Mathematics Curriculum Guide

Precalculus

2017-18

Topic 6: Sequences, Induction, and Probability

In this unit students will begin by learning about sequences and summation notation to find particular terms of a sequence from the general term. They will use recursion formulas, factorial notation, and summation notation. They will continue with arithmetic and geometric sequences and series where they will find either the common difference or the common ratio, and find the terms of a sequence. Afterwards, students will learn about the binomial theorem in order to evaluate a binomial coefficient, expand a binomial raised to a power, and find a particular term in a binomial expansion. Students will then learn about counting principles, permutations, and combinations. Finally, students will compute empirical and theoretical probability.

Topic 6: Sequences, Induction, and Probability

Transfer Goals

Transfer Goals	
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.	**Timeframe:** 15 days **Start Date:** January 22, 2018 **Assessment Dates:** Feb. 9, 2018

Standards

F-IF 3

F-BF 2

A-APR 5

S-CP 2

S-CP 3

S-CP 6

S-CP 9

Meaning-Making

Understandings

Students will understand that…

- An infinite sequence $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or terms, are represented by $a_1, a_2, \ldots, a_{n,\ldots}$
- Sequences can be defined using recursion formulas that define the *nth* term as a function of the previous term.
- In a arithmetic sequence, each term after the first differs from the preceding term by a constant, the common difference. Subtract any term from the term that directly follows to find the common difference.
- In a geometric sequence, each term after the first is obtained by multiplying the preceding term by a nonzero constant, the common ratio. Divide any term after the first by the term that directly precedes it to find the common ratio.
- An annuity is a sequence of equal payments made at equal time periods. The value of an annuity is the sum of all deposits made plus all interest paid.
- If we use binomial coefficients and the pattern for the variable part of each term, a formula called the Binomial Theorem can be used to expand any positive integral power of a binomial.
- The Fundamental Counting Principle: The number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur.
- A permutation from a group of items occurs when no item is used more than once and the order of arrangement makes a difference. A combination from a group of items occurs when no item is used more than once and the order of items makes no difference.
- Empirical probability applies to situations in which we observe the frequency of the occurrence of an event.
- Theoretical probability applies to situations in which the sample space of all equally likely outcomes is known.
- Events can be independent, dependent, or mutually exclusive.

Essential Questions

Students will keep considering…

- How do we find particular terms of a sequence of numbers?
- How can we find specific terms in an arithmetic sequence?
- How can we find specific terms in a geometric sequence?
- How can we calculate the growth of annuities?
- What are some efficient methods for raising binomials to higher powers?
- How can we calculate the number of ways in which a series of things can occur?
- How does the relationship of events affect empirical and theoretical probability?

Acquisition

Knowledge

Students will know…

Vocabulary: Fibonacci sequence, infinite sequence, finite sequence, graph of a sequence, factorial notation, summation notation, index of summation, upper limit of summation, lower limit of summation, arithmetic sequence, common difference, nth partial sum, geometric sequence, common ratio, nth partial sum, annuity, value of an annuity, infinite geometric series, multiplier effect, binomial coefficient, Binomial Theorem, tree diagram, permutation, combination, permutation

Procedures for:

- Using: Recursion Formulas, Factorial Notation, Summation Notation, Properties of Sums
- General Terms of an Arithmetic Sequence and a Geometric Sequence
- The Sum of the First n Terms of an Arithmetic Sequence and a Geometric Sequence
- Calculating the Value of an Annuity: Interest Compounded n Times per Year
- Binomial Theorem
- Fundamental Counting Principle
- Methods for counting possible outcomes
- Empirical and Theoretical Probability

Skills

Students will be skilled at and able to do the following…

- Find particular terms of a sequence from the general term.
- Use recursion formulas, factorial notation, and summation notation.
- Find the common difference and write the terms for an arithmetic sequence.
- Find the common ratio and write the terms of a geometric sequence.
- Use the formulas for the general term and the sum of the first n terms of an arithmetic sequence and geometric sequence. Use the formula for the sum of an infinite geometric series.
- Find the value of an annuity.
- Evaluate a binomial coefficient, and expand a binomial raised to a power.
- Find a particular term in a binomial expansion.
- Use the Fundamental Counting Principle.
- Use the permutations formula and the combinations formula.
- Distinguish between permutation problems and combination problems.
- Compute empirical and theoretical probability.
- Find the probability of an event will not occur, and of one event and/or a second event occurring.

