Interactive Classroom

Glencoe

ALGEBRA 2

LESSON 3–3 Optimization with Linear Programming

Lesson Menu

Five-Minute Check (over Lesson 3–2)

CCSS

Then/Now

New Vocabulary

Key Concept: Feasible Regions

Example 1: Bounded Region

Example 2: Unbounded Region

Key Concept: Optimization with Linear Programming

Example 3: Real-World Example: Optimization with Linear Programming

5-Minute Check

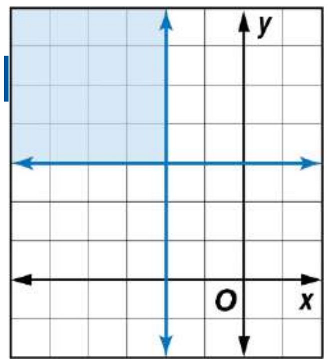
Over Lesson 3–2

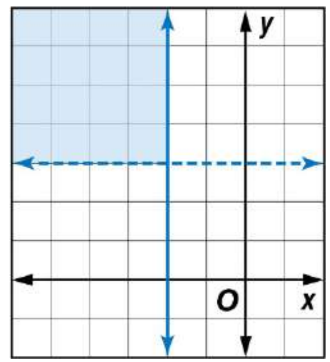
1 **Solve the system of inequalities by graphing.**

$x \leq 2$

$y > 3$

A.





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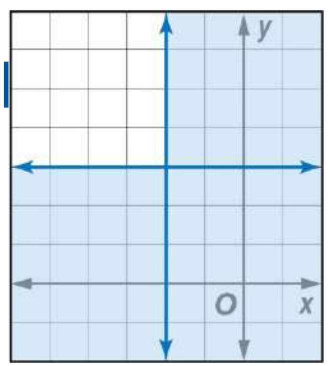
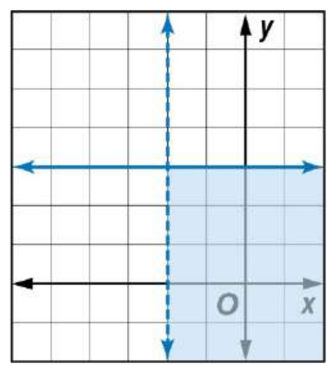




5-Minute Check

Over Lesson 3–2

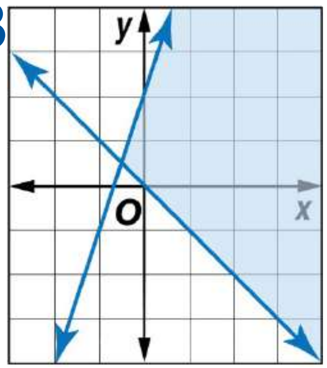
CheckPoint

2 Solve the system of inequalities by graphing.

$y \leq 3x + 2$

$y > -x$

A.



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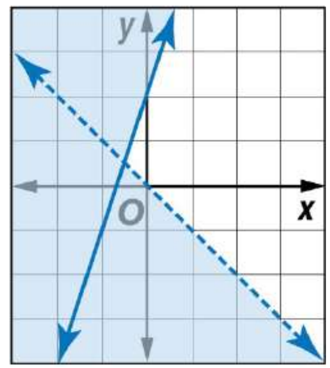
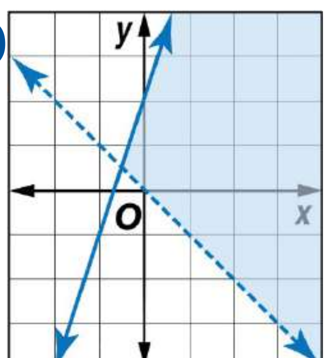


C.



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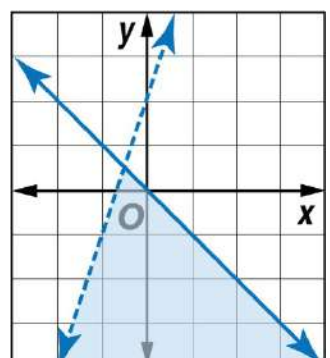


