Mathematics Curriculum Guide

Plane Geometry ~ Senior Campus

2017-18

Topic 7: Two-Dimensional Area & Three-Dimensional Volume

Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Essential Questions:

- How do you find the perimeter and area of polygons on the coordinate plane?
- How do perimeters and areas of similar polygons compare?
- How do you apply density based on area in modeling situations?
- How can ratios be used to compare the perimeters and areas of similar figures?
- How can you determine the intersection of a solid and a plane?
- How do you find the volume of three-dimensional figures such as: a prism, a cylinder, a pyramid, a cone, and a sphere?
- How do the volumes of similar solids compare?

Standards: G-GPE-7, G-MG-2, G-GMD 1, G-GMD 3, G-GMD 4, G-GMD-5

Suggested Timeframe: 4 weeks/20 days
Start Date: March 26, 2018
Assessment Dates: April 26-27, 2018

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 day	**Opening Activity:** *See pages 614-615 #1-14 Concept Byte: Transforming to Find Area*					
1 day	**Lesson 1-8: Perimeter, Circumference, and Area** (pp. 59-67) SMP: 1,3,4,7 G-GPE 7	**Focus Question:** • Explain two different ways we can measure geometric figures and what information you need about the figure to find those measures. **Inquiry Question:** p. 59 Solve It!	• The perimeter of a polygon is the sum of the lengths of its sides. • The area of a polygon is the number of square units it encloses. • Formulas can be used to find the perimeter (or circumference) and area. • Distance / length is measured in units while area is measured in square units. • Area of an irregular shape can be found by adding the areas of non-overlapping regular shapes contained within it.	**Vocab:** perimeter, circumference, area, π **Concepts:** • Perimeter and Area formulas for rectangles and triangles • Circumference and Area formulas for circles • Distance Formula	• Determine the perimeter and area of rectangles and triangles (including problems on the coordinate plane that may require distance formula). • Determine the circumference and area of circles (including problems on the coordinate plane that may require distance formula). • Calculate area of an irregular shape by breaking it into smaller, regular shapes and adding those areas together. • Use appropriate units with calculations.	**Common Core Problems:** #4,5,6, 34-37, 38, 39, 40, 41, 42, 44-46, 47, 48, 53, 55, 56, 60, 61 **Thinking Maps:** Tree Map divided into two branches for perimeter and area. Leave extra room on the area side to add formulas throughout the topic.

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days	**Lesson 10-1: Areas of Parallelograms and Triangles** SMP: 3,4,5,6 (pp. 616-622) **G-GPE 7**	**Focus Question:** • What dimensions of a triangle or parallelogram are needed to calculate its area? **Inquiry Question:** p. 616 Solve It!	• The area of a parallelogram or a triangle can be found when the length of its base and its height are known. • The area of a triangle is found using the formula $A = \frac{bh}{2}$. • The area of a parallelogram is found using the formula $A = bh$. • The area of an equilateral triangle can be derived from 30-60-90 triangle or $A = \frac{s^2\sqrt{3}}{4}$.	**Vocab:** height, base, area, perimeter, altitude, hypotenuse **Concepts:** • Pythagorean Theorem • Special Right Triangles • Distance Formula • Formula for the area of a triangle and parallelogram	• Use Pythagorean Theorem, Distance Formula or Special Right Triangles to find the height or base of a triangle or parallelogram. • Calculate the area of a triangle. • Calculate the area of an equilateral triangle given the formula (*Not in this section but necessary for Volume in Topic 7) Calculate the area of a parallelogram.	**Common Core Problems:** #5,6,22, 23, 31, 36 **Thinking Maps:** Add to the Tree Map created in the first lesson of this topic.
3 Days	**Lesson 10-2: Areas of Trapezoids, Rhombuses, and Kites** SMP: 1,3,4,6 (pp. 623-628) **G-GPE 7**	**Focus Question:** • What dimensions of a trapezoid, rhombus or kite are needed to calculate its area? **Inquiry Question:** p. 623 Solve It!	• The height of a trapezoid is the perpendicular distance between the bases • The area of a trapezoid is found using the formula: $A = \frac{1}{2}h(b_1 + b_2)$ • The area of a kite is found using the formula: $A = \frac{d_1 d_2}{2}$ • The area of a rhombus is found using the formula: $A = \frac{d_1 d_2}{2}$	**Vocab:** base, height, diagonal, altitude, trapezoid **Concepts:** • Pythagorean Theorem • Special Right Triangles • Distance Formula • Formula for the area of a trapezoid, kite, and rhombus	• Identify the necessary dimensions for finding the area of a kite, trapezoid or rhombus. • Use Pythagorean theorem, distance formula or special right triangles to find: 1) the height or bases of a trapezoid, and 2) the diagonals of a kite or rhombus. • Calculate the area of a kite, trapezoid or rhombus.	**Common Core Problems:** #7-10, 26, 28, 40, 41 **Thinking Maps:** Add to the Tree Map created in the first lesson of this topic.
1 Day	**Lesson 10-4: Perimeters and Areas of Similar Figures** SMP: 1,3,4,5,7,8 (pp. 635-641) **G-GPE 7, G-GMD 5**	**Focus Question:** • If the scale factor between two similar figures is a/b then what is the ratio of their perimeters? • What is the ratio of their areas? **Inquiry Question:** p. 635 Solve It!	• Ratios can be used to compare the perimeters and areas of similar figures. • If the scale factor between two similar figures is a/b, then the ratio of the perimeters is a/b and the ratio of the areas is $\frac{a^2}{b^2}$.	**Vocab:** ratio, area, proportion, perimeter, similar, scale factor **Concepts:** • Formula for the area of a regular polygon and an equilateral triangle	• Use a scale factor to find ratio of areas or ratio of perimeters. • Find the perimeters and areas of similar polygons.	**Common Core Problems:** #5, 6, 7, 8, 31, 32, 41, 46, 47 **STEM:** #40, 47 **Thinking Maps:** Add to the Tree Map created in the first lesson of this topic.

