only two outcomes, such as having students flip a coin to model the probability of the two outcomes of a coin flip.
- Put various numbers of three different objects in a bag and have students pull them out. Record the results on a chart that contains an additional object. Discuss the likelihood of obtaining data for the fourth object.
- Throw a bean bag toward a circle target and record the results. Identify the most likely result.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the numerical probability of simple events.	Determine the probability of simple events as possible, likely, or impossible and in some cases, most likely.	Identify an event as possible or impossible.	Recognize that flipping a coin has two possible outcomes.

Grade: 8

Content Area: Mathematics
Strand: The Number System
Objective: Know that there are numbers that are not rational, and approximate them by rational numbers.

General Education Standards

M.8.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

M.8.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Alternate Achievement Standards

M.AAS.8.1 Identify the decimal equivalents of common fractions as repeating or non-repeating (i.e., halves, thirds, fourths, fifths).

M.AAS.8.2 Given the decimal approximation of irrational numbers located on a number line; compare the sizes of the irrational numbers.

Achievement Elements

Students be able to identify the decimal equivalents of common fractions (i.e., ¼, ½, ¾, all tenths, and all fifths).
Students will be able to give the decimal approximation of irrational numbers using a number line.

Key Vocabulary

decimal, equivalent, approximate, irrational numbers

Teaching and Learning Progressions

- Compare the decimal approximation of irrational numbers located on a number line.
- Compare decimal numbers placed on a number line to determine which one is larger or smaller.
- Identify decimal numbers placed on a number line.
- Know that when two numbers are placed on a number line, the number on the right will be the larger number and the number on the left will be the smaller number.

- Know that the scale on a number line increases as one moves from left to right on the number line.
- Determine the scale used in a given number line.
- Identify a number line.
- Use division to find the decimal equivalents for a given fraction.
- Know that to find a decimal equivalent for a fraction, the numerator is divided by the denominator.
- Be able to express a whole number in decimal form.
- Know that decimals represent parts of whole units in another form.
- Recognize that fractions represent parts of whole units.

Application of the Alternate Achievement Standards

- Use transparency overlays to compare decimal equivalents of rational and irrational numbers.
- Use fraction/decimal manipulatives to identify common fraction equivalents. The manipulatives can be purchased, teacher-made, or student-made.
- Use painter's tape on the floor to create a number line to practice locating rational and irrational number equivalents.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify decimal equivalents of fractions.	Identify the decimal equivalents of common fractions (i.e., ¼, ½, ¾, all tenths, and all fifths).	Identify the decimal equivalents for ¼, ½, and ¾ using money.	Identify a quarter as ¼ of a dollar, two quarters as ½ of a dollar, and three quarters as ¾ of a dollar.
Using a number line, give the approximate decimal value for mixed numbers up to 5.	Using a number line, give the approximate decimal for an irrational number.	Locate a point between two given points on a number line.	Locate a point on a number line given a verbal cue.
Compare decimal approximations for mixed numbers up to 5.	Compare the decimal approximations for irrational numbers using a number line.	Compare decimal equivalents for common fractions using a number line.	Compare whole numbers up to 20 using a number line.

Grade: 8

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Work with radicals and integer exponents.

General Education Standards

M.8.3 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

M.8.4 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

Alternate Achievement Standards

M.AAS.8.3 Calculate the square of numbers 1 to 10 using a calculator.

M.AAS.8.4 Find the square root of the perfect squares up to 100 using a calculator.

Achievement Elements

Students will be able to square and find the square root of numbers 1 to 10 using a calculator.

Key Vocabulary

square, square root, radical, perfect square

Teaching and Learning Progressions

- Know how to use the "guess and check" method and calculating perfect squares method to find the square root of a given perfect square up to 100.
- Recognize that the number being squared is the *square root* of the perfect square.
- Calculate the perfect squares for integers 2–10.
- Identify x^2 as a *perfect square*.
- Understand *a number squared*.
- Square numbers 1–10.
- Pair *square* with multiply a number by itself.

- Multiply a number from 1–10 by itself.
- Multiply numbers up to 10 by each other.

Application of the Alternate Achievement Standards

- Use muffin tins to understand repeated multiplication (put the same number of objects into each space and then calculate the total number of objects).
- Reverse the muffin tin activity to determine the square root (given a perfect square number of objects, divide them out into the same number in each cup as the number of cups). (Realize there is a certain amount of guess and check here.)

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Uses x^2 in a numerical equation.	Square numbers 1 to 10 and recognize the symbol for a number squared using a calculator.	Multiply numbers 1 to 12 using a calculator.	Use repeated addition to multiply numbers 1 to 6.
Determine the square root of the perfect squares up to 225 using a calculator.	Determine the square root of the perfect squares 1 to 100 using a calculator.	Find the factors of a number 1 to 20.	Know factor families for numbers 1 to 10.

Grade: 8

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Understand the connections between proportional relationships, lines, and linear equations.

General Education Standards

M.8.7 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

M.8.8 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Alternate Achievement Standards

M.AAS.8.7 Determine slopes of 1/2, 1, and 2 from the graphs of proportional relationships.

M.AAS.8.8 Identify the slope of a line using the rise and run from the associated triangle on the coordinate plane to determine the slopes of the line.

Achievement Elements

Students will be able to identify the slope of a line.

Key Vocabulary

slope, coordinate plane, proportional relationships, steepness, vertical, horizontal

Teaching and Learning Progressions

- Identify the slope of a line segment on a coordinate plane or grid using the rise and run of the associated triangle.
- Know that slope can be mathematically determined as the *rise* over the *run*.
- Pair *vertical length in units* with *rise* and horizontal *length in units* with *run*.
- Understand *slope* as the steepness of a given line segment drawn on a coordinate grid.
- Count the lengths of the vertical and horizontal sides of a given triangle drawn on a coordinate grid.
- Locate the vertical and horizontal sides of a given triangle drawn on a coordinate grid.

- Recognize a triangle formed by drawing lines vertically and horizontally from the endpoints of a given line segment.
- Represent slope as the change in *y* over the unit change (1) in *x*.
- Determine the change in *y* at a point.
- Locate a point on the *x*-axis that shows a change of one unit.
- Recognize a line drawn on a coordinate grid beginning at the origin.
- Count out given unit values on the *x*- and *y*-axes.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, x-*axis*, y-*axis*, *horizontal*, and *vertical*.
- Recognize a coordinate grid.

Application of the Alternate Achievement Standards

- Use a painter's tape grid on the floor to have students determine the rise and run of a given line.
- Have students use geoboards to create lines with various slopes.
- Using grid paper, have students use two different-colored pencils to draw the vertical and horizontal lines from the ends of a line segment to create a reference triangle.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the slope of a line segment on a coordinate plane.	Determine the slope of a line segment on a coordinate plane (limited to slopes of ½, 1, and 2).	Recognize line segments on a coordinate plane as having different slopes.	Recognize slope as the steepness of a line in relation to the horizontal.

Grade: 8

Content Area: Mathematics
Strand: Expressions and Equations
Objective: Analyze and solve linear equations and pairs of simultaneous linear equations.

General Education Standards

M.8.10 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
c. Solve real-world and mathematical problems leading to two linear equations in two variables.

Alternate Achievement Standard

M.AAS.8.10 Identify the labeled point of intersection on graphs of two linear equations as a solution for both equations.

Achievement Elements

Students will be able to solve one-step equations with one variable, using addition, subtraction, multiplication, or division.

Key Vocabulary

add, subtract, multiply, divide, variable, solve

Teaching and Learning Progressions

- Given an equation of the form $a + b = c$, $a - b = c$, $ab = c$, or $a/b = c$, solve for the unknown quantity when given
  - b and c
  - a and c
  - a and b
- Understand substitution of two numbers into a simple equation.
- Understand substitution of a number into a simple equation.
- Recognize a mathematical equation.

- Recognize a mathematical expression.
- Understand that letters (variables) in mathematics stand for numbers (unknowns).
- Fluently add, subtract, multiply, and divide numbers.

Application of the Alternate Achievement Standards

- Model real-world situations with algebraic equations to demonstrate constants and variables.
- Calculate amounts in a savings account given a starting amount.
- Use manipulatives and operation symbols to model problem solving.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Recognize and solve a one-step algebraic equation with one variable that represents a real-world event.	Solve a simple one-step algebraic equation with one variable using addition, subtraction, multiplication, or division for numbers < 100.	Solve a simple one-step algebraic equation written with one variable representing the answer of an addition or subtraction problem.	Recognize a simple one-step equation that represents a number sentence.

Grade: 8

Content Area: Mathematics
Strand: Functions
Objective: Define, evaluate, and compare functions.

General Education Standards

M.8.11 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

M.8.13 Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

Alternate Achievement Standards

M.AAS.8.11 Identify a missing number in a function table that contains at least two complete ordered pairs by determining and applying the rule for the function (limited to linear functions; values limited to 10 or less).

M.AAS.8.13 Given a set of graphs, identify which graph is linear.

Achievement Elements

Students will be able to identify a linear graph.
Students will be able to identify a missing number in a simple function table.

Key Vocabulary

linear, nonlinear, graph, function, rule, solution

Teaching and Learning Progressions

- Given a set of graphs, identify which graph is linear.
- Given a linear graph and a nonlinear graph, identify the linear graph.
- Identify a linear graph by recognizing it as a straight line on the coordinate grid.
- Identify a coordinate grid.
- Given a table of at least three ordered pairs with one ordered pair missing one number, find the missing number.
- Given a table of ordered pairs that are the solution for a linear function, determine the rule for the function.

- Understand that each ordered pair on a given table is a solution for the given function.
- Create a table of ordered pairs that contains the solutions for a linear function.
- Use a rule and up to three given *x*-values to find at least three solutions to a given function.
- Pair *function* and *equation*.
- Given an equation of the form: $a + b = c$, $a - b = c$, $ab = c$ or $a/b = c$, find c when given
    - b and c
    - a and c
    - a and b
- Understand substitution of two numbers into a simple equation.
- Understand substitution of a number into a simple equation.
- Recognize a mathematical equation.
- Understand substitution of a number into a simple expression.
- Recognize a mathematical expression.

Application of the Alternate Achievement Standards

- Create real-world context functions and determine the meaning of each part of the function.
- Model functions in a real-world context and demonstrate solutions. (e.g., Have two students divide up a given number of objects in various ways: There are 10 red counting bears and Kia has 3 and Jay has 7, or Kia has 5 and Jay has 5, or Kia has 6 and Jay has 4.)
- Create tables of ordered pairs with data from a real-world context and have students discover the rule.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Discriminate between linear and nonlinear graphs using mathematical vocabulary.	Given a set of two or more graphs, identify the linear graph.	Identify a linear graph.	Identify a straight line.
Given a rule and the *x*-value of two or more ordered pairs, complete the ordered pairs.	Given a table of three or more ordered pairs with one pair missing one number, find the missing number (values 1–10).	Find the missing number in an ordered pair when given the rule.	Identify the ordered pair for a given point on the coordinate grid.

Grade: 8

Content Area: Mathematics
Strand: Functions
Objective: Use functions to model relationships between quantities.

General Education Standards

M.8.14 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two *(x,y)* values, including reading these from a table or from a graph. Interpret the rate of change and initial value of linear function in terms of the situation it models and in terms of its graph or a table of values.

M.8.15 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Alternate Achievement Standards

M.AAS.8.14 Given a graph of a function, determine if it is linear and identify the *y*-intercept.

M.AAS.8.14a Count out the rise over the run of a line on a graph to show the change from point to point on a line.

M.AAS.8.15 Given the graph of a linear function, determine whether it is increasing or decreasing.

Achievement Elements

Students will be able to determine whether a function is linear, identify the *y*-intercept from the graph, and determine whether the function is increasing or decreasing.

Key Vocabulary

linear, function, *y*-intercept, graph, increasing, decreasing, slope

Teaching and Learning Progressions

- Apply knowledge to a given graphed linear function.
- Know *increasing*, *decreasing*, and *staying the same* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up*.