Topic 6: Sequences, Induction, and Probability

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 6: Sequences, Induction, and Probability

The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Selected Standards	Examples
• Find particular terms of a sequence from the general term. • Use recursion formulas, factorial notation, and summation notation. • Find the common difference and write the terms for an arithmetic sequence. • Find the common ratio and write the terms of a geometric sequence. • Use the formulas for the general term and the sum of the first n terms of an arithmetic sequence and geometric sequence. Use the formula for the sum of an infinite geometric series. • Find the value of an annuity. • Evaluate a binomial coefficient, and expand a binomial raised to a power. • Find a particular term in a binomial expansion. • Use the Fundamental Counting Principle. • Use the permutations formula and the combinations formula. • Distinguish between permutation problems and combination problems. • Compute empirical and theoretical probability. • Find the probability of an event will not occur, and of one event and/or a second event occurring.	Create equations that describe numbers or relationships. (1,2) Represent and solve equations graphically. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**F-IF 3** – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers… **F-BF 2** – Write arithmetic and geometric sequences both recursively and with an explicit formula… **A-APR 5** - (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)n in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's triangle. **S-CP 2** – Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S-CP 3** – Understand the conditional probability of A given B as P(A and B)/P(b), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **S-CP 6** – Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **S-CP 9** – Use permutations and combinations to compute probabilities of compound events and solve problems.	**A.APR.C.5 (+) Item 1** Expand $(a+b)^7$ based on the Binomial Theorem. Answer: $1(a^7)+7(a^6b)+21(a^5b^2)+35(a^4b^3)+35(a^3b^4)+21(a^2b^5)+7(ab^6)+1(b^7)$ **A.APR.C.5 (+) Item 2** Expand $(x^2+3)^6$ based on the Binomial Theorem. Answer: $x^{12}+18x^{10}+135x^8+540x^6+1215x^4+1458x^2+729$

11. A random sample of 200 teenagers participated in a taste test. Each teenager sampled four choices of fruit drink (labeled A, B, C, and D), and then were asked to pick a favorite. The table shows the results of this taste test.

	A	B	C	D	Total
Boys	45	25	30	20	120
Girls	25	10	30	15	80
Total	70	35	60	35	200

Based on the information given, which of the given statements are true?

Select **all** that apply.

Ⓐ 40% of the participants were girls.

Ⓑ 70% of the participants preferred A.

Ⓒ $\frac{20}{120}$ of the boys preferred D.

Ⓓ $\frac{10}{35}$ of the participants who preferred B were girls.

Ⓔ The proportion of boys who preferred C is equal to the proportion of girls who preferred C.

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Essential Questions: • How do we find particular terms of a sequence of numbers? • How can we find specific terms in an arithmetic sequence? • How can we find specific terms in a geometric sequence? • How can we calculate the growth of annuities? • What are some efficient methods for raising binomials to higher powers? • How can we calculate the number of ways in which a series of things can occur? • How does the relationship of events affect empirical and theoretical probability?	**Standards:** F-IF 3, F-BF 2, A-APR 5, S -CP 2, S-CP 3, S-CP 6, S-CP 9, S-MD 3 **Timeframe:** 15 days **Start Date:** January 22, 2018 **Assessment Dates:** February 9, 2018

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 day	**Lesson 10.1: Sequences and Summation Notation** SMP: 1,2,3,5,6 (pp. 1002-1013) **F-IF 3**	**Focus Question:** • How do we find particular terms of a sequence of numbers? **Inquiry Question:** Page 994 Problems #55-57	• An infinite sequence $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or terms, are represented by $a_1, a_2, \ldots, a_{n,\ldots}$ • Sequences can be defined using recursion formulas that define the *nth* term as a function of the previous term.	**Vocabulary:** Fibonacci sequence, infinite sequence, finite sequence, graph of a sequence, factorial notation, summation notation, index of summation, upper limit of summation, lower limit of summation **Concepts:** • Recursion Formulas • Factorial Notation • Summation Notation • Properties of Sums	• Find particular terms of a sequence from the general term. • Use recursion formulas. • Use factorial notation. • Use summation.	**Common Core Problems:** **10.1:** #1-8, 69-72, 73-80, 93-101 **Test Prep:** **10.1:** #1-10, 13-16