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day	**Performance Task:** *MKL Population Density (see pages attached to this document)*					
1 Day	**Review Lessons 1.8, 10.1, 10.2, 10.4 Concepts and Skills** Teachers may want to focus on … complex figures such as the one used on Pg. 620 #24 – 30, and polygons on the coordinate plane if that hasn't been interwoven throughout lessons already for population/crop density (sample lesson below)					
1 Day	**Review Topic 6 Test**					
1 Day	**Opening Activity:** How Much Money is That? http://robertkaplinsky.com/work/drug-money/					
1 Day	**Lesson 11-1: Space Figures and Cross Sections** SMP: 1,2,3,4,5,7 (pp. 688-695) **G-GMD 4**	**Focus Question:** • How are the number of faces, F, vertices, V, and edges, E, of a polyhedron related to each other? • What is a cross section? **Inquiry Question:** p. 688 Solve It!	• A three-dimensional figure can be analyzed by describing the relationships among its vertices, edges, and faces. • A cross section is the intersection of a three-dimensional figure and a plane.	**Vocab:** polyhedron, face, edge, vertex, cross section, net **Concepts:** • Euler's Formula	• Examine and describe cross sections of polyhedra and recognize their parts and cross sections. • Apply Euler's Formula to verify two-dimensional nets of three-dimensional figures.	**Common Core Problems:** #4, 5, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40 **Thinking Maps:** A Circle Map to record learning in this lesson.
1 Day	**Lesson 11-4: Volumes of Prisms and Cylinders** SMP: 1,3,4,6,7 (pp. 717-724) **G-GMD 1, G-GMD 3**	**Focus Question:** • What does volume measure? • How do you find the volume of a prism or cylinder? • How are the formulas similar? How are they different? **Inquiry Question:** p. 717 Solve It!	• The volume of a prism is found using the formula: $\boldsymbol{V = Bh;\ B = base\ area}$ • The volume of a cylinder is found using the formula: $\boldsymbol{V = Bh;\ B = base\ area}$ **or** $\boldsymbol{V = \pi r^2 h}$	**Vocab:** volume, base area, height, cubic units **Concepts:** • Area of a circle • Area of a triangle • Area of a rectangle • Area of a parallelogram • Radius of a circle is half the diameter • Slant height vs altitude • Volume of a Prism and Cylinder	• Find the volume of a prism using the formula: $\boldsymbol{V = Bh;}$ $\boldsymbol{B = base\ area}$ • Find the volume of a cylinder using the formula: $\boldsymbol{V = \pi r^2 h}$	**Common Core Problems:** #21, 22, 26, 28, 29, 30, 35, 45 **STEM:** #28, 44 **Thinking Maps:** Begin a Tree Map to identify the volume formulas for this chapter.