- Pair *decreasing* with *going down*.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, x-*axis*, y-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.
- Pair the point of intersection with *y*-intercept.
- Locate the point on a linear graph where the line intersects with the *y*-axis.
- Identify a linear graph by identifying it as a straight line on the coordinate grid.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.
- Recognize an equation of the form $y = mx + b$ as linear.

Application of the Alternate Achievement Standards

- Have students compare graphs of various functions (e.g., linear, quadratic) on a coordinate grid.
- Use a picture bingo game to identify parts of a coordinate grid such as origin, *x*-axis, *y*-axis, point, line, curve, and quadrant 1.
- Create a floor grid and have students use material such as string, streamers, or yarn to illustrate linear and nonlinear graphs.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe the graph of a function that models a real-world concept using appropriate vocabulary.	Identify a function as linear, identify the *y*-intercept from the graph, and determine whether the function is increasing or decreasing.	Identify a line graph and determine whether it is increasing or decreasing.	Identify increasing and decreasing slopes of lines.

Grade: 8

Content Area: Mathematics
Strand: Geometry
Objective: Understand congruence and similarity using physical models, transparencies, or geometry software.

General Education Standards

M.8.16 Verify experimentally the properties of rotations, reflections, and translations:
- a. Lines are taken to lines, and line segments are taken to line segments of the same length.
- b. Angles are taken to angles of the same measure.
- c. Parallel lines are taken to parallel lines.

M.8.17 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.18 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

M.8.20 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Alternate Achievement Standards

M.AAS.8.16 Recognize translations and reflections of a non-equilateral rectangle or triangle.

M.AAS.8.17 Given a geometric figure and a vertical or horizontal translation and reflection across a vertical or horizontal line, identify the components (lines/sides, angles) of the two figures that are congruent (limited to non-equilateral rectangles and triangles).

M.AAS.8.18 Recognize the reflection (across the *x*- or *y*-axis) and translation (across quadrants) of a two-dimensional figure on a coordinate plane (limited to non-equilateral rectangles and triangles).

M.AAS.8.20 Compare any angle to a right angle using greater than, less than, or congruent to the right angle.

Achievement Elements

Students will be able to recognize given non-equilateral rectangles or triangles as they are translated or reflected.
Students will be able to compare angles to right angles using greater than, less than, and congruent.

Key Vocabulary

translation, reflection, compare, greater than, less than, congruent

Teaching and Learning Progressions

- Given a right angle and another angle, determine whether the measure of the arc is greater than, less than, or congruent (equal) to the right angle.
- Given various angles, identify the arc/measure of the angle.
- Given various angles, identify the right angle.
- Identify a right triangle as an angle with a 90-degree arc or one formed by two rays that are perpendicular to each other.
- Recognize the measure of an angle as the length of the arc inside the angle.
- Locate the arc of an angle.
- Identify an angle as a figure formed by two rays (lines) that share a common endpoint.
- Know *congruent*.
- Know *greater than* and *less than*.
- Given a figure on a coordinate grid, identify a translation of that figure.
- Given a figure on a coordinate grid, identify a reflection of that figure.
- Pair *reflection* with *flip*.
- Pair *translation* with *shift*
- Know *horizontal* and *vertical*.
- Know *left* and *right*.
- Recognize a given figure in various orientations on a coordinate grid.
    - Know that the coordinates of the vertices of the figure change, but the lengths of the sides and the measures of the angles do not change.
- Recognize characteristics of a figure placed on a coordinate grid.
- Recognize a given figure in various orientations as the same figure.
- Given a geometric figure and its reflection or translation, identify the congruent components/parts.
- Given a figure on a coordinate grid, recognize a translation of that figure.
- Given a figure on a coordinate grid, recognize a reflection of that figure.
- Pair *reflection* with *flip*.
- Pair *translation* with *shift*.
- Know *horizontal* and *vertical*.
- Know *left* and *right*.

- Recognize a given figure in various orientations on a coordinate grid.
    - Know that the coordinates of the vertices of the figure change, but the lengths of the sides and the measures of the angles do not change.
- Recognize characteristics of a figure placed on a coordinate grid.
- Recognize a given figure in various orientations as the same figure.
- Given a non-equilateral rectangle or triangle, identify the reflection of the figure.
- Given a non-equilateral rectangle or triangle, identify the translation of the figure.
- Pair *reflection* with *flipping.*
- Pair *translation* with *shifting.*
- Know *horizontal* and *vertical*.
- Know *left* and *right*.
- Recognize a given non-equilateral rectangle or triangle in various orientations on a coordinate grid.
    - Know that the coordinates of the vertices of the figure change, but the lengths of the sides and the measures of the angles do not change.
- Recognize characteristics of a non-equilateral rectangle or triangle placed on a coordinate grid.
- Recognize a non-equilateral rectangle or triangle in various orientations as the same triangle.

Application of the Alternate Achievement Standards

- Give students multiple opportunities to manipulate triangles and rectangles by placing the shapes in different orientations to demonstrate translation and reflection.
- Have students create artwork using translations and reflections of non-equilateral triangles and rectangles.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Reflect and translate given rectangles and triangles over the *x*- and *y*-axes on a coordinate plane.	Recognize given non-equilateral rectangles or triangles as they are translated or reflected.	Match non-equilateral rectangles or triangles given in various orientations as the same object.	Identify rectangles and triangles.
Measure angles in a given enclosed figure.	Compare angles to right angles using *greater than, less than*, and *congruent.*	Identify an angle given in an enclosed figure.	Identify a line, angle, and point.

Grade: 8

Content Area: Mathematics
Strand: Geometry
Objective: Understand and apply the Pythagorean theorem.

General Education Standard

M.8.22 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

Alternate Achievement Standard

M.AAS.8.22 Use a diagram of two similar right triangles with a simple multiple to find the measure of a missing side length.

Achievement Elements

Students will be able to solve real-world and mathematical problems for perimeter of right triangles given the formula for perimeter**.**

Key Vocabulary

perimeter, length, side, right triangle, Pythagorean Theorem

Teaching and Learning Progressions

- Apply either addition or multiplication to a given perimeter problem.
  - $P = s + s + s$ or $P = 3s$
- Identify the measurement of the given sides of a right triangle.
- Identify the lines that represent the perimeter/boundaries of a given triangle.
- Know that perimeter is the measurement, in units, of the boundary of a two-dimensional shape.

Application of the Alternate Achievement Standards

- Mark off a floor grid using painter's tape. Have students select points for the vertices of a right triangle, count the measurement of the sides, and calculate the perimeter of the triangle they have formed.
- Use geoboards to have students construct right triangles, determine the measure of each side, and calculate each triangle's perimeter.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Given the Pythagorean theorem, solve real-world and mathematical problems using the perimeter of right triangles.	Given the formula for perimeter, solve real-world and mathematical problems for the perimeter of right triangles.	Identify the measurement of a given side of a right triangle.	Identify the sides of a right triangle.

Grade: 8

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Investigate patterns of association in bivariate data.

General Education Standards

M.8.25 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

M.8.28 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Alternate Achievement Standards

M.AAS.8.25 Given a simple scatter plot of points in a straight line, identify a pattern.

M.AAS.8.28 Recognize a display or table constructed from given categorized data; given a two-column table of data, recognize a display or table constructed from that data.

Achievement Elements

Students will be able to recognize a graphic display of a two-column table of data and identify a pattern from the data**.**

Key Vocabulary

scatter plot, data, pattern

Teaching and Learning Progressions

- Make a comparison of the data categorized in a graph or table.
- Identify a real-world situation related to the display or table of data.
- Identify the categories in a display or table of data.
- Recognize a display or table of data.

- Apply knowledge to a given plot of points.
- Know *increasing*, *decreasing*, and *staying the same* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up*.
- Pair *decreasing* with *going down*.
- Know that *y*- values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.

Application of the Alternate Achievement Standards

- Demonstrate creating a scatter plot using student-collected data.
- Have students display data they have collected in a scatter plot by using large grid paper and stickers. Then have students describe the data as increasing, decreasing, or staying the same.
- Using scatter plots from newspapers or magazines, have students explore what the graphs communicate and the trends of the data.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Create a scatter plot using gathered data and answer questions about the data.	Recognize a scatter plot display of a given two-column table of data and identify a pattern from the data.	Identify a given data plot on a three-point scatter plot.	Identify a scatter plot trend as increasing or decreasing.

Grade: 9

Content Area: Mathematics
Strand: Number and Quantity
Objective: The Real Number System:
Extend the properties of exponents to rational exponents.

General Education Standards

M.NS.HS.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

M.NS.HS.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Alternate Achievement Standards

M.AAS.NS.HS.1 Recognize that a number raised to the ½ power is the square root of that number; similarly, a number raised to the 1/3 power is the cube root of that number. Identify the root of a number when given the fractional notation. Limit base values for square roots to 9, 16, 25. Limit base values for cube roots to 8, 27.

M.AAS.NS.HS.2 Determine the value of an expression squared (base values 1-15) or cubed (base values 1-10) using a calculator.

Achievement Elements

Students will be able to square and cube values up to 15 and determine the square root of the perfect squares up to 225.

Key Vocabulary

square, cube

Teaching and Learning Progressions

- Solve expressions of the form x^2 and x^3 cubed.
- Pair x^2 with x times x times x.
- Pair x^2 with x times x.
- Multiply numbers up to 15 by each other.
- Using mathematical tools, use the "guess and check" method and the "calculating perfect squares method" to find the square root of a given perfect square.
- Recognize that the number being squared is the *square root* of the perfect square.

- Calculate the perfect squares for integers 2 through 15.
- Identify x^2 as a *perfect square*.
- Understand *a number squared.*

Application of the Alternate Achievement Standards

- Have students use mathematical tools to calculate squares and cubes of numbers up to 15.
- Create arrays to demonstrate squaring and cubing numbers.
- Use a multiplication table to locate numbers multiplied by themselves and connect to squaring and square roots.

Levels of Standards

Level 4: Exceed Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the value of *x* raised to a power not to exceed 5 using a calculator.	Square and cube whole numbers 1–15 using a calculator.	Multiply a number times itself up to three times using a calculator.	Multiply a number times itself using a calculator.
Determine the square root of numbers up to 100 using a mathematical tool and rounding to the hundredths, using a calculator.	Determine the square roots of perfect squares up to and including 225 using a calculator.	Using mathematical tools, determine the square root of perfect squares up to and including 100 using a calculator.	Using mathematical tools, determine the square root of perfect squares up to 25 using a calculator.

Grade: 9

Content Area: Mathematics
Strand: Number and Quantity
Objective: The Real Number System:
Use properties of rational and irrational numbers.

General Education Standard

M.NS.HS.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Alternate Achievement Standard

M.AAS.NS.HS.3 Identify rational and irrational numbers within 1 to 20 (irrational numbers limited to pi, √2, √3, √5).

Achievement Elements

Students will be able to identify a number as rational or irrational.

Key Vocabulary

rational, irrational, pi

Teaching and Learning Progressions

- Identify a given number as rational or irrational (using common irrational numbers).
- Identify common irrational numbers 1–20.
- Know that irrational numbers cannot be made by dividing two integers a/b, where b is not 0.
- Know that a rational number is one that can be made by dividing two integers a/b, where b is not 0.
- Know that an integer is a number with no fractional part.

Application of the Alternate Achievement Standards

- Use a bingo game to practice rational and irrational numbers.
- Use a number line to order numbers up to 20 including whole numbers, fractions, square roots, and pi.
- Have students compare fractions and decimals.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Express an irrational number as a decimal rounded to hundredths.	Identify a number as rational or irrational.	Identify whole numbers, decimals, fractions, and square roots using a number line.	Identify fractional representations of whole numbers.

Grade: 9

Content Area: Mathematics
Strand: Quantities
Objective: Reason quantitatively and use units to solve problems.