5-Minute Check

Over Lesson 3–2

CheckPoint

3 Find the coordinates of the vertices of the figure formed by the system of inequalities.

$x \geq 0$

$y \leq 0$

$-3x + y = -6$

A. (0, 0), (1, 0), (–3, 0)

B. (0, 0), (2, 0), (0, –6)

C. (0, 0), (–3, 0), (0, –6)

D. (0, 0), (2, 0), (–3, 0)

5-Minute Check

Over Lesson 3–2

CheckPoint

4 Find the coordinates of the vertices of the figure formed by the system of inequalities.

$y \leq 3$
$y \geq 0$
$x \geq 0$
$2y + 3x \leq 12$

A. (0, 3), (0, 6), (2, 12)

B. (0, 0), (0, 3), (0, 6), (2, 3)

C. (0, 0), (0, 3), (2, 3), (3, 2)

D. (0, 0), (0, 3), (2, 3), (4, 0)

5-Minute Check

Over Lesson 3–2

CheckPoint

Standardized Test Practice

5 Which point is *not* a solution of the system of inequalities $y \le 4$ and $y > |x - 3|$?

A. (2, 2)

B. (4, 2)

C. (3, 0)

D. (3, 4)

CCSS

Content Standards

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Mathematical Practices

4 Model with mathematics.

8 Look for and express regularity in repeated reasoning.

Then

You solved systems of linear inequalities by graphing.

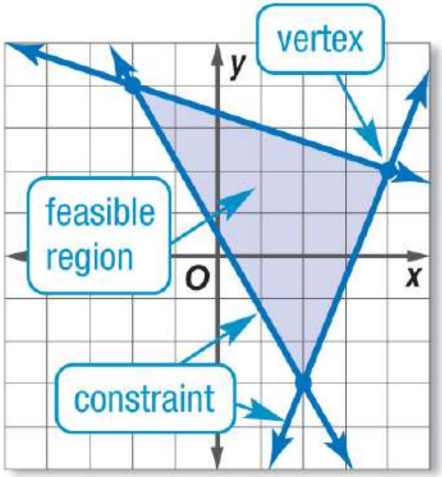
Now

- Find the maximum and minimum values of a function over a region.
- Solve real-world optimization problems using linear programming.

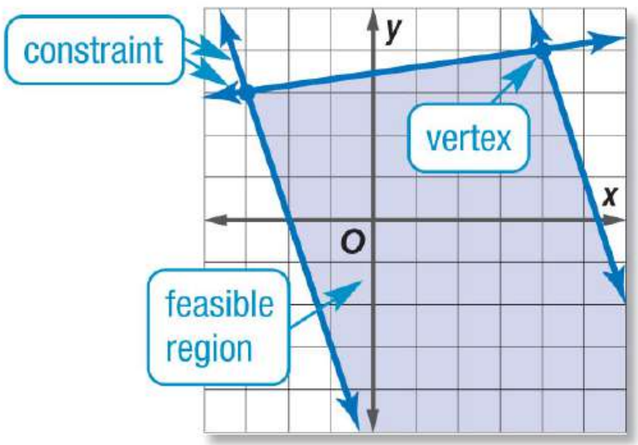
New Vocabulary

- linear programming
- feasible region
- bounded
- unbounded
- optimize

KeyConcept Feasible Regions



The feasible region is enclosed, or **bounded**, by the constraints. The maximum or minimum value of the related function *always* occurs at a vertex of the feasible region.



The feasible region is open and can go on forever. It is **unbounded**. Unbounded regions have either a maximum or a minimum.

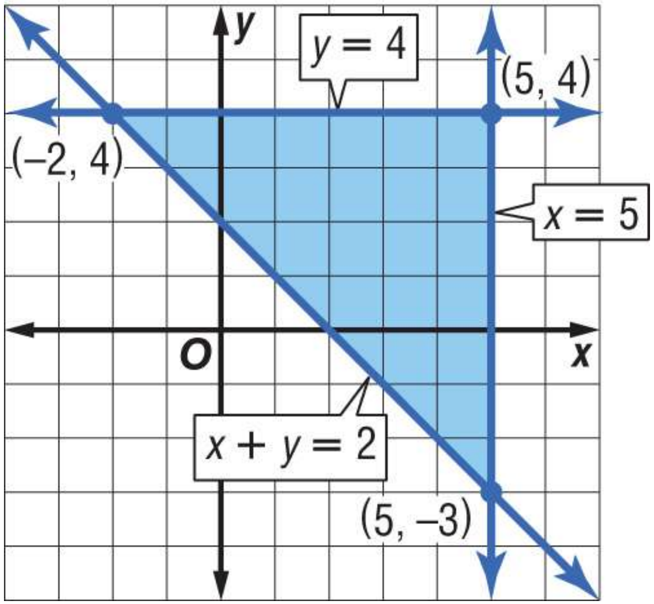
EXAMPLE 1 **Bounded Region**

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x – 2y$ for this region.

$x \leq 5$

$y \leq 4$

$x + y \geq 2$



Step 1 Graph the inequalities.

The polygon formed is a triangle with vertices at (–2, 4), (5, –3), and (5, 4).

EXAMPLE 1 Bounded Region

Step 2 Use a table to find the maximum and minimum values of $f(x, y)$. Substitute the coordinates of the vertices into the function.