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day	**Lesson 11-5: Volumes of Pyramids and Cones** SMP: 1,3,4,7 (pp. 726-732) **G-GMD 3**	**Focus Question:** • How do you find the volume of a pyramid or cone? • How do the volumes of right solids compare to the volume of oblique solids with the same dimensions? **Inquiry Question:** p. 726 Solve It!	• The volume of a pyramid is found using the formula: $V = \frac{1}{3}Bh;\ B = base\ area$ • The volume of a cone is found using the formula: $V = \frac{1}{3}Bh;\ B = base\ area$ **or** $V = \frac{1}{3}\pi r^2 h$	**Vocab:** volume, slant height, altitude, base area, radius, diameter, cubic units **Concepts:** • Area of a circle • Radius of a circle is half the diameter. • Area of a rectangle • Area of triangle • Special Right Triangles • Slant height vs altitude • Volume of a Pyramid and Cone	• Find the volume of a pyramid using the formula: $V = \frac{1}{3}Bh;$ $B = base\ area$ • Find the volume of a cone using the formula: $V = \frac{1}{3}\pi r^2 h$	**Common Core Problems:** #5, 15, 16, 21, 27, 29 **STEM:** #15, 16, 27, 28 **Thinking Maps:** Add to the Tree Map started in lesson 11-4.
1 Day	**Lesson 11-6: Surface Areas and Volumes of Spheres** SMP: 1,3,4,6,7,8 (pp. 733-740) **G-GMD 1**	**Focus Question:** • How do you find the surface area of a sphere? • How do you find the volume of a sphere? **Inquiry Question:** p. 733 Solve It!	• The surface area of a sphere is found using the formula: $S.A. = 4\pi r^2$ • The volume of a sphere is found using the formula: $V = \frac{4}{3}\pi r^3$	**Vocab:** sphere, radius, diameter **Concepts:** • Radius of a circle is half the diameter • Surface Area Formula for a Sphere • Volume Formula for a Sphere	• Find the volume of a sphere using the formula: $V = \frac{4}{3}\pi r^3$	**Common Core Problems:** #30, 32,43,46,47,48,52 **STEM:** #31, 52, 53 **Thinking Maps:** Add to the Tree Map started in lesson 11-4.
2 days	**Review Topic 7 Concepts and Skills**					
2 days	**Topic 7 Assessment**					

Common Core Practices

☐ Instruction in the Standards for Mathematical Practices
☐ Use of Talk Moves
☐ Note-taking
☐ Use of Manipulatives
☐ Use of Technology
☐ Use of Real-world Scenarios
☐ Project-based Learning
☐ Thinking Maps

UnCommon Core Mission Statement
We're real public school teachers, creating lessons that reinforce Common Core standards and honor the unique perspectives of your students.

Lesson Description
An estimated 300,000 marchers listened to Martin Luther King, Jr.'s "I Have a Dream" speech on August 28, 1963. But just how densely packed was the National Mall that day?

In this assignment, students use a scaled drawing of the National Mall to determine its actual area. They must measure the sides (and radii) of various shapes, calculate area, convert units, use scale factors, and interpret their findings.

High school geometry students should be able to do this assignment with little help from the teacher. The assignment is appropriate for middle school classes when taught using the scaffolding ideas presented in the lesson plan.

Along with the handout and lesson plan, a key is also provided.

I am very excited about Common Core's "Standards for Mathematical Practice," which aim to make our students competitive in college and the workplace. This particular assignment addresses many of those standards:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Of course, Domain Standards are also addressed. They are listed with the lesson plan.

March for the Dream

Objective: Students use geometry to calculate the crowd density of people present at Dr. King's historic "I Have a Dream" speech. Students will demonstrate an understanding of measurement, area, scale factors, ratios, and unit conversion.

Target Age Group: High school geometry students should be able to do this assignment with little scaffolding from the teacher. The assignment is appropriate for middle school classes when taught using the suggested scaffolding ideas (see next page).

Materials needed: Copies of the *March for the Dream* handout; *March for the Dream* answer key; rulers

Standards

Grade 6

Geometry – Solve real-world and mathematical problems involving area, surface area, and volume

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Grade 7

Geometry

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing…

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles,

Lesson Plan

1. Engage students by showing a YouTube clip of Dr. King's speech. Pause the video during a shot of the audience, and ask students to comment. You might ask your class the following questions: How many people do you think there were that day? What were they marching for? Have you ever been in a crowd that big?

2. Pass out *March for the Dream*. Advanced students should be able to complete the handout without additional information. It can be given as an in-class partner project, solo task, or homework assignment. For lower grade (and skill) levels, use the scaffolding ideas on the next page.