General Education Standards

M.Q.HS.4 Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.Q.HS.5 Define appropriate quantities for the purpose of descriptive modeling.
M.Q.HS.6 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Alternate Achievement Standards

M.AAS.Q.HS.4 Using real world models, express quantities of measurement to the given precision. (limited to measurements of length (inch, 1/2 inch, 1/4 inch), weight (pounds, kilograms (tenth of a unit), volume (cup, 1/2 cup, 1/4 cup, 1/3 cup, liter), temperature (degree), velocity (mph, kmph).
M.AAS.Q.HS.5 Recognize units of weight (ounces, pounds, grams, kilograms), length (inch, foot, mile, centimeter, meter, kilometer), area (square inches in^2, square feet ft^2, square centimeters cm^2, square meters m^2) and capacity (cubic inches in^3, cubic feet ft^3, cubic centimeters cm^3, cubic meters m^3).
M.AAS.Q.HS.6 Estimate to the nearest 1, 10, and 100 when adding, subtracting, multiplying, or dividing; include units with estimates.

Achievement Elements

Students will be able to recognize and express quantities to the appropriate precision of measurement in real-world context.

Key Vocabulary

round, rounding, weight, pound, ounce, length, inch, foot, yard, capacity, cup, liter, quart, gallon

Teaching and Learning Progressions

- Round given measurements to the given whole, decimal, or fractional part.
- Read the appropriate scales for each measuring tool.
- Recognize appropriate measuring tools for each type of measurement.
- Know common units of measure related to length, weight, and volume.
- Recognize the common units of volume.
- Know that volume (capacity) is the amount of space within a container.
- Recognize the common units of weight.
- Know that weight is the heaviness of an object.
- Recognize the common units of measurement.
- Know that length is the measurement of something from end to end (objects, time).
- Perform a given operation on estimates.
- Given a set of numbers to add, subtract, multiply, or divide, round each number to the nearest indicated value.
- Estimate whole numbers to the nearest 100.
- Estimate whole numbers to the nearest 10.
- Estimate mixed numbers and decimals to the nearest whole number.
- Understand what fractional parts represent.
- Understand what decimal parts represent.
- Add, subtract, multiply, and divide fluently.

Application of the Alternate Achievement Standards

- Use tens bars to estimate values between 1 and 100 to the nearest 10.
- Use fraction/decimal bars to estimate values to the nearest 1, including mixed numbers.
- Demonstrate measurement to the appropriate precision using real-world objects in the classroom.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Make appropriate measurements in a real-world context and express them to the appropriate precision of measurement.	Recognize and express quantities to the appropriate precision of measurement in real-world context.	Recognize the appropriate unit and subunits of measure for weight, length, and volume.	Recognize the appropriate tools of measurement for weight, length, and volume.
Estimate the results of addition, subtraction, multiplication, or division problems and compare to the actual calculation.	Estimate to the nearest 1, 10, and 100 in addition, subtraction, multiplication, or division problems.	Estimate quantities to the nearest 1, 10, and 100.	Estimate values between 1 and 100 to the nearest 10.

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Seeing structure in expressions.
Interpret the structure of expressions.

General Education Standards

M.A.HS.7 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Alternate Achievement Standard

M.AAS.A.HS.7 Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.

Achievement Elements

Students should identify a one-step algebraic expression representing a real-world problem.

Key Vocabulary

Expression, variable, constant, coefficient

Teaching and Learning Progressions

- Determine the relationship between the information given (arithmetic operation to be used).
- Identify which (if any) information is unknown.
- Identify which information is known.
- In a real-world scenario, identify the important information needed to solve the problem.
- Understand that variables (letters) represent numbers in math.

Application of the Alternate Achievement Standards

- Create a real-world context and expression and determine the meaning of each part of the expression.
- Calculate amounts using savings accounts given a starting amount as constant.
- Model real-life situations (e.g., job earnings).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the meanings of coefficients, variables, terms, and constants in an expression based on real-world contexts.	Identify a one-step algebraic expression representing a real-world problem.	Identify the meaning of the unknown in a real-world context.	Given an expression, identify a part (variable, term, constant).

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Seeing structure in expressions.
Write expressions in equivalent forms to solve problems.

General Education Standards

M.A.HS.9 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal and explain properties of the quantity represented in the expression.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Determine a quadratic equation when given its graph or roots.
d. Use the properties of exponents to transform expressions for exponential functions.

Alternate Achievement Standard

M.AAS.A.HS.9 Identify the expression that is the same as the one shown. (limit to two operations e.g. x^2 + 3x is the same as x(x+3).

Achievement Elements

Students will be able to solve simple algebraic equations with one variable using multiplication and division**.**

Key Vocabulary

equation, variable, multiply, divide, substitution

Teaching and Learning Progressions

- Know that solving an equation involving division involves multiplying both sides by the denominator.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Understand the terms in the equation as "x divided by a number (p) equals a new number q."
- Recognize $x/p = q$ as an equation.
- Understand the terms in the equation as "a number (p) multiplied by x equals a new number q."
- Recognize p as the coefficient of x.
- Recognize $px = q$ as an equation.

Application of the Alternate Achievement Standards

- Create a real-world context and equation and determine the meaning of each part of the equation.
- Use manipulatives and symbols to represent simple equations and solve those equations.
- Demonstrate real-world contextual equations and how to solve them using items and context of interest to the student.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve simple two-step algebraic equations.	Solve simple algebraic equations with one variable using multiplication and division.	Solve simple algebraic equations with one variable using addition and subtraction.	Use substitution to simplify an algebraic expression with one variable.

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Arithmetic with Polynomials and Rational Expressions: Rewrite rational expressions

General Education Standard

M.A.HS.11 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Alternate Achievement Standard

M.AAS.A.HS.11 Add or subtract two polynomial expressions (limit to 2 terms each) with one variable.

Achievement Elements

Students will be able to identify the coordinates of a point on a line on a coordinate grid.

Key Vocabulary

coordinates, point, line, grid, ordered pair

Teaching and Learning Progressions

- Given the graph of a line on a coordinate grid, identify the coordinates of the indicated point on that line.
- Given an ordered pair, locate the point on a labeled coordinate grid.
- Given the *x*- and *y*-values in an ordered pair, locate the point on a labeled coordinate grid.
- Know that the second number in an ordered pair is the number of units moved on the *y*-axis to be in line with the given point.
- Know that the first number in an ordered pair is the number of units moved on the *x*-axis to be in line with the given point.
- Pair *ordered pair* with the notation (#, #).
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.

Application of the Alternate Achievement Standards

- Create a coordinate grid and a line on the floor with painter's tape to demonstrate finding various points on the line.
- Have students create a grid map of the area around their school or home and locate various landmarks.
- Have students use colored pencils to draw the rise and run associated with a given point on a line on the coordinate grid.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify the graph of a simple equation in two variables.	Identify the coordinates of a point on a line on a coordinate grid.	Recognize that a line is made up of points.	Locate points on a coordinate grid given the coordinates of the point.

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Reasoning with Equations and Inequalities:
Solve equations and inequalities in one variable.

General Education Standards

M.A.HS.17 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

M.A.HS.18 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation.

Alternate Achievement Standards

M.AAS.A.HS.17 Solve an equation of the form $ax + b = c$ where a, b, and c are positive whole numbers and the solution, x, is a positive whole number to represent a real-world problem.

M.AAS.A.HS.18 Solve an equation of the form $x^2 = p$, where p is a perfect square less than or equal to 225 using a calculator.

Achievement Elements

Students will be able to solve an equation of the form $ax + b = c$.
Students will be able to solve an equation of the form $x^2 = p$.

Key Vocabulary

solve, equation, perfect square

Teaching and Learning Progressions

- Recognize the result as the solution, $x =$___.
- Use division on the equation to solve the second step of the equation.

- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Use subtraction on a given equation to solve the first step in a two-step equation.
- Know that solving an equation involving addition involves subtracting b from both sides.
- Understand the terms in the equation as "a number times a given number x plus a number b equals a new number c."
- Solve equations of the form $x^2 = p$.
- Pair x^2 with x times x.
- Know that x represents a given number and p represents the number that results when x is squared.
- Recognize $x^2 = p$ as an equation.
- Know that the letters x and p can represent numbers in math.
- Multiply numbers up to 15 by each other.
- Given a one-step equation $px = q$, identify an equivalent equation of the form $x = q/p$.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Understand the terms in the equation as "a number (p) multiplied by x equals a new number q."
- Given a one-step equation $x - p = q$, identify an equivalent equation of the form $s = p + q$.
- Know that solving an equation involving subtraction involves adding p to both sides.
- Understand the terms in the equation as "x minus a number (p) equals a new number q."
- Given a one-step equation $x + p = q$, identify an equivalent equation of the form $x = q - p$.
- Know that solving an equation involving addition involves subtracting p from both sides.
- Understand the terms in the equation as "x plus a number (p) equals a new number q."

Application of the Alternate Achievement Standards

- Use a variety of transparency/computer program overlays to show how to solve an equation.
- Have students use a variety of resources, including a calculator, to multiply a number times itself.
- Have students use a variety of resources, including a calculator, to divide a number by two.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve equations of the form $x + b = c$, $x - b = c$, $ax + b = c$, and $x/a + b = c$.	Solve simple equations of the form(s): $x + b = c$, $ax + b = c$, and $x^2 = p$ in real-world context.	Solve simple equations of the form $x + b = c$ in real-world context.	Find solutions for the equation $a + b = c$ when a and c are given.
Solve an equation of the form x^2.	Solve an equation of the form $x^2 = p$ where p is a perfect square.	Multiply a number by itself.	Understand that x^2 means *x times x.*

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Reasoning with Equations and Inequalities:
Solve systems of equations.

General Education Standards

M.A.HS.19 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

M.A.HS.20 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

M.A.HS.21 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Alternate Achievement Standards

M.AAS.A.HS.19 Given a pair of equations, identify a coordinate pair that is the solution of both equations. (Limit to one-step equations: ($x + y = 8$, $y = x + 1$, or $x + y = 15$, $y = 3$.)

M.AAS.A.HS.20 Name the coordinate pair of the intersection of two lines in a coordinate plane.

M.AAS.A.HS.21 Identify the coordinate pairs of the solutions of the graph of an intersecting quadratic function and linear function in Quadrant 1.

Achievement Elements

Students will be able to solve simple one- and two-step equations.
Students will be able to identify points of intersection on a coordinate plane.
Students will be able to identify the coordinate pairs of the solution of the graph of an intersecting quadratic function and linear function.

Key Vocabulary

solve, equation, solution, intersection, coordinate pairs, quadratic function, linear function

Teaching and Learning Progressions

- Recognize the result as the solution, $x =$___.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Use subtraction on a given equation to solve the first step in a two-step equation.
- Know that solving an equation involving addition involves subtracting b from both sides.
- Pair an equation of the form $ax + b = c$ with a real-world problem.
- Understand the terms in the equation as "a number times a given number x plus a number b equals a new number c."
- Solve equations of the form $x^2 = p$.
- Pair x^2 with x times x.
- Know that x represents a given number and p represents the number that results when x is squared.
- Recognize $x^2 = p$ as an equation.
- Know that the letters x and p can represent numbers in math.
- Multiply numbers up to 15 by each other.
- Given a one-step equation $px = q$, identify an equivalent equation of the form $x = q/p$.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Understand the terms in the equation as "a number (p) multiplied by x equals a new number q."
- Given a one-step equation $x – p = q$, identify an equivalent equation of the form $s = p + q$.
- Know that solving an equation involving subtraction involves adding p to both sides.
- Understand the terms in the equation as "x minus a number (p) equals a new number q."
- Given a one-step equation $x + p = q$, identify an equivalent equation of the form $x = q - p$.
- Know that solving and equation involving addition involves subtracting p from both sides.
- Understand the terms in the equation as "x plus a number (p) equals a new number q."
- Given two intersecting lines on a coordinate grid, name the point of intersection by its corresponding ordered pair.
- Given two intersecting lines on a coordinate grid, locate point(s) of intersection.
- Know *intersect* and *intersection*.
- Identify lines on a coordinate grid.
- Given an ordered pair, locate a point on a labeled coordinate grid.
- Given the x- and y-values in an ordered pair, locate a point on a labeled coordinate grid.
- Know that the second number in an ordered pair is the number of units moved on the y-axis to be in line with the given point.
- Know that the first number in an ordered pair is the number of units moved on the x-axis to be in line with the given point.
- Pair *ordered pair* with the notation (#, #).
- Locate point(s) of intersection on a coordinate grid, given a graph of two linear functions on the same grid.