(x, y)	$3x - 2y$	$f(x, y)$	
$(-2, 4)$	$3(-2) - 2(4)$	-14	← **minimum**
$(5, -3)$	$3(5) - 2(-3)$	21	← **maximum**
$(5, 4)$	$3(5) - 2(4)$	7	

Answer: The vertices of the feasible region are (–2, 4), (5, –3), and (5, 4). The maximum value is 21 at (5, –3). The minimum value is –14 at (–2, 4).

EXAMPLE 1 Check Your Progress

CheckPoint

Graph the following system of inequalities. What are the maximum and minimum values of the function $f(x, y) = 4x - 3y$ for the feasible region of the graph?

$x \leq 4y \leq 5$ $x + y \geq 6$

A. maximum: $f(4, 5) = 5$
minimum: $f(1, 5) = -11$

B. maximum: $f(4, 2) = 10$
minimum: $f(1, 5) = -11$

C. maximum: $f(4, 2) = 10$
minimum: $f(4, 5) = 5$

D. maximum: $f(1, 5) = -11$
minimum: $f(4, 2) = 10$

EXAMPLE 2 **Unbounded Region**

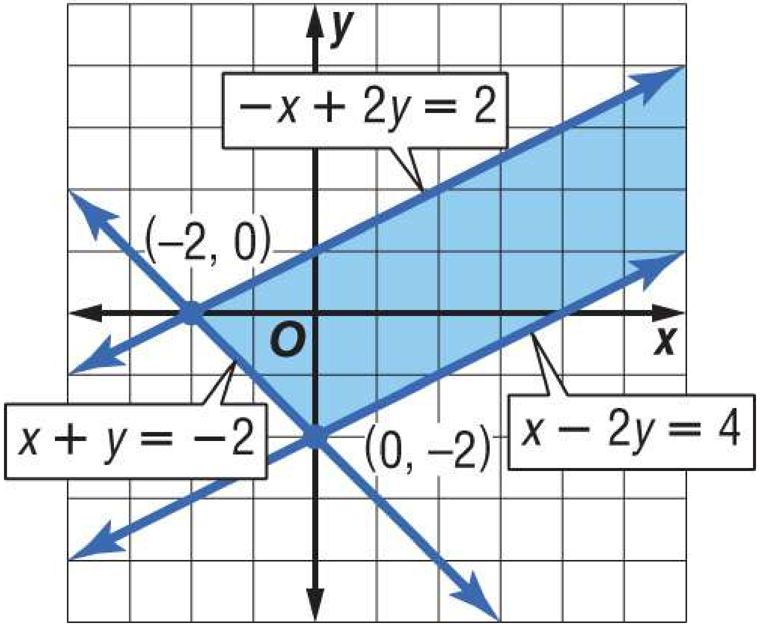
Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for this region.

$-x + 2y \leq 2$

$x - 2y \leq 4$

$x + y \geq -2$

Graph the system of inequalities. There are only two points of intersection, (–2, 0) and (0, –2).



EXAMPLE 2 Unbounded Region

(x, y)	$2x + 3y$	$f(x, y)$
$(-2, 0)$	$2(-2) + 3(0)$	-4
$(0, -2)$	$2(0) + 3(-2)$	-6

The minimum value is –6 at (0, –2). Although *f*(–2, 0) is –4, it is not the maximum value since there are other points that produce greater values. For example, *f*(2,1) is 7 and *f*(3, 1) is 9. It appears that because the region is unbounded, *f*(*x*, *y*) has no maximum value.

Answer: The vertices are at (–2, 0) and (0, –2).
There is no maximum value.
The minimum value is –6 at (0, –2).

EXAMPLE 2 Check Your Progress

CheckPoint

Graph the following system of inequalities. What are the maximum and minimum values of the function $f(x, y) = x + 2y$ for the feasible region of the graph?
$x + 3y \leq 6$ $-x - 3y \leq 9$ $2y - x \geq -6$

A. maximum: no maximum
minimum: $f(6, 0) = 6$

B. maximum: $f(6, 0) = 6$
minimum: $f(0, -3) = -6$

C. maximum: $f(6, 0) = 6$
minimum: no minimum

D. maximum: no maximum
minimum: $f(0, -3) = -6$

KeyConcept Optimization with Linear Programming

Step 1 Define the variables.

Step 2 Write a system of inequalities.

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

Step 5 Write a linear function to be maximized or minimized.

Step 6 Substitute the coordinates of the vertices into the function.

Step 7 Select the greatest or least result. Answer the problem.