3. Grade the assignment using the attached key. Remember that answers are approximate, as we are dealing with millimeters. The final density answer should be between 5 and 5.3 feet^2.

quadrilaterals, polygons, cubes, and right prisms.

Common Core Standards, Geometry

Modeling with Geometry – Apply geometric concepts in modeling situations

> *1. Use geometric shapes, their measures, and their properties to describe objects*
>
> *3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)*

Scaffolding Ideas

A. Write the necessary area formulas on the board:

Circle: $A = \pi (radius)^2$ **Rectangle:** $A = length \bullet width$

Trapezoid: $A = \frac{1}{2} \bullet height \bullet (base_1 + base_2)$

or students can dissect the trapezoid into a rectangle and two triangles ($A = \frac{1}{2} \bullet base \bullet height$)

B. Provide the unit conversions for your students:

1,000 millimeter = 1 meter 1,000,000 millimeters^2 = 1 meter^2

1 foot^2 = 10.7639 meters^2

C. Younger students may only be able to find the area of the diagram. Complete the rest of the handout as a class.

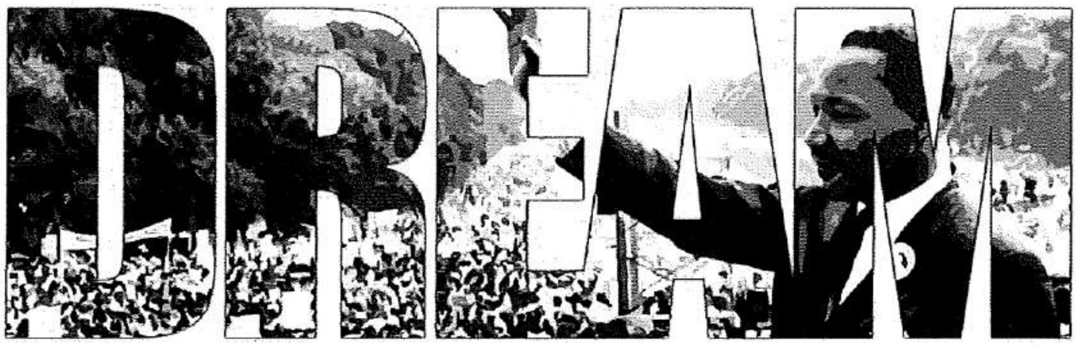
D. Students may have a hard time understanding the difference between length ratios and area ratios.

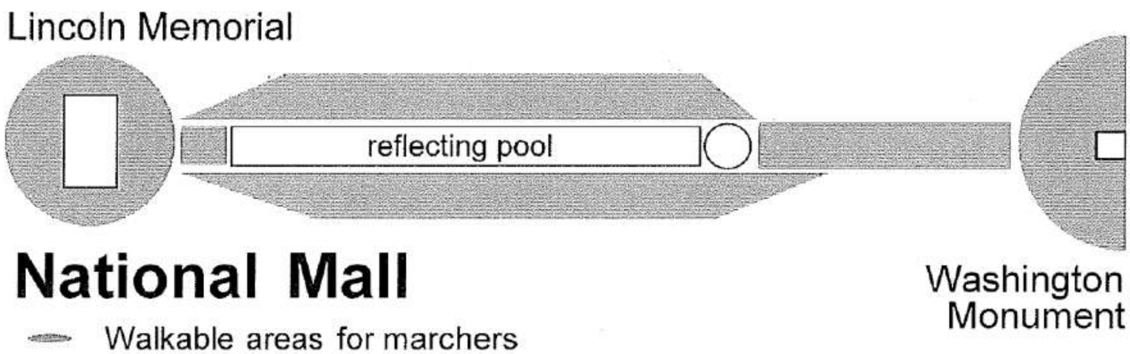
Explain that you obtain $\frac{\text{Area in Diagram}}{\text{Area in Real Life}}$ by squaring $\frac{\text{Length in Diagram}}{\text{Length in Real Life}}$

For this assignment they should do the following calculation:

$$\left(\frac{1 \text{ mm}}{10 \text{ m}}\right)^2 = \frac{1 \text{ mm}^2}{100 \text{ m}^2}$$

March for the





On August 28, 1963, Martin Luther King, Jr. delivered his famous "I Have a Dream" speech on the steps of the Lincoln Memorial. King was speaking to over 300,000 participants of the *March on Washington for Jobs and Freedom.* Look at the above map and determine how packed the crowds were that historic day.