- Identify point(s) of intersection on a coordinate grid, given a graph of a linear function and a graph of a quadratic function on the same grid.
- Recognize that linear function straight lines and quadratic function curved lines can be graphed on a coordinate grid.
- Recognize a coordinate grid.
- Identify a point on the graph that is a solution to an equation.
- Know that all the points on a line represent solutions to an equation.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Identify the point or shaded region that is a solution to a given system of equations on a coordinate grid.
- Identify the intersection of shaded regions on a coordinate grid as the solution to a system of two linear equations using < or >.
- Identify a shaded region of a coordinate grid as the solution of a linear equation graph using < or >.
- Identify the point of intersection of two lines representing linear equations as a solution to the system of equations.

Application of the Alternate Achievement Standards

- Use a variety of transparency/computer program overlays to illustrate intersection of lines on a coordinate plane.
- Use classroom and real-world events to model equations and demonstrate solving those equations step by step.
- Use a variety of graphs to identify where two lines intersect on a coordinate plane.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Solve equations of the form $x + b = c$, $x - b = c$, $ax + b = c$, and $x/a + b = c$.	Solve simple equations of the form(s): $x + b = c$, $ax + b = c$, and $x^2 = p$ in real-world context.	Solve simple equations of the form $x + b = c$ in real-world context.	Find solutions for the equation $a + b = c$ when a and c are given.
Identify the points of intersection for any two geometric figures on a coordinate grid.	Identify the points of intersection of a quadratic function and a linear function.	Identify the point where two lines on the coordinate grid intersect as a coordinate pair.	Identify the point where two lines on the coordinate grid intersect.

Grade: 9

Content Area: Mathematics
Strand: Algebra
Objective: Reasoning with Equations and Inequalities:
Represent and solve equations and inequalities graphically.

General Education Standards

M.A.HS.22 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

M.A.HS.23 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

M.A.HS.24 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Alternate Achievement Standards

M.AAS.A.HS.22 Given the graph of a linear equation in quadrant 1, identify a point on the graph and its corresponding ordered pair that is a solution to the equation.

M.AAS.A.HS.23 Identify the point of intersection and its corresponding ordered pair for two lines graphed on a coordinate grid.

M.AAS.A.HS.24 Given the graph of a linear system of inequalities (limited to two inequalities), identify a point that represents a solution in the shaded region of the graph.

Achievement Elements

Students will be able to identify the following on a coordinate plane: point(s) of intersection, points that are a solution to an equation, and shaded regions that are a solution to a system of equations.

Key Vocabulary

graph, linear, equation, point, ordered pair, solution, equation, intersection, coordinate grid, inequalities

Teaching and Learning Progressions

- Recognize the result as the solution, $x =$___.
- Use division on the equation to solve the second step of the equation.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Use subtraction on a given equation to solve the first step in a two-step equation.
- Know that solving an equation involving addition involves subtracting b from both sides.
- Pair an equation of the form $ax + b = c$ with a real-world problem.
- Understand the terms in the equation as "a number times a given number x plus a number b equals a new number c."
- Solve equations of the form $x^2 = p$.
- Pair x^2 with x times x.
- Know that x represents a given number and p represents the number that results when x is squared.
- Recognize $x^2 = p$ as an equation.
- Know that the letters x and p can represent numbers in math.
- Multiply numbers up to 15 by each other.
- Given a one-step equation $px = q$, identify an equivalent equation of the form $x = q/p$.
- Know that solving an equation involving multiplication involves dividing both sides by the coefficient of x.
- Understand the terms in the equation as "a number (p) multiplied by x equals a new number q."
- Given a one-step equation $x – p = q$, identify an equivalent equation of the form $s = p + q$.
- Know that solving an equation involving subtraction involves adding p to both sides.
- Understand the terms in the equation as "x minus a number (p) equals a new number q."
- Given a one-step equation $x + p = q$, identify an equivalent equation of the form $x = q – p$.
- Know that solving an equation involving addition involves subtracting p from both sides.
- Understand the terms in the equation as "x plus a number (p) equals a new number q."
- Given two intersecting lines on a coordinate grid, name the point of intersection by its corresponding ordered pair.
- Given two intersecting lines on a coordinate grid, locate point(s) of intersection.
- Know *intersect* and *intersection*.
- Identify lines on a coordinate grid.
- Given an ordered pair, locate a point on a labeled coordinate grid.

- Given the *x*- and *y*-values in an ordered pair, locate a point on a labeled coordinate grid.
- Know that the second number in an ordered pair is the number of units moved on the *y*-axis to be in line with the given point.
- Know that the first number in an ordered pair is the number of units moved on the *x*-axis to be in line with the given point.
- Pair *ordered pair* with the notation (#, #).
- Identify point(s) of intersection on a coordinate grid, given a graph of a linear function and a graph of a quadratic function on the same grid.
- Locate point(s) of intersection on a coordinate grid, given a graph of two linear functions on the same grid.
- Recognize that linear function (straight line) and quadratic function (curved line) can be graphed on a coordinate grid.
- Recognize a coordinate grid.
- Identify a point on the graph that is a solution to an equation.
- Know that all the points on a line represent solutions to an equation.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Identify the point or shaded region that is a solution to a given system of equations on a coordinate grid.
- Identify the intersection of shaded regions on a coordinate grid as the solution to a system of two linear equations using < or >.
- Identify a shaded region of a coordinate grid as the solution of a linear equation graph using < or >.
- Identify the point of intersection of two lines representing linear equations as a solution to the system of equations.

Application of the Alternate Achievement Standards

- Use a variety of transparency/computer program overlays to illustrate intersection of various functions, including < and >.
- Use classroom and real-world events to model equations and demonstrate solving those equations step by step.
- Use a variety of graphs and have students identify the point at which the lines intersect.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Find the solution to a system of linear equations using < or >.	Identify the solution (point or shaded area) to a system of linear equations (<, >, =).	Identify the point where two lines on the coordinate grid intersect as a coordinate pair.	Identify the point where two lines on the coordinate grid intersect.
Solve equations of the form $x + b = c$, $x - b = c$, $ax + b = c$, and $x/a + b = c$.	Solve simple equations of the form(s): $x + b = c$, $ax + b = c$, and $x^2 = p$ in real-world context.	Solve simple equations of the form $x + b = c$ in real-world context.	Find solutions for the equation $a + b = c$ when a and c are given.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Interpreting Functions:
Understand the concept of a function and use function notation.

General Education Standards

M.F.HS.25 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

M.F.HS.26 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

M.F.HS.27 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Alternate Achievement Standards

M.AAS.F.HS.25 Identify graphs of functions.

M.AAS.F.HS.26 Substitute *x*-values into one-step linear equations in two variables (*y* = *x* + *p* or *y* = *px)* and solve for the *y*-values. (this could include the original information listed above and have students represent in data table)

M.AAS.F.HS.27 Given a sequence of numbers, identify the rule that will give you the next number in the sequence. (Limit to expressions with simple arithmetic (adding or subtracting) or geometric (multiplying or dividing) operations.

Achievement Elements

Students will be able to apply a function rule to a set of data to create a data table (limited to addition/subtraction and numbers less than 10).

Students will be able to substitute *x*-values into one-step linear equations involving multiplication and addition in two variables and solve for *y*.

Students will be able to describe the rule in a simple arithmetic (+/-) or geometric (multiplying) sequence (limited to positive numbers).

Key Vocabulary

data table, function, solve, substitute, arithmetic sequence, geometric sequence

Teaching and Learning Progressions

- Complete a data table by selecting *x*-values less than 10, applying the function rule, and determining the *y*-value.
- Create a three-column data table (*x*, function rule, *y*)
- Recognize $y = x + p$ and $y = px$ as *input*/*output* functions where *x*-values are the input which result in the output, the *y*-values.
- Recognize that letters may stand for numbers in math.
- Input *x*-values to obtain *y*-values in given one-step linear equations.
- Recognize $y = x + p$ and $y = px$ as *input*/*output* functions where *x*-values are the input which result in the output, the *y*-values.
- Recognize that letters may stand for numbers in mathematics.
- Describe the rule for a simple geometric sequence.
- Determine the multiplication factor of a simple, given geometric sequence.
- Recognize a geometric sequence.
- Describe the rule for a simple arithmetic sequence.
- Determine the interval and operation in a simple arithmetic sequence.
- Recognize a simple arithmetic sequence.
- Compare numbers in a list to the number that precedes and follows a given number (smaller/larger, greater/less).
- Fluently add, subtract, and multiply whole numbers.

Application of the Alternate Achievement Standards

- Have students draw a number chip from a bag and apply a simple addition or subtraction rule to create a data table.
- Have students generate a number pattern for *x*-values (e.g., odd numbers, even numbers, multiples of 2) on a data table and apply a simple addition or subtraction rule (up to 10) to identify the matching *y*-values.
- Have students measure their hand-span for the *x*-values on a data table and apply a simple addition or subtraction rule to identify the matching *y*-values.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Apply a function rule to a set of data to create a data table and display the data in a graph.	Apply a function rule to a set of data to create a data table (limited to addition/subtraction and numbers less than 10).	Recognize a function rule in a data table.	Given a data table, identify the set of *x*-values.
Determine the rule for a given data table and find missing *x*-values.	Substitute *x*-values into one-step linear equations involving multiplication and addition in two variables and solve for *y*.	Determine whether two expressions are equal (commutative, associative properties).	Determine the solution for a simple expression using addition and subtraction.
Create a sequence of no less than five items by applying a geometric rule to a given starting point.	Describe the rule in a simple arithmetic (+/-) or geometric (multiplying) sequence (limited to positive numbers).	Describe the rule in a simple arithmetic sequence limited to numbers less than 50.	Create a sequence using + 1, +2, or +3 with a given starting point less than 20.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Interpreting Functions:
Interpret functions that arise in applications in terms of the context.

General Education Standards

M.F.HS.28 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

M.F.HS.29 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

M.F.HS.30 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Alternate Achievement Standards

M.AAS.F.HS.28 Given a linear graph, identify characteristics of the line in the graph (limit to *y*-intercept, *x*-intercept, increasing, decreasing).

M.AAS.F.HS.29 Given the graph of a linear function with a finite domain evident in the graph, identify the domain (limit to first quadrant values between 0 and 10).

M.AAS.F.HS.30 Given two points on the graph of the line, describe how the *y*-values change compared to the *x*-values for a given rate of change.

Achievement Elements

Students will be able to compare the rate of change of linear functions (faster/slower).
Students will be able to identify the domain of a given graph of a linear function.
Students will be able to compare the values of *x* and *y* in two points on a graph of a linear function over the interval.

Key Vocabulary

linear function, rate of change, domain, interval

Teaching and Learning Progressions

- Pair the comparison to the appropriate interpretation for comparisons such as faster/slower and higher/lower.
- Compare the *steepness* of two given lines with regard to *more* and *less*.
- Recognize that two lines may have different slope (i.e., *steepness*).
- Recognize the *steepness* of a given line.
- Construct lines on a coordinate grid given the *x*- and *y*-values in a chart/table or ordered pairs.
- Recognize that more than one line can be drawn on the same coordinate grid.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Given the *x*- and *y*-values in a chart/table or ordered pairs, locate points on a labeled coordinate grid.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Pair the number of units of change in the *x*-values with *domain*.
- Determine the number of units of change in the *x*-values.
- Identify the *x*-values that correspond to the beginning point and ending point on a coordinate grid.
- Locate the beginning and ending points of a line segment on a coordinate grid.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Describe how the *y*-values change compared to the *x*-values for a given interval.
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up*, *positive*, and *getting larger*.
- Pair *decreasing* with *going down*, *negative*, and *getting smaller*.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Given the *x*- and *y*-values in an ordered pair, locate a point on a labeled coordinate grid.
- Know that the second number in an ordered pair is the number of units moved on the *y*-axis to be in line with the given point.
- Know that the first number in an ordered pair is the number of units moved on the *x*-axis to be in line with the given point.
- Pair *ordered pair* with the notation (#, #).
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.