Common Core Practices

☐ Instruction in the Standards for Mathematical Practices
☐ Use of Talk Moves
☐ Note-taking
☐ Use of Manipulatives
☐ Use of Technology
☐ Use of Real-world Scenarios
☐ Project-based Learning
☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 days	**Lesson 10.2: Arithmetic Sequences** SMP: 1,2,5,6 (pp. 1013-1023) **F-BF 2**	**Focus Question:** • How can we find specific terms in an arithmetic sequence? **Inquiry Question:** Page 1013 Problems # 103-105	• In a arithmetic sequence, each term after the first differs from the preceding term by a constant, the common difference. Subtract any term from the term that directly follows to find the common difference. • General term or nth term: $a_n = a_1 + (n-1)d$. The first term in a_1 and the common difference is d. • Sum of the first n terms: $S_n = \frac{n}{2}(a_1 + a_n)$	**Vocabulary:** arithmetic sequence, common difference, nth partial sum **Concepts:** • General Term of an Arithmetic Sequence • The Sum of the First n Terms of an Arithmetic Sequence	• Find the common difference for an arithmetic sequence. • Write terms of an arithmetic sequence. • Use the formula for the general term of an arithmetic sequence. • Use the formula for the sum of the first n terms of an arithmetic sequence.	**Common Core Problems:** **10.2:** #1-5, 61-71, 72-75, 78-84 **Test Prep:** **10.2:** #27-30, 35-38, 45-50
2 days	**Lesson 10.3: Geometric Sequences and Series** SMP: 1,3,5,6 (pp. 1023-1039) **F-BF 2**	**Focus Question:** • How can we find specific terms in a geometric sequence? • How can we calculate the growth of annuities? **Inquiry Question:** Page 1023 Problems #85-87	• In a geometric sequence, each term after the first is obtained by multiplying the preceding term by a nonzero constant, the common ratio. Divide any term after the first by the term that directly precedes it to find the common ratio. • An annuity is a sequence of equal payments made at equal time periods. The value of an annuity is the sum of all deposits made plus all interest paid. The deposit made at the end of each period, the annual interest rate, compounded n times per year, and the number of years deposits have been made.	**Vocabulary:** geometric sequence, common ratio, nth partial sum, annuity, value of an annuity, infinite geometric series, multiplier effect **Concepts:** • General Term of a Geometric Sequence • Sum of the First n Terms of a Geometric Sequence • Value of an Annuity: Interest Compounded n Times per Year	• Find the common ratio of a geometric sequence. • Write terms of a geometric sequence. • Use the formula for the general term of a geometric sequence. • Use the formula for the sum of the first n terms of a geometric sequence. • Find the value of an annuity. • Use the formula for the sum of an infinite geometric series.	**Common Core Problems:** **10.3:** #1-10, 65-87, 88-96, 101-110 **Test Prep:** **10.3:** #1-10, 25-27, 43-44

Common Core Practices

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- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 day	**Lesson 10.5: The Binomial Theorem** SMP: 1,3,5,6 (pp. 1048-1056) **A-APR 5**	**Focus Question** • What are some efficient methods for raising binomials to higher powers? **Inquiry Question** Pg.1048 Problems #46-48	• If we use binomial coefficients and the pattern for the variable part of each term, a formula called the Binomial Theorem can be used to expand any positive integral power of a binomial.	**Vocabulary:** binomial coefficient, Binomial Theorem **Concepts:** • Binomial Theorem	• Evaluate a binomial coefficient. • Expand a binomial raised to a power. • Find a particular term in a binomial expansion.	**Common Core Problems:** **10.5:** #1-7, 57-58, 59-66, 73-85 **Test Prep:** **10.5:** #11-16, 39-42
2 days	**Lesson 10.6: Counting Principles, Permutations, and Combinations** SMP: 1,3,5,6 (pp. 1056-1067) **S-CP 9**	**Focus Question** • How can we calculate the number of ways in which a series of things can occur? **Inquiry Question:** Pg. 1056 Problems # 86-87	• The Fundamental Counting Principle: The number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur. • A permutation from a group of items occurs when no item is used more than once and the order of arrangement makes a difference. • A combination from a group of items occurs when no item is used more than once and the order of items makes no difference.	**Vocabulary:** tree diagram, permutation, combination, permutation **Concepts:** • Fundamental Counting Principle • Methods for counting possible outcomes	• Use the Fundamental Counting Principle. • Use the permutations formula. • Distinguish between permutation problems and combination problems. • Use the combinations formula.	**Common Core Problems:** **10.6:** #1-5, 29-72, 73-80, 83-93 **Test Prep:** **10.6:** #29-38, 41-48