"Crowd-counting professionals" use the following rule of thumb:

mosh pit	very dense	dense	light density
1 person per 2.5 ft^2	1 person per 4.5 ft^2	1 person per 7 ft^2	1 person per 10 ft^2

For point of reference, the reflecting pool is approximately 50 meters wide and 610 meters long.

Find the following two ratios *(hint: they aren't the same)*

$$\frac{\textbf{Length in Diagram}}{\textbf{Length in Real Life}} = \_\_\_\_\_\_\_\_ \quad \& \quad \frac{\textbf{Area in Diagram}}{\textbf{Area in Real Life}} = \_\_\_\_\_\_\_\_$$

Area of the shaded regions on the diagram (in millimeters^2): ________

Area of the walkable regions (in meters^2): ____________

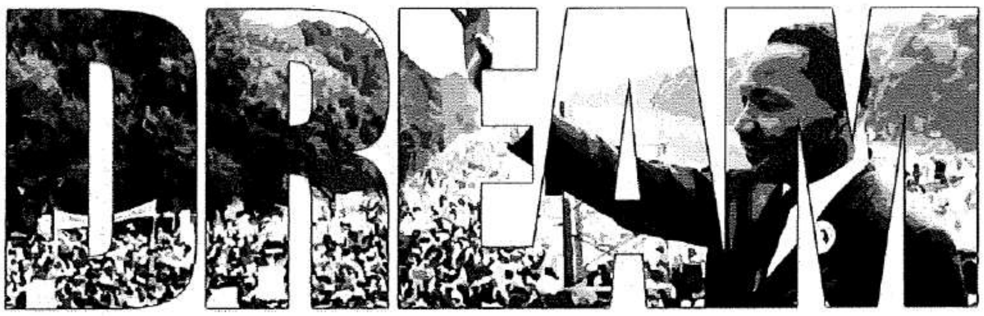
Area of the walkable regions (in feet^2): _________

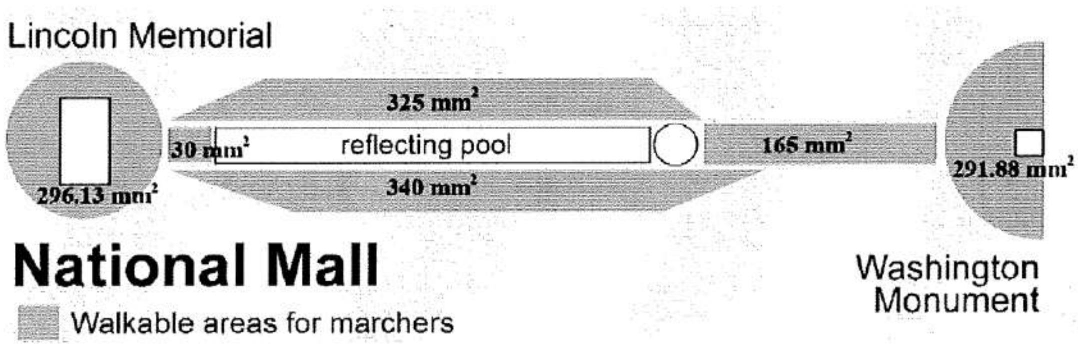
Average crowd density was 1 person for every ___________ feet^2.

Where does this fall on the crowd density spectrum?

KEY

March for the





On August 28, 1963, Martin Luther King, Jr. delivered his famous "I Have a Dream" speech on the steps of the Lincoln Memorial. King was speaking to over 300,000 participants of the *March on Washington for Jobs and Freedom.* Look at the above map and determine how packed the crowds were that historic day.

"Crowd-counting professionals" use the following rule of thumb:

mosh pit	very dense	dense	light density
1 person per 2.5 ft²	1 person per 4.5 ft²	1 person per 7 ft²	1 person per 10 ft²

For point of reference, the reflecting pool is approximately 50 meters wide and 610 meters long.

Find the following two ratios (*hint: they aren't the same*)

$$\frac{\textbf{Length in Diagram}}{\textbf{Length in Real Life}} = \frac{\textbf{5 mm}}{\textbf{50 m}} = \frac{\textbf{1 mm}}{\textbf{10 m}} \quad \& \quad \frac{\textbf{Area in Diagram}}{\textbf{Area in Real Life}} = \frac{\textbf{1 mm}^2}{\textbf{100 m}^2}$$

Area of the shaded regions on the diagram (in millimeters²): **1,448 mm²**

Area of the walkable regions (in meters²): **144,800 m²**

Area of the walkable regions (in feet²): **1,558,610 ft²**

Average crowd density was 1 person for every $\frac{\textbf{1,558,610 ft}^2}{\textbf{300,000}} = \textbf{5.2 ft}^2$

Where does this fall on the crowd density spectrum? **Between very dense and dense.**

INQUIRY QUESTION ...