- Know *origin, x*-axis, *y*-axis, *horizontal*, and *vertical.*
- Recognize a coordinate grid.

Application of the Alternate Achievement Standards

- Color code the horizontal (*x*-values) and vertical (*y*-values) on a coordinate grid. Have students plot points to represent a rate of change of a linear function (representing a real-world situation with each student's data different) in a third color. Have students "meet and greet" and compare their graphs to identify faster/slower.
- Demonstrate real-world rate of change of a linear function, and identify the domain.
- Using a painter's tape coordinate grid, demonstrate comparing the *x*- and *y*-values of two given points by having students move in the appropriate horizontal or vertical directions.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Answer the questions "How much faster/slower" when comparing the rate of change of two linear functions.	Compare the rate of change of linear functions (faster/slower) shown by a graphic display.	Compare the slopes of two lines shown on a coordinate grid.	Recognize the slope of a line.
Identify the domain and range of a given graph of a linear function.	Identify the domain of a given graph of a linear function.	Identify the lowest and greatest numbers in the *x*-values of a data set of ordered pairs.	Identify the starting and ending points of the *x*-values on a given graph.
Given a graph of a linear function, determine the slope of the line.	Compare the values of *x* and *y* in two points on a graph of a linear function over the interval.	Graph two lines on the same coordinate grid given two points for each line.	Draw a line given two points (ordered pairs) on the line.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Interpreting Functions:
Analyze functions using different representations.

General Achievement Standards

M.F.HS.31 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions, and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

M.F.HS.32 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions for exponential functions.

M.F.HS.33 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Alternate Achievement Standards

M.AAS.F.HS.31 Given the graph of a quadratic function, identify characteristics of the parabola in the graph (limit to first quadrant, maximum, minimum, *x*-intercepts).

M.AAS.F.HS.32 Identify the *y*-intercept of a linear equation in the form of $y=mx+b$ as $(0, b)$.

M.AAS.F.HS.33 Compare the *y*-intercept, slope (increasing/decreasing), or domain of two linear functions represented by a table or a graph.

Achievement Elements

Students will be able to identify the *y*-intercept when given a graph of a linear function and know that a linear function of the form $y = mx + b$ will have the *y*-intercept of (0, *b*).
Students will be able to compare two linear functions by considering tables or graphs.

Key Vocabulary

graph, linear function, *y*-intercept, compare, parabola, maximum, minimum

Teaching and Learning Progressions

- Compare the slope of two lines presented in table or graphic form.
- Calculate the slope of a linear function presented in a table.
- Identify the slope of a linear function presented as a graph.
- Know slope.
- Identify a linear function (from a graph or table).
- Pair *y*-intercept with *b* in a linear equation.
- Know that the point where a line crosses the *y*-axis is called the *y*-intercept.
- Locate the point where the line crosses the *y*-axis.
- Pair *slope* with *m* in a linear equation.
- Recognize that lines can be represented by linear equations of the form $y = mx + b$.
- Express slope as *rise/run.*
- Use a grid to determine run (how much it changes from left to right).
- Use a grid to determine rise (how much it changes from bottom to top).
- Know *increasing* and *decreasing* in relation to moving across a coordinate grid from left to right.
- Pair *increasing* with *going up* and *positive*.
- Pair *decreasing* with *going down* and *negative*.
- Recognize that a line can be drawn between two given points on a coordinate grid.
- Given the *x*- and *y*-values in an ordered pair, locate the point on a labeled coordinate grid.
- Know that the second number in an ordered pair is the number of units moved on the *y*-axis to be in line with the given point.
- Know that the first number in an ordered pair is the number of units moved on the *x*-axis to be in line with the given point.
- Pair *ordered pair* with the notation (#, #).

Application of the Alternate Achievement Standards

- Highlight and classify the slope of a line on graphs and in equations.
- Create a matching game using *y*-intercept cards and graphic representations of linear functions cards.
- A *y*-intercept search: present students with tables representing linear functions. As each student draws a card of the ordered pair representing the *y*-intercept of a line, have the student find the *y*-intercept in a table.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe what the *y*-intercept means when given a graph of a linear function that represents a real-world situation.	Identify the *y*-intercept when given a graph of a linear function.	Identify the point where two lines intersect on a coordinate grid.	Identify a point on a line.
Know that *m* represents the slope in a linear function represented as $y = mx + b$.	Know that the *y*-intercept of a linear function represented as $y = mx + b$ is the point $(0, b)$.	Know that a linear function can be represented as $y = mx + b$.	Identify the equation of a simple function.
Explain how two linear functions are the same and/or different using slope and *y*-intercept.	Compare two linear functions in terms of their *y*-intercepts and slopes by considering tables or graphs.	Compare two linear functions in terms of slope by considering a table.	Compare two linear functions in terms of slope by considering a graph.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Building functions:
Build a function that models a relationship between two quantities.

General Education Standards

M.F.HS.34 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

M.F.HS.35 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Alternate Achievement Standards

M.AAS.F.HS.34 Select the appropriate graphical representation (first quadrant) given a situation involving a constant rate of change (slope).

M.AAS.F.HS.35 Determine an arithmetic sequence with whole numbers when provided a recursive rule. (limit rule to whole numbers involving addition/subtraction or multiplication or division - e.g., start with the number 4. Each term in the sequence is found by taking the previous term and adding 8. Find the next 3 terms.)

Achievement Elements

Students will be able to select the appropriate graph (in the first quadrant) of a situation representing change of rate.
Students will be able to determine an arithmetic sequence with whole numbers when provided a starting point and recursive rule.

Key Vocabulary

sequence, rate of change, recursive rule, graph, function

Teaching and Learning Progressions

- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.
- Understand and apply a recursive rule.
- Fluently add or subtract whole numbers.
- Apply previous knowledge to graphical representations of real-world data.
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up* and *positive*.
- Pair *decreasing* with *going down* and *negative*.
- Recognize a graph/scatter plot of data related to a real-world context.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, x-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.

Application of Alternate Achievement Standards

- Describe and discuss real-world graphs showing rate of change from such sources as newspapers, magazines, or the Internet.
- Create arithmetic sequences and discuss *n*th terms in the sequence in relation to the starting point using objects (e.g., chips, counting blocks).
- Create tables of values for $f(x)$ and then $f(x) + 1$, 2, or 3. Show what happens to the graph using transparency overlays.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Using a real-world situation, create the data table for a given situation representing rate of change.	Select the appropriate graph (in the first quadrant) of a situation representing rate of change.	Identify a graph showing an increasing or decreasing slope.	Given two points on a coordinate grid, draw the line.
Determine the *n*th term (up to 10) when provided a starting point and a recursive rule.	Determine an arithmetic sequence with whole numbers when provided a starting point and recursive rule.	Match a rule to a given sequence.	Create a three-term sequence using +1, +2, and +3.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Building functions:
Build new functions from existing functions.

General Education Standard

M.F.HS.36 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Alternate Achievement Standard

M.AAS.F.HS.36 Given the graph of a linear function $f(x)$, identify $f(x) + k$.

Achievement Elements

Students will be able to identify the graph of a linear function $f(x) + k$ when given the graph of $f(x)$.

Key Vocabulary

graph, linear, function

Teaching and Learning Progressions

- Understand that $f(x) + k$ will move the graph k units up on the coordinate grid.
- Identify the graph of $f(x)$.
- Know that y-values increase vertically from bottom to top.
- Know that x-values increase horizontally from left to right.
- Know *origin*, x-axis, y-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.
- Apply previous knowledge to graphical representations of real-world data.
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.

- Pair *increasing* with *going up* and *positive*.
- Pair *decreasing* with *going down* and *negative*.
- Recognize a graph/scatter plot of data related to a real-world context.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*.
- Recognize a coordinate grid.

Application of Alternate Achievement Standards

- Create tables of values for $f(x)$ and then $f(x) + 1$, 2, or 3.
- Show what happens to the graph using transparency overlays for the increased values of k.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Create the graph of $f(x) + k$ given the graph of $f(x)$.	Identify the graph of a linear function $f(x) + k$ when given the graph of $f(x)$.	Identify the graph of a linear function $f(x)$.	Identify a line on a coordinate grid.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Linear, quadratic, and exponential models:
Construct and compare linear, quadratic, and exponential models and solve problems.

General Education Standards

M.F.HS.37 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one-quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

M.F.HS.38 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

M.F.HS.39 Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Alternate Achievement Standards

M.AAS.F.HS.37 Recognize real-world situations that are modeled with linear functions.
M.AAS.F.HS.38 Identify three points defined by a linear function from a table of values from 0 to 10.
M.AAS.F.HS.39 Given the graph of two functions, identify which function has a greater *y*-value for a specific *x*-value.

Achievement Elements

Students will be able to recognize real-world situations that are modeled with linear or exponential functions.
Students will be able to plot three points defined by a linear function from a table.
Students will be able to identify which of two functions has the greater *y*-value for a specific *x*-value on a given graph.

Key Vocabulary

linear, exponential, table, function

Teaching and Learning Progressions

- Apply previous knowledge to identify which function has a greater *y*-value for a given *x*-value.
- Recognize the graphs of functions.
- Identify points that have a greater *y*-value for the same *x*-value.
- Know *greater* in the context of the coordinate grid.
- Describe a point on the grid by its *x*-value and its *y*-value (ordered pair).
- Given the *x*- and *y*-values on a table, locate the point on a labeled coordinate grid.
- Given the *x*- and *y*-values, locate the point on a labeled coordinate grid.
- Identify characteristics of exponential functions and real-world applications.
- Identify characteristics of linear functions and real-world applications.
- Identify exponential functions graphed on a coordinate grid.
- Identify linear functions graphed on a coordinate grid.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin, x*-axis, *y*-axis*, horizontal,* and *vertical.*
- Recognize a coordinate grid.

Application of Alternate Achievement Standards

- Draw attention to real-world situations that can be modeled with linear or exponential functions.
- Use real-world graphs, constructed graphs, or floor grids to locate lines and points on a line and discuss the meaning of the data.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine which graph represents a given real-world situation modeled by an exponential function.	Recognize real-world situations that are modeled with linear or exponential functions.	Recognize a real-world situation that cannot be modeled with a graph of a linear function (straight line).	Differentiate graphs that are straight lines and curves.
Determine a simple function given a table of values for that function (up to 10).	Plot three points defined by a linear function from a table of values up to 10.	Identify up to three points on a given graph of a linear function, given the ordered pair.	Identify a point in the first quadrant of a coordinate grid, given the ordered pair.
Compare the graphs of linear and exponential functions.	Identify which of two functions has the greater *y*-value for a specific *x*-value on a given graph.	Identify two functions on a given graph where the *x*- and *y*-values are equal.	Recognize two lines on a graph that have the same or different slopes.

Grade: 10

Content Area: Mathematics
Strand: Functions
Objective: Interpret expressions for functions in terms of the situations they model.

General Education Standard

M.F.HS.40 Interpret the parameters in a linear or exponential function in terms of a context.

Alternate Achievement Standard

M.AAS.F.HS.40 Identify rate of change (slope) and starting value (*y*-intercept) in context.

Achievement Elements

Students will be able to identify rate of change (slope) and starting value (*y*-intercept) in context.

Key Vocabulary

rate of change, slope, starting value, *y*-intercept

Teaching and Learning Progressions

- Describe the *y*-intercept/starting value.
- Locate the point where the line crosses the *y*-axis.
- Express slope as *rise/run.*
- Use a grid to determine run (how much it changes from left to right).
- Use a grid to determine rise (how much it changes from bottom to top).
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up* and *positive*.
- Pair *decreasing* with *going down* and *negative*.
- Recognize a graph/scatter plot of data related to a real-world context.
- Know that *y*-values increase vertically from bottom to top.

- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical*
- Recognize a coordinate grid.

Application of Alternate Achievement Standards

- Create a variety of slopes using rulers and stacks of books (various heights/stacks) and compare the slopes.
- Using a painter's tape grid on the floor, have students create paper strip lines with various starting points and slopes.
- Identify real slopes and starting points in the environment (e.g., roof lines, leaning ladders).

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Calculate the rate of change and starting value for various real-world situations.	Identify rate of change (slope) and starting value (*y*-intercept) in context.	Identify real-world rate of change situations (e.g., miles per hour, price per pound).	Identify non-numerical slope in a real-world context.

Grade: 10

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Interpreting Categorical and Quantitative Data:
Summarize, represent, and interpret data on a single count or measurement variable.

General Education Standards

M.SP.HS.41 Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.SP.HS.42 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.SP.HS.43 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Alternate Achievement Standards

M.AAS.SP.HS.41 Given data, construct a simple graph (line, pie, bar, picture) or table, and interpret the data in terms of range, mode, and median, mean.
M.AAS.SP.HS.42 Given two dot plots representing two different data sets, identify which data set has the greater maximum, median, or range.
M.AAS.SP.HS.43 Interpret general trends on a graph or chart (increase/decrease).

Achievement Elements

Students will be able to construct simple graphs from given data and answer questions about the data.
Students will be able to identify a data set that has a greater maximum, median, or range when given two dot plots representing different data.
Students will be able to interpret general trends on a graph or chart.

Key Vocabulary

line graph, pie chart, bar graph, dot plot, pictograph, data table, data maximum, data median, data range, increasing, decreasing

Teaching and Learning Progressions

- Recognize data on a graph or chart as *increasing* or *decreasing*.
- On a chart, know *increasing* and *decreasing* in relation to moving across from left to right or down from top to bottom.
- On a graph, know *increasing* and *decreasing* in relation to moving across the graph from left to right.
- Pair *increasing* with *going up*.
- Pair *decreasing* with *going down*.
- On a graph, know that dependent values increase vertically from bottom to top.
- On a graph, know that independent values increase horizontally from left to right.
- On a graph, know *horizontal* and *vertical*
- Recognize a graph or chart representing collected data.
- Identify the greater maximum, median, or range when two dot plots are compared.
- Know *greater* and *less* in the context of dot plot data.
- Know that the range is the difference between the largest number and the smallest number in a set of data.
- Know that the median of a set of data is the middle number when the data is placed in value order.
- Know that the maximum of a set of data is the largest number.
- Know the construction of a dot plot (parts and their meaning).
- Know that two dot plots can represent data collected about the same conditions.
- Recognize that a dot plot represents collected data.
- Explain the data in response to an appropriate question.
- Construct a graph using appropriate tools.
- Select the best graph (line, pie, bar, or picture) to represent the data.
- Organize data into a two-column chart/table.
- Recognize the elements of data given in paragraph or list form.

Application of Alternate Achievement Standards

- Have students determine the median, maximum, and range in various classroom situations (e.g., maximum height of students, median place in a line of students, or given length of string, range of shoe sizes).
- Demonstrate the construction of a dot plot from a given set of data and discuss what the data tells.
- Using real-world graphs from, e.g., magazines, newspapers, or the Internet, discuss range and the maximum and median values of the data shown.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Gather data for a real-world situation, construct an appropriate graph, and interpret the data.	Construct simple graphs from given data and answer questions about the data.	Construct a simple graph, given data.	Construct a simple graph, given at least five data points.
Compare and contrast data sets that have different maximum, median, or range when given two different dot plots.	Identify a data set that has a greater maximum, median, or range when given two dot plots representing different data.	Given two dot plots, identify which data set has the greater maximum value.	Given a dot plot, identify the range of the data.
Predict outcomes based on the general trends on a graph or chart.	Interpret general trends on a graph or chart.	Identify general trends on a graph or chart.	Identify increasing or decreasing data on a graph.

Grade: 10

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Summarize, represent, and interpret data on two categorical and quantitative variables.

General Education Standards

M.SP.HS.44 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.SP.HS.45 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Alternate Achievement Standards

M.AAS.SP.HS.44 Calculate the mean of a given data set (number of data points limited to fewer than five, values of less than 10).

M.AAS.SP.HS.45 Given a scatter plot with data with a line of best fit that can be represented by a linear function, describe what is happening to the *y*-values in reference to the *x*-values (x and y values limited positive numbers).

Achievement Elements

Students will be able to calculate the mean of a data set of five points fewer than five and within ten.
Students will be able to describe what is happening to the *y*-values in reference to the *x*-values limited to positive numbers.

Key Vocabulary

x-values, *y*-values, increasing, decreasing, mean

Teaching and Learning Progressions

- Identify what happens to the *y*-values as the *x*-values increase.
- Recognize data as *increasing* or *decreasing*.
- Recognize a scatter plot as representing data.
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up*.
- Pair *decreasing* with *going down*.
- Know that *y*-values increase vertically from bottom to top.
- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis*, y*-axis*, horizontal*, and *vertical*.
- Recognize a coordinate grid.
- Calculate the mean of a given data set.
- Divide the sum by the number of numbers summed.
- Determine the number of numbers summed.
- Sum no more than five numbers with values of less than 10.

Application of Alternate Achievement Standards

- Have three to five students select up to ten items each (e.g., chips, pennies) and find the average number of items.
- Have students measure their hand span to the nearest whole number and find the mean hand span of a student in the class.
- Use real-world examples to demonstrate data where a dependent variable increases as the independent variable increases.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Represent data on a scatter plot and describe how the variables are related.	Describe what is happening to the *y*-values in reference to the *x*-values on a scatter plot limited to positive numbers.	Identify the scatter plot, given a data table.	Describe a scatter plot as increasing or decreasing.
Calculate the mean and median and determine the range of a data set.	Calculate the mean of a data set of five points.	Divide a two-digit number by a one-digit number.	Sum five single-digit numbers.

Grade: 10

Content Area: Mathematics
Strand: Statistics and Probability
Objective: Interpret linear models.

General Education Standard

M.SP.HS.46 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Alternate Achievement Standard

M.AAS.SP.HS.46 Given a graph that describes a set of linear data, identify the rate of change (slope) and constant term (*y*-intercept). (Use context of data—the total price of the stamps is calculated by increasing 50 cents for every stamp purchased or the cost if no stamps are purchased is $0.)

Achievement Elements

Students will be able to describe the relationship between the *x*- and *y*-values in context on a given scatter plot.

Key Vocabulary

x-value, *y*-value, scatter plot, increasing, decreasing, staying the same, slope

Teaching and Learning Progressions

- Apply knowledge of *increasing* and *decreasing* in scatter plots to a context.
- Identify what happens to the *y*-values as the *x*-values increase.
- Recognize data as *increasing* or *decreasing*.
- Recognize a scatter plot as representing data.
- Know *increasing* and *decreasing* in relation to moving across the coordinate grid from left to right.
- Pair *increasing* with *going up*.
- Pair *decreasing* with *going down*.
- Know that *y*-values increase vertically from bottom to top.

- Know that *x*-values increase horizontally from left to right.
- Know *origin*, *x*-axis, *y*-axis, *horizontal*, and *vertical.*
- Recognize a coordinate grid.

Application of Alternate Achievement Standards

- Collect real-world graphs and discuss the meaning and data of those graphs.
- Describe real-world situations where as the independent variable increases, the dependent variable increases, and vice versa.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Predict outcomes in context using the data on a given scatter plot.	Describe the relationship between the *x*- and *y*-values in context on a given scatter plot.	Identify the scatter plot, given a data table.	Describe a scatter plot as increasing or decreasing.

Grade: 11

Content Area: Mathematics
Strand: Geometry
Objective: Congruence:
Experiment with transformations in the plane.

General Education Standards

M.G.HS.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.G.HS.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.G.HS.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

M.G.HS.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.G.HS.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Alternate Achievement Standards

M.G.AAS.HS.1 Compare properties of perpendicular lines, parallel lines, line segments, angles, and circles.

M.G.AAS.HS.2 Given a triangle on a coordinate grid, recognize the image of the triangle after a vertical or horizontal translation.

M.G.AAS.HS.3 Identify the reflection of a polygon.

M.G.AAS.HS.4 Given a geometric figure of a reflection or a translation of that figure, identify if the geometric figure is a reflection or translation.

M.G.AAS.HS.5 Given a figure and that figure after a vertical or horizontal translation, identify the vertical or horizontal translation.

Achievement Elements

Students will be able to identify perpendicular lines, parallel lines, line segments, angles, and circles.
Students will be able to recognize a triangle after a vertical or horizontal translation.

Students will be able to recognize the reflection of a polygon.
Students will be able to differentiate between a reflection and translation of a given figure.
Students will be able to identify the vertical or horizontal translation of a figure.

Key Vocabulary

reflection, translation, polygon, triangle, vertical, horizontal, perpendicular, parallel, line segment, angle, circle

Teaching and Learning Progressions

- Pair the vocabulary perpendicular line, parallel line, line segment, angle, and circle with a two-dimensional representation.
- Given a triangle, identify the vertical translation of the figure.
- Given a triangle, identify the horizontal translation of the figure.
- Know *left* and *right*.
- Recognize a given triangle in various orientations on a coordinate grid.
- Know that the coordinates of the figure vertices change, but the lengths of the sides and the measures of the angles do not change.
- Recognize characteristics of a triangle placed on a coordinate grid.
- Recognize a triangle in various orientations as the same triangle.
- Given a polygon, identify its reflection.
- Pair *reflection* with *flip*.
- Recognize a given figure in various orientations as the same figure.
- Given a figure on a coordinate grid, identify a translation of that figure.
- Given a figure on a coordinate grid, identify a reflection of that figure.
- Pair *translation* with *shift*.
- Know *horizontal* and *vertical*.
- Recognize a given figure in various orientations on a coordinate grid.
- Recognize characteristics of a figure placed on a coordinate grid.
- Recognize a given figure in various orientations as the same figure.

Application of the Alternate Achievement Standards

- Have students use manipulatives to explore the orientation of figures on a coordinate grid.
- Have students recognize figures in different orientations in the classroom or school.
- Have students compare reflections in a mirror with reflections of an object over a line.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe and compare perpendicular lines, parallel lines, line segments, angles, and circles on a coordinate grid.	Identify perpendicular lines, parallel lines, line segments, angles, and circles.	Identify lines/line segments, circles, angles, and intersecting lines.	Identify lines/line segments, and circles.
Recognize vertical or horizontal translations of shapes on a coordinate grid.	Recognize a triangle after a vertical or horizontal translation and the reflection of a polygon on a coordinate grid.	Recognize a given triangle or polygon in various orientations on a coordinate grid.	Discriminate triangles and given polygons from other shapes.
Compare and contrast reflections and translations of a given figure on a coordinate grid.	Differentiate between a reflection and translation of a given figure on a coordinate grid.	Recognize a given shape in different orientations on a coordinate grid.	Discriminate regular closed figures.
Describe the vertical or horizontal translation of a figure on a coordinate grid.	Identify the vertical or horizontal translation of a figure on a coordinate grid.	Translate a given triangle or square vertically or horizontally two units on a coordinate grid.	Move a given triangle or square vertically or horizontally.

Grade: 11

Content Area: Mathematics
Strand: Geometry
Objective: Congruence:
Understand congruence in terms of rigid motions.

General Education Standards

M.G.HS.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.G.HS.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.G.HS.8 Explain how the criteria for triangle congruence, angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), follow from the definition of congruence in terms of rigid motions.

Alternate Achievement Standards

M.G.AAS.HS.6 Identify corresponding congruent parts of transformed shapes (squares, rectangles, triangles, pentagons).

M.G.AAS.HS.7 Given two congruent triangles and side lengths of one of the triangles, identify the side lengths of the other triangle.