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
3 days	**Lesson 10.7: Probability** SMP: 1,3,5,6 (pp. 1067-1082) **S-CP 2**, **S-CP 3**, **S-CP 6**, **S-MD 3**	**Focus Question** • How does the relationship of events affect empirical and theoretical probability? **Inquiry Question:** Pg. 1067 Problems # 95-97	• Empirical probability applies to situations in which we observe the frequency of the occurrence of an event. • Theoretical probability applies to situations in which the sample space of all equally likely outcomes is known. • If it is impossible for events A and B to occur simultaneously, the events are mutually exclusive. • Two events are independent if the occurrence of either of them has no effect on the probability of the other. • The probability of a succession of independent events is the product of each of their probabilities.	**Vocabulary:** empirical probability, experiment, sample space, event, theoretical probability, mutually exclusive, independent events **Concepts:** • Computing: ○ Empirical Probability ○ Theoretical Probability ○ Probability of an Event Not Occurring ○ Probabilities with Mutually Exclusive Events ○ Probabilities with Events Not Mutually Exclusive ○ Probabilities with Independent Events	• Compute empirical and theoretical probability. • Find the probability that: an event will not occur, and of one event and/or a second event occurring.	**Common Core Problems:** **10.7:** #1-9, 25-53, 54-63, 64-73 **Test Prep:** **10.7:** #1-10, 29-30, 31-36, 37-42
1 day	**Topic 6 Performance Task** (p. 1086 #83-88)					
2 days	**Review Topic 6 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
1 day	**Topic 6 Assessment** (Created and provided by PUSD/Teachers)					

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Name ___________________________ Per____

Performance Task Activity: Chapter 10

Instructions: In order to get full credit, you MUST write your answers in complete sentences.

TASK 1

Read and solve the following problems. Then answer the questions.

1. A company offers a starting yearly salary of $33,000 with raises of $2,500 per year. Find the total salary over a ten-year period

 ____

2. A union contract species that each worker will receive a 5% pay increase each year for the next 30 years. One worker is paid $20,000 the first year. What is this person's total lifetime salary over a 30-year period?

 ____

I. Which is an example of arithmetic sequences? How do you know?
Explain____
____

II. Which is an example of geometry sequences? How do you know? Is it a finite or infinite?
Explain____
____

Task 2

You evaluate six overhead light fixtures to find the intensity of light at work statations that are about 2 m from the fixture. Use the data below. Roung to the nearest whole percent, if necessary.

Trial Number	1	2	3	4	5	6
Intensity of Light (lux)	102	99	105	97	100	98
Distance from the Light Fixture (m)	2.1	2.0	1.7	2.2	2.1	1.9

a. What is the probability that a light fixture selected at random is more that 2 m from a work startion? Has intensity of less than 99 lux?

b. What is the probability that a light fixture selected at ramdom is more than 2 m from a work station and has and intensity of more than 100 lux?

c. What is the probability that a light fixture selected at ramdom has an intesity of more than 100 lux or less than 99 lux?

d. Which of the above answers is an example of an independet/dependent probability? How do you know?

e. Which of the above answers is an example of mutually exclusive/non mutually exclusive probability? How do you know

Task 3: Cassie is planning her outfits for school.

a. She has seven skirts, five blouses, and ten pairs of shoes. How many possible outfits can she wear?

b. Cassie decides that four of her skirts should not be worn to school. How many possible outfits can she wear to school today?

c. Two of Cassie's friends come over and share her clothes. In how many different ways can the three girls wear the seven skirts?

d. Cassie has six different colors bracelets. In how many different ways can she wear the back on Monday, the red one on Tuesday, and the white one on Wednesday?

e. Which of the above questions is/ are examples of Combinations and Permutations? Explain your thinking.