The city of Sierra Vista, AZ has a population of 45,129 and is 153.5 square miles in land area.

The city of Paramount, CA has a population of 54,908 and is 4.73 square miles in land area.

The city of Diamond Bar, CA has a population of 56,449 and is 14.8 square miles in land area.

Based on the information above, which city do you think would be most crowded? Least crowded? Why? Be prepared to explain your reasoning.

____

____

____

____

POPULATION DENSITY

Population density is a measure of population per unit area. It can be applied to crops (# of plants per acre), people living in a defined area (# of people per square mile), etc.

Use the information above in the "Inquiry Question" to find the population density for each city.

Sierra Vista, AZ	Paramount, CA	Diamond Bar, CA

Compare the population densities found above with some other cities around the world ...

	Population	Land Area (mi^2)	Population Density
Dhaka, Bangladesh	15,414,000	134	
Buenos Aires, Argentina	13,639,000	1,020	
Mexico City, Mexico	19,463,000	790	
Manila, Philippines	21,951,000	550	

CROP DENSITY OF SUNFLOWERS

In a study conducted at the University of Minnesota, sunflowers were planted in different fields and studied to determine the best way farmers could produce as much sunflower seed as possible while using the fewest number of plants possible. (Link at bottom of note guide)

Mark has a field in the shape of a trapezoid with vertices S (0, 0), N (1480, 0), F (1480, 1285), and R (740, 1285) where each unit on the coordinate plane represents one foot. After reading the study done at the University of Minnesota, Mark determines that he needs to plant 15,000 sunflower plants per acre to have the maximum yield.

1) What is the area of Mike's field to the nearest hundredth of an acre? (HINT: 1 acre equals 43,560 ft^2)

2) Approximately how many sunflower plants should Mark plant on his field to have the maximum yield?

http://www.extension.umn.edu/garden/yard-garden/flowers/sunflower-plant-population-and-its-arrangement/

INQUIRY QUESTION ...

The city of Sierra Vista, AZ has a population of 45,129 and is 153.5 square miles in land area.
The city of Paramount, CA has a population of 54,908 and is 4.73 square miles in land area.
The city of Diamond Bar, CA has a population of 56,449 and is 14.8 square miles in land area.
Based on the information above, which city do you think would be most crowded? Least crowded? Why? Be prepared to explain your reasoning. Answers will vary

POPULATION DENSITY

Population density is a measure of population per unit area. It can be applied to crops (# of plants per acre), people living in a defined area (# of people per square mile), etc.

Use the information above in the "Inquiry Question" to find the population density for each city.

Sierra Vista, AZ	Paramount, CA	Diamond Bar, CA
274 people per mi^2	11,608 people per mi^2	3,814 people per mi^2

Compare the population densities found above with some other cities around the world ...

	Population	Land Area (mi^2)	Population Density
Dhaka, Bangladesh	15,414,000	134	115,030 people per mi^2
Buenos Aires, Argentina	13,639,000	1,020	13,372 people per mi^2
Mexico City, Mexico	19,463,000	790	24,637 people per mi^2
Manila, Philippines	21,951,000	550	39,911 people per mi^2

CROP DENSITY OF SUNFLOWERS

In a study conducted at the University of Minnesota, sunflowers were planted in different fields and studied to determine the best way farmers could produce as much sunflower seed as possible while using the fewest number of plants possible. (Link at bottom of note guide)

Mark has a field in the shape of a trapezoid with vertices S (0, 0), N (1480, 0), F (1480, 1285), and R (740, 1285) where each unit on the coordinate plane represents one foot. After reading the study done at the University of Minnesota, Mark determines that he needs to plant 15,000 sunflower plants per acre to have the maximum yield.

1) What is the area of Mike's field to the nearest hundredth of an acre? (HINT: 1 acre equals 43,560 ft^2)

$$\frac{1}{2}(1285)(740+1480)$$

1,426,350 ft^2

32.74 acres

2) Approximately how many sunflower plants should Mark plant on his field to have the maximum yield?

Approximately 491,100 plants

http://www.extension.umn.edu/garden/yard-garden/flowers/sunflower-plant-population-and-its-arrangement/