M.G.AAS.HS.8 Given two congruent triangles and angle measures of one of the triangles, identify the angle measures of the other triangle.

Achievement Elements

Students will be able to identify corresponding congruent and similar parts of shapes (squares, rectangles, triangles, pentagons).
Given two congruent triangles and side lengths of one of the triangles, students will be able to identify the side lengths of the other triangle.
Given two congruent triangles and angle measures of one of the triangles, students will be able to identify the angle measures of the other triangle.

Key Vocabulary

corresponding, congruent, similar, side length, angle measure

Teaching and Learning Progressions

- Given two congruent triangles and the angle measures of one, identify the angle measures of the other triangle.
- Identify that the corresponding angles of congruent triangles are congruent.
- Identify corresponding angles of congruent triangles.
- Compare and contrast triangles that are and are not congruent.
- Identify congruent angles when given a group of angles.
- Know that congruent angles have the same measure.
- Given two congruent triangles and the side lengths of one, identify the side lengths of the other triangle.
- Identify that the corresponding sides of congruent triangles are congruent.
- Identify corresponding parts of congruent figures.
- Identify corresponding parts of similar figures.
- Compare and contrast figures that are and are not similar.
- Know that similar figures have the same corresponding angle measure but may have different side lengths (corresponding sides have the same ratio).
- Know that congruent figures have the same corresponding angle measure and the same side length.
- Know that congruent angles have the same measure.
- Know that congruent lines have the same length.
- Pair *congruence* with *exactly the same*.
- Pair *similar* with *same shape*.
- Identify angles of the same measure given a group of angles of different measures.
- Identify line segments of the same length given a group of lines of various lengths.
- Identify two-dimensional figures in various orientations.

Application of the Alternate Achievement Standards

- Have students overlay two-dimensional cutouts of squares, rectangles, triangles, and pentagons to demonstrate congruence.
- Use matching exercises to practice congruence and similarity.
- Use computer software to have students practice recognition of congruence and similarity.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify corresponding congruent and similar parts of shapes.	Identify corresponding congruent and similar parts of shapes (squares, rectangles, triangles, pentagons).	Identify the congruent parts of squares and triangles.	Identify congruent squares and triangles.
Given the side lengths of a triangle, construct a congruent triangle.	Given two congruent triangles and side lengths of one of the triangles, identify the side lengths of the other triangle.	Recognize congruent triangles of various sizes and identify the sides of a given triangle.	Recognize similar triangles.
Given two congruent triangles and two angle measures of one of the triangles, identify the angle measures of the other triangle.	Given two congruent triangles and angle measures of one of the triangles, identify the angle measures of the other triangle.	Recognize congruent triangles of various sizes and identify the angles of a given triangle.	Recognize similar triangles.

Grade: 11

Content Area: Mathematics
Strand: Geometry
Objective: Congruence:
Prove geometric theorems.

General Education Standards

M.G.HS.9 Prove theorems about lines and angles. *Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

M.G.HS.10 Prove theorems about triangles. *Theorems include measures of interior angles of a triangle sum to 180º, base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, and the medians of a triangle meet at a point.*

M.G.HS.11 Prove theorems about parallelograms. *Theorems include opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals.*

Alternate Achievement Standards

M.G.AAS.HS.9 Given the intersection of two non-perpendicular lines and the measure of one angle, identify the measure of its vertical angle.

M.G.AAS.HS.10 Given a measure of a leg or base angle of an isosceles triangle, identify the measure of the other leg or other base angle.

M.G.AAS.HS.11 Given the measure of one side or one angle of a parallelogram, identify the measure of the opposite side or opposite angle.

Achievement Elements

Students will be able to identify vertical angles.
Students will be able to identify missing measures of legs or base angles in an isosceles triangle.
Students will be able to identify the measure of the opposite side or angle given the measure of one side or angle in a parallelogram.

Key Vocabulary

intersection, vertical angle, leg, base angle, isosceles triangle, parallelogram

Teaching and Learning Progressions

- Pair the terms *opposite* and *vertical*
- In a drawing of intersecting lines, identify the pairs of opposite/vertical angles.
- In a drawing of intersecting lines, identify the angles.
- In a drawing of intersecting lines, identify the lines.
- Identify angles in various orientations.
- Identify lines in various orientations.
- Recognize that the angle formed by one leg and the base has the same measure as the angle formed by the other leg and the base.
- Know that an isosceles triangle is a triangle with two sides of the same length. Identify the two sides of the same length as *legs* and the third side as the *base*.
- Identify angles of the same measure, given a group of angles of different measures.
- Identify line segments of the same length, given a group of lines of various lengths.
- Identify triangles in various orientations.
- Know that a triangle is a figure with three sides.
- Recognize that the measure of the opposite (paired) angle of a parallelogram has the same measure as the given angle.
- Identify the given measure of an inside angle of a parallelogram.
- Recognize that the pairs of opposite inside angles of a parallelogram have the same measure.
- Identify the pairs of opposite inside angles of a parallelogram in various orientations.
- Identify the inside angles of a parallelogram.
- Recognize that the measure of the opposite (paired) side of a parallelogram has the same measure as the given side.
- Identify the given length of a side of a parallelogram.
- Identify the pairs of parallel sides of a parallelogram.
- Know that a parallelogram is a figure with four sides and opposite sides that are parallel line segments.
- Identify parallel lines.
- Identify angles of the same measure given a group of angles of different measures.
- Identify line segments of the same length given a group of lines of various lengths.

Application of the Alternate Achievement Standards

- Identify parallelograms and triangles in surroundings and discuss lines (base, side) and angles.
- Manipulate (fold) paper isosceles triangles to discover congruent sides and angles.
- Construct vertical angles of various measures using toothpicks.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Identify vertical, opposite, and adjacent angles, given a parallelogram.	Identify vertical angles.	Identify that angles are formed when two lines intersect.	Recognize intersecting lines.
Identify that the segment joining midpoints of two sides of an isosceles triangle is parallel to the third side and half the length.	Identify missing measures of legs or base angles in an isosceles triangle.	Identify that the sum of the angles of a triangle is 180 degrees.	Identify the characteristics of an isosceles triangle.
Know that the diagonals of a parallelogram bisect each other and that rectangles are parallelograms with congruent diagonals.	Identify the measure of the opposite side or opposite angle in a parallelogram, given a measure of one side or angle.	Identify the opposite sides and the opposite angles in a parallelogram.	Identify a parallelogram.

Grade: 11

Content Area: Mathematics
Strand: Geometry
Objective: Congruence:
Make geometric constructions.

General Education Standards

M.G.HS.12 Make formal geometric constructions with a variety of tools and methods such as compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software. *Constructions include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

M.G.HS.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Alternate Achievement Standards

M.G.AAS.HS.12 Given a drawing with angles and a protractor overlay, determine which angles are congruent. Sample image below.

M.G.AAS.HS.13 Identify an equilateral triangle from a set of triangles or identify a regular hexagon from a set of hexagons. Make sure sides/angles are marked so that students can identify congruence.

Achievement Elements

Students will be able to identify the appropriate tools to make geometric constructions.
Students will be able to identify an equilateral triangle and a regular hexagon from a set of triangles and hexagons, respectively.

Key Vocabulary

equilateral triangle, regular hexagon, geometric construction, compass, straightedge, reflective device, paper folding

Teaching and Learning Progressions

- Pair geometric construction tools with their resulting products (e.g., straightedge and straight line; compass and circle)
- Pair geometric construction tools with their names.

- Know that other two-dimensional shapes (rectangles, hexagons) are regular if they have all sides of the same length and all angles of the same measure.
- Know that an equilateral triangle is regular, with three sides of the same length and three angles of the same measure.
- Identify angles of the same measure given a group of angles of different measures.
- Identify line segments of the same length given a group of lines of various lengths.
- Identify triangles in various orientations.
- Know that a triangle is a figure with three sides.

Application of the Alternate Achievement Standards

- Using manipulatives, demonstrate selecting an equilateral triangle from a group of triangles and a regular hexagon from a group of polygons.
- Use technology (software programs) to demonstrate the construction of closed figures.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use appropriate tools to make geometric constructions of lines, angles, and triangles.	Identify the appropriate tools to make geometric constructions.	Identify a compass as a tool to draw a circle and an angle.	Identify a straightedge as a tool to draw a line.
Construct an equilateral triangle.	Identify an equilateral triangle and a regular hexagon from a set of triangles and hexagons, respectively.	Identify an equilateral triangle. Identify a regular hexagon.	Identify a triangle and a hexagon.

Grade: 12

Content Area: Mathematics
Strand: Geometry
Objective: Expressing Geometric Properties with Equations:
Translate between the geometric description and the equation for a conic section.

General Education Standard

M.G.HS.29 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Alternate Achievement Standard

M.G.AAS.HS.29 Given a circle graphed on the coordinate plane and a point on the edge of the circle, determine the length of the radius of the circle.

Achievement Elements

Students will be able to identify points on a circle that are equal distance from the center.

Key Vocabulary

circle, equal, distance, center, radius, circumference

Teaching and Learning Progressions

- Recognize that circumference is a measure, in units, of the boundary of a circle while perimeter is the measure, in units, of the boundary of a figure with straight lines.
- Recognize that all of the drawn points on a circle are the same distance from the center.
- Using a measurement tool, measure radii of a circle.
- Given a point, draw the boundary of various radii of a circle.
- Recognize that a line is composed of points.
- Identify the center point of a circle.

- Identify a circle both on and off a coordinate grid.
- Identify a point.

Application of the Alternate Achievement Standards

- Use jar lids of different sizes to demonstrate circles of various sizes in a real-world context.
- Have students construct circles of various sizes with standard and nonstandard tools.
- Punch out the center point of various sizes of circles and have students put them on a pencil or straw to compare the circumferences.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Construct circles, given a center point and a radius.	Identify points on a circle that are equal distance from the center.	Identify the center, border, and radius of a circle.	Identify the center point and border of a circle.

Grade: 12

Content Area: Mathematics
Strand: Geometry
Objective: Expressing Geometric Properties with Equations:
Use coordinates to prove simple geometric theorems algebraically.

General Education Standards

M.G.HS.30 Use coordinates to prove simple geometric theorems algebraically.
M.G.HS.31 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
M.G.HS.32 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
M.G.HS.33 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
M.G.HS.34 Determine areas and perimeters of regular polygons, including inscribed or circumscribed polygons, given the coordinates of vertices or other characteristics

Alternate Achievement Standards

M.G.AAS.HS.30 Given a rectangle plotted on a coordinate grid with sides parallel to the *x*- and *y*-axis, determine the lengths of the sides to verify that opposite sides have the same length.
M.G.AAS.HS.31 Given a set of parallel lines in a coordinate plane and the slope of one of the lines, identify the slope of the other line.
M.G.AAS.HS.32 Identify the midpoint between two points on a vertical or horizontal line.
M.G.AAS.HS.33 Given a graph of a square, a rectangle, or a right triangle in the first quadrant, find the area and perimeter of the figure. (limit to squares/rectangles with sides parallel to axes and right triangles with legs parallel to axes.)
M.G.AAS.HS.34 Find the perimeter of an equilateral triangle, square, or regular pentagon given the length of one side.

Achievement Elements

Students will be able to demonstrate that the opposite sides of a rectangle plotted on a coordinate grid with sides parallel to the *x*- and *y*-axis have the same length.
Students will be able to identify that parallel lines have the same slope.
Students will be able to identify the midpoint between two points on a vertical or horizontal line.

Students will be able to find the perimeters of squares (given one side), rectangles (given a length and width), and equilateral triangles (given one side) in mathematical problems and to solve real-world problems.