Real-World Example 3

Optimization with Linear Programming

LANDSCAPING A landscaping company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew is scheduled for no more than 2 pruning jobs per day. Each crew's schedule is set up for a maximum of 9 hours per day. On the average, the charge for mowing a lawn is $40 and the charge for pruning shrubbery is $120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day from one of its crews.

Real-World Example 3

Optimization with Linear Programming

Step 1 Define the variables.

m = the number of mowing jobs

p = the number of pruning jobs

Real-World Example 3

Optimization with Linear Programming

Step 2 Write a system of inequalities.

Since the number of jobs cannot be negative, *m* and *p* must be nonnegative numbers.

$m \geq 0, p \geq 0$

Mowing jobs take 1 hour. Pruning jobs take 3 hours. There are 9 hours to do the jobs.

$1m + 3p \leq 9$

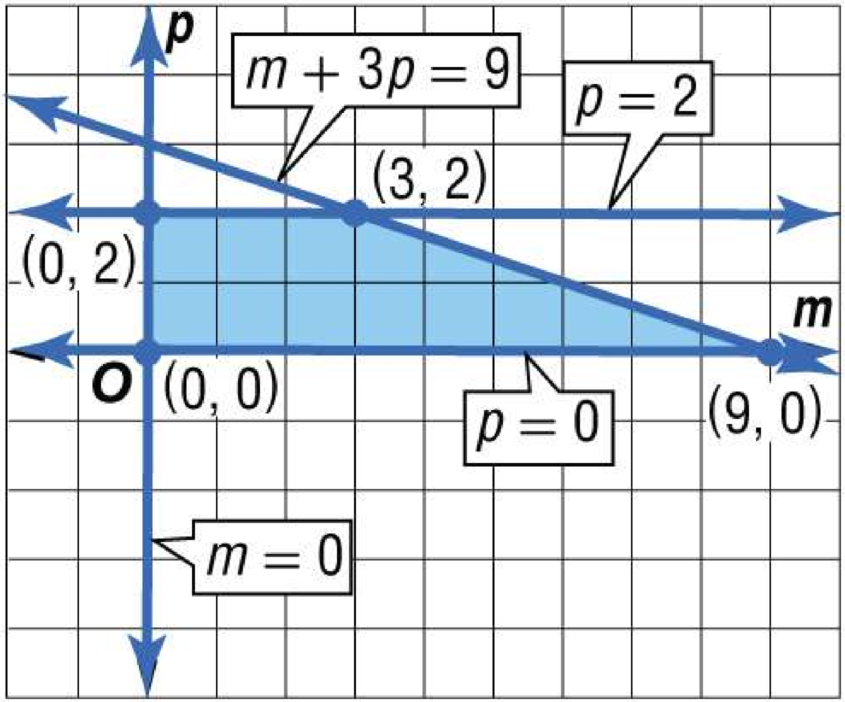
There are no more than 2 pruning jobs a day.

$p \leq 2$

Real-World Example 3

Optimization with Linear Programming

Step 3 Graph the system of inequalities.



Real-World Example 3

Optimization with Linear Programming

Step 4 Find the coordinates of the vertices of the feasible region.

From the graph, the vertices are at (0, 2), (3, 2), (9, 0), and (0, 0).

Step 5 Write the function to be maximized.

The function that describes the income is $f(m, p) = 40m + 120p$. We want to find the maximum value for this function.

Real-World Example 3

Optimization with Linear Programming

Step 6 Substitute the coordinates of the vertices into the function.

(m, p)	$40m + 120p$	$f(m, p)$
(0, 2)	40(0) + 120(2)	240
(3, 2)	40(3) + 120(2)	360
(9, 0)	40(9) + 120(0)	360
(0, 0)	40(0) + 120(0)	0

Step 7 Select the greatest amount.

Real-World Example 3

Optimization with Linear Programming

Answer: The maximum values are 360 at (3, 2) and 360 at (9, 0). This means that the company receives the most money with 3 mowings and 2 prunings or 9 mowings and 0 prunings.

Real-World Example 3 Check Your Progress CheckPoint

LANDSCAPING A landscaping company has crews who rake leaves and mulch. The company schedules 2 hours for mulching jobs and 4 hours for raking jobs. Each crew is scheduled for no more than 2 raking jobs per day. Each crew's schedule is set up for a maximum of 8 hours per day. On the average, the charge for raking a lawn is $50 and the charge for mulching is $30.

Real-World Example 3 — Check Your Progress — CheckPoint

What is a combination of raking leaves and mulching that will maximize the income the company receives per day from one of its crews?

A. 0 mulching; 2 raking

B. 4 mulching; 0 raking

C. 0 mulching; 4 raking

D. 2 mulching; 0 raking

Click the mouse button to return to the lesson menu.