Key Vocabulary

rectangle, square, triangle, equilateral, midpoint

Teaching and Learning Progressions

- Determine, by using grid lines, the length and width of a given rectangle.
- Recognize a rectangle placed on a coordinate grid.
- Determine the length of vertical and horizontal line segments placed on a coordinate grid.
- Know the meaning of grid lines on a coordinate grid.
- Know that parallel lines/line segments have the same slope.
- Identify *slope* as a characteristic of a line in various orientations.
- Identify parallel lines on a coordinate grid.
- Know that parallel lines/line segments do not intersect.
    - Identify intersecting and nonintersecting lines.
    - Identify line(s) and line segments on a coordinate plane (vs. grids, points, curves, figures).
- Apply the meaning of *halfway* using multiplication (multiply by ½) or division (divide by 2) to obtain the value of the midpoint.
- Know that midpoint means *halfway*.
- Identify the given length, in units, of both vertical and horizontal lines.
- Identify a line in both vertical and horizontal orientation.
- Apply either addition or multiplication to solve a given perimeter problem.
    - $P = s + s + s + s$ or
    - $P = 4s$
    - $P = 2l + 2w$ to solve a problem.
- Identify the measurement of the given sides of a square or rectangle.
- Identify the lines that represent the perimeter/boundaries of a given square or rectangle.
- Know that perimeter is the measurement, in units, of the boundary of a two-dimensional shape.
- Apply either addition or multiplication to solve a given perimeter problem.
    - $P = s + s + s$ or $P = 3s$
    - $P = s + s + s + s$ or $P = 4s$

- Recognize that a measurement for one side of an equilateral applies to all sides of that equilateral figure.
- Identify the measurement for one side of a given equilateral triangle or square.
- Identify the lines that represent the perimeter/boundaries of a given equilateral triangle or square.
- Know that a square has four sides of the same length.
- Know that an equilateral triangle has three sides of the same length.

Application of the Alternate Achievement Standards

- Use paper folding to demonstrate that rectangles have two sets of congruent sides and squares and equilateral triangles have congruent sides.
- Use transparency overlays, manipulatives (folding, measuring), or other tools to demonstrate the meaning of a midpoint on a line.
- Find parallel and perpendicular lines in the environment and describe the reasons for the labels.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Describe how one knows that the sides of a square on a coordinate grid with sides parallel to the *x*- and *y*-axes have the same length.	Demonstrate that the opposite sides of a rectangle plotted on a coordinate grid with sides parallel to the *x*- and *y*-axes have the same length.	Know that the opposite sides of a rectangle have the same measure.	Recognize the characteristics of a rectangle.
Determine whether two lines are parallel by calculating their slopes.	Identify that parallel lines have the same slope.	Identify the slope of a line.	Identify parallel lines.
Calculate the midpoint between two points on a vertical or horizontal line.	Identify the midpoint between two points on a vertical or horizontal line.	Identify a point between two other points on a line.	Identify at least three points on a line.
Find the areas of squares, rectangles, and equilateral triangles using formulas in mathematical problems to solve real world problems.	Find the perimeters of squares (given one side), rectangles (given a length and width), and equilateral triangles (given one side) in mathematical problems to solve real-world problems.	Find the perimeter of a square, rectangle, and triangle, given the measure(s) of the sides.	Identify the measure(s) of the sides of given squares, rectangles, and triangles.

Grade: 12

Content Area: Mathematics
Strand: Geometry
Objective: Geometric Measurement and Dimensions:
Explain volume formulas and use them to solve problems.

General Education Standards

M.G.HS.35 Give an informal argument for the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone. *Use dissection arguments*, *Cavalieri's principle, and informal limit arguments.*
M.G.HS.36 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
M.G.HS.37 Determine the relationship between surface areas of similar figures and volumes of similar figures

Alternate Achievement Standards

M.G.AAS.HS.35 Make a prediction about the volume of a container, the area of a figure, or the perimeter of a figure -e:g., how many cubes will go in one figure vs. another. Limit to cylinders and circles.
M.G.AAS.HS.36 Given a cylinder and a cone with the same height and radius, identify that the volume of the cone will be one-third the volume of the cylinder.
M.G.AAS.HS.37 Identify that a rectangle with sides that are two times as large as another rectangle will have an area that is four times as large by using models.

Achievement Elements

Students will be able to make a prediction about the volume of a figure, the area of a figure, or the perimeter of a figure.
Students will be able to compare the volumes of a cone and a cylinder (with the same height and radius) and conclude that the volume of the cone is one-third the volume of the cylinder.
Students will be able to identify (with models) that a rectangle with sides that are twice as long as another rectangle will have an area that is four times as large.

Key Vocabulary

prediction, area, perimeter, volume, cone, cylinder, compare, similar

Teaching and Learning Progressions

- Compare the volume, area, or perimeter of two containers or figures.
- Know and use comparative vocabulary (e.g., *more, less, larger, smaller*).
- Predict the perimeter, area, and volume of given figures or containers.
- Use models to determine the perimeter, area, and volume of figures or containers.
- Recognize cubic units as the measure of volume.
- Know that volume is the amount of space that an object occupies or that is enclosed within a container.
- Recognize square units as the measure of volume.
- Know that area is the amount of space inside the boundary of a flat (two-dimensional) object.
- Recognize units as the measure of perimeter.
- Know that perimeter is the continuous line forming the boundary of a closed geometric figure.
- Conclude that the volume of a given cone will have a volume of one-third the volume of a cylinder with the same height and radius of the base.
- Using models and calculations, calculate the volume of a given cylinder and a given cone that have the same height and radius of the base.
- Using models and calculations, calculate the volume of given cylinders and cones.
- Compare the volumes of given cylinders and cones.
- Know that volume is the amount of space that an object occupies or the space that is enclosed in a cylinder and cone.
- Identify the radius of the base of a three-dimensional figure: cylinder and cone.
- Identify the base of a three-dimensional figure: cylinder and cone.
- Identify the height of a three-dimensional figure: cylinder and cone.
- Identify the three-dimensional figures of a cylinder and cone.
- Using models, identify that a rectangle with sides that are two times as large as another rectangle will have an area that is four times as large.
- Know and use comparative vocabulary, particularly “two times as much” and “four times as much.”
- Use models and numbers to calculate the areas of various given rectangles.
- Know the relationship between the lengths of the sides of a rectangle and the area of the rectangle: $l \times w = A$.
- Show an understanding of area as the amount of space inside the boundary of a given rectangle by using models.
- Identify the dimensions (length and width) of a given rectangle.

Application of the Alternate Achievement Standards

- Demonstrate predictions in a variety of real-world and mathematical situations.
- Have students construct and compare two rectangles, one of which is twice the size, in side lengths, of the other rectangle.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Make a prediction about the volume of a figure, the area of a figure, or the perimeter of a figure and compare it to a calculated measurement.	Make a prediction about the volume of a figure, the area of a figure, or the perimeter of a figure.	Compare volumes, areas, and perimeters of given figures.	Recognize volume, area, and perimeter as measurements.
Using a formula, calculate and compare the volume of a cone and a cylinder with the same height and radius.	Compare the volume of a cone and a cylinder (with the same height and radius) and conclude that the volume of the cone is one-third the volume of the cylinder.	Compare the volume of a cone and a cylinder with the same height and radius.	Identify a cone and cylinder.
Calculate and compare the areas of two similar rectangles, one of which has sides that are twice as long as the other rectangle.	Identify (with models) that a rectangle with sides that are twice as long as another rectangle will have an area that is four times as large.	Identify a rectangle that has sides that are twice as long as another rectangle.	Identify similar rectangles.

Grade: 12

Content Area: Mathematics
Strand: Geometry
Objective: Geometric Measurement and Dimensions:
Visualize relationships between two-dimensional and three-dimensional objects.

General Education Standard

M.G.HS.38 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Alternate Achievement Standard

M.G.AAS.HS.38 Identify the shapes of two-dimensional cross-sections of three-dimensional objects (limited to sphere, rectangular prism, triangular prism).

Achievement Elements

Students will be able to identify the shapes of two-dimensional cross sections of three-dimensional objects: sphere, rectangular prism, triangular prism.

Key Vocabulary

two-dimensional, cross section, three-dimensional, sphere, rectangular prism, triangular prism, pyramid, cone, base

Teaching and Learning Progressions

- Identify the shapes of two-dimensional cross sections of sphere, rectangular prism, and triangular prism.
- Know and describe the term *cross section*.
- Name the base of a given three-dimensional prism.
- Identify the base of a given three-dimensional prism.
- Name three-dimensional objects: sphere, rectangular prism, and triangular prism.
- Recognize three-dimensional objects.

Application of the Alternate Achievement Standards

- Using three-dimensional objects, demonstrate the cross section of each object.
- Have students manipulate three-dimensional models of a sphere, rectangular prism, and triangular prism that come apart to show cross sections.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Determine the shapes of two-dimensional cross sections of three-dimensional objects.	Identify the shapes of two-dimensional cross sections of three-dimensional objects: sphere, rectangular prism, and triangular prism.	Identify the cross section of a rectangular prism and a sphere.	Identify a sphere, rectangular prism, and triangular prism.

Grade: 12

Content Area: Mathematics
Strand: Geometry
Objective: Modeling With Geometry:
Apply geometric concepts in modeling situations.

General Education Standards

M.G.HS.39 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

M.G.HS.40 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, British Thermal Units (BTUs) per cubic foot).

M.G.HS.41 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios).

Alternate Achievement Standards

M.G.AAS.HS.39 Identify objects that have a similar shape to a cylinder, rectangular prism, and cone.

M.G.AAS.HS.40 Perform computation operations (addition, subtraction, multiplication, division) in context-based problems about weight, length, or capacity using units.

M.G.AAS.HS.41 Solve a context-based problem involving area of rectangles.

Achievement Elements

Students will be able to use properties of geometric shapes to describe real-world objects.

Students will be able to perform computation operations in context-based problems about weight, length, or capacity and use appropriate units.

Students will be able to solve context-based problems involving the area of rectangles.

Key Vocabulary

names of geometric shapes, area, weight, length, capacity

Teaching and Learning Progressions

- Describe real-world objects using basic geometric properties.
- Identify basic geometric properties in common geometric shapes identified in the real world.
- Know the basic properties of each common geometric shape.
- Identify common shapes in the real world.
- Identify common geometric shapes.
- Solve context-based problems about weight, length, or capacity using units.
- Determine the appropriate computation for a given context-based problem.
- Perform computations using addition, subtraction, multiplication, and division using appropriate tools.
- Apply comparative vocabulary to context-based problems.
- Know the meaning of comparative vocabulary (e.g., *more, less, shorter, longer, heavier, lighter, larger, smaller, twice, two times*).
- Associate a given volume with common objects.
- Associate a given weight with common objects.
- Associate a given length with lines and common objects.
- Recognize the common units of volume.
- Know that volume (capacity) is the amount of space within a container.
- Recognize the common units of weight.
- Know that weight is the heaviness of an object.
- Recognize the common units of measurement.
- Solve a real-world problem involving the area of a rectangle.
- Know that area of a real-world object can be calculated by multiplying its length by its width.
- Know that area is the amount of space covered by a two-dimensional object.
- Know that length and width are descriptions of the measurement of the sides of a rectangle.

Application of the Alternate Achievement Standards

- Design three rectangular flower beds/gardens for a yard and calculate the area of each bed/garden.
- Play “I Spy” Bingo while taking a walk around the school or school yard to find a variety of shapes in the environment.
- Demonstrate real-world problem solving that involves addition, subtraction, multiplication, and division in weight, length, or capacity.

Levels of Standards

Level 4: Exceeds Standards	Level 3: Meets Standards	Level 2: Developing	Level 1: Emerging
Use geometric shapes and their measurement to describe real-world objects.	Use properties of geometric shapes to describe real-world objects.	Identify a shape in a real-world object.	Identify shapes.
Compute the volume of real-world objects/shapes given their area and volume.	Compute in context-based problems about weight, length, or capacity and use appropriate units.	Solve simple weight and length problems using appropriate units.	Know the common units of measurement for weight and length.
Solve a design problem involving the area of rectangles and triangles.	Solve context-based problems involving the area of rectangles using a calculator.	Determine the area of a rectangle using a grid.	Identify the length and width of a rectangle.