Mathematics Foundations
Number Sense

A California Department of Education
Child Development Division Presentation

Icebreaker

Use an icebreaker of your choice. There are two icebreaker slides at the end of the presentation for optional use.

Take a Stand

Did you like math when you were in school?

Outcomes

- Become familiar with the research in early childhood and mathematics
- Describe what the implications of this research and instruction are for young children and teachers
- Become familiar with the preschool learning foundations in mathematics with an emphasis on number sense
- Understand the links between the mathematics research in California's Child Development Division's initiatives (Preschool Learning Foundations-Mathematics, PreK Guidelines & Desired Results)
- Identify classroom practices that support all children's mathematical growth and development

Norms

- Start on time and end on time.
- Turn off cell phones or to vibrate.
- Help the group stay on task.
- Listen to thoughts and ideas of others.
- Contribute your thoughts and ideas.

Parking Lot

Please write questions on post-its and place them on chart paper titled "Parking Lot."

Agenda

Research

- What research tells us about the importance of early mathematics
- National Council of Teachers of Mathematics - 5 Big Ideas

California's Preschool Initiatives

- The Prekindergarten Learning and Development Guidelines
- The Early Childhood Environment Rating Scale
- The Desired Results Developmental Profile-Revised
- Preschool Learning Foundations

Research to Practice

- Exploring Number Sense in the classroom

Lessons Learned from Research

- Gaps are striking
- Less is more
- Connect informal and school math
- Meet the needs of all learners
- Include geometry
- Use learning trajectories

Doug Clements, 2007 CPIN presentation

Gaps are Striking

School Mathematics is improving but Not Working Well Enough for Enough Students

Internationally our students are not mathematically competitive

United States

	1994-95	2007
HIGHER	20 countries	9 countries
SAME	14 countries	2 countries
LOWER	7 countries	37 countries

Third International Mathematics and Science Study:Grade 8, 1994–1995 and 2007

Comparisons: SES & Ethnicity

Low SES have gaps - Level 2:

79% of children with mothers with bachelor's degree passed

vs.

32% of those whose mothers with less than a high school degree

Ethnic groups

70% of Asian and 66% of non-Hispanic white children passed

vs.

42% of African American, 40% of Hispanic, 48% of Hawaiian Native or Pacific Islander, and 34% of American Indian or Alaska Native

International Comparisons

There are cultural differences: High income and low income Chinese children performed higher than high income U.S. children.

Using Research

Early mathematics research helps teachers be more effective and have more fun teaching math.

Core Mathematic Content Areas PreK - 8th grade

Number and Operations

Geometry

Measurement

Algebra

Data Analysis & Probability

National Council Teachers of Mathematics (NCTM)

NCTM Focal Points

- Number and Operations
- Geometry
- Measurement

National Council Teachers of Mathematics

NCTM Focal Points for Prekindergarten

Handout 1

Research Bites

- In the table groups, open the research bite.
- Share information and discuss how this informs participant's work.
- There will be an opportunity to share insights with the large group.

Research Bite #1

Infants are born with the ability to understand numerical ideas

"Children possess and build mathematical competencies from their first year and keep on learning mathematical ideas throughout their preschool years."

Clements, D.H. & Sarama, J. "Creative Pathways to Math," *Scholastic Early Childhood Today Journal, 2003.*

For example…

Research Bite # 2

"High quality teaching in mathematics is about challenge and joy, not imposition and pressure. Math instruction includes providing a lot of unit blocks, along with loads of time to use them. It invites children to experience mathematics as they play in, describe, and think about their world."

Clements, D.H., "Mathematics in the Preschool," *Teaching Children,* 2001.

Research Bite #3

"Teachers should provide time, materials, and support for children to engage in play, to nourish their interest in exploration and manipulation of mathematical ideas."

National Association for the Education of Young Children (NAEYC) & National Council of Teachers of Mathematics (NCTM) 2002. *Early Childhood Mathematics: Promoting Good Beginnings, A joint position statement of NAEYC & NCTM*, Washington D.C.

Research Bite #4

"Preschool children explore a variety of mathematical ideas during play including comparison, estimation, patterns, symmetry, and spatial relationships."

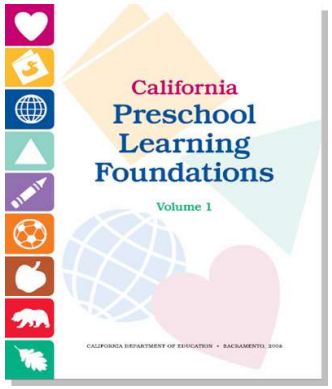
K.-H., Seo. "What Children's Play Tells Us about Teaching Mathematics," *Young Children,* January, 2003.

Research Bite #5

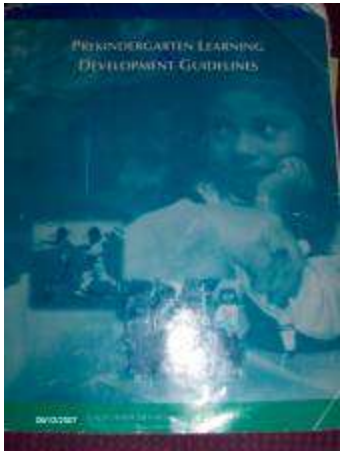
- Find the mathematics in, and develop mathematics from, children's activity.
- Help children extend and "mathematize" every day activities from building blocks, art, songs, and puzzles.
- Create invitations to mathematical activities based on children's experiences and interests.

Clements & Sarama, *Building Blocks Real Math Pre K Math Program*, 2002.

About Children How we teach How we asses

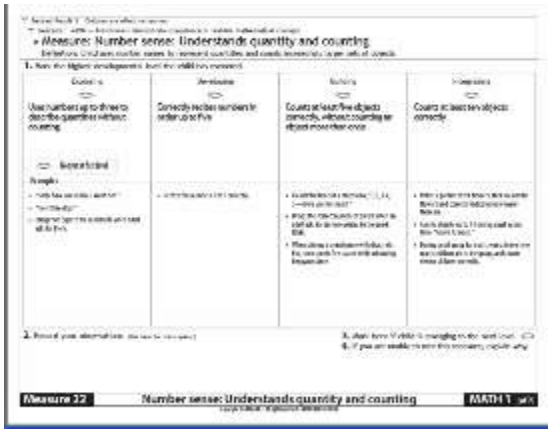


California Preschool Learning Foundations, Volume 1

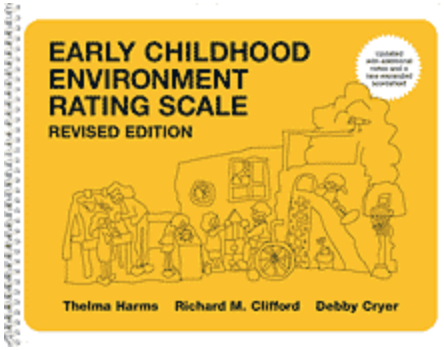


PrekGuidelines

Preschool Framework, Volume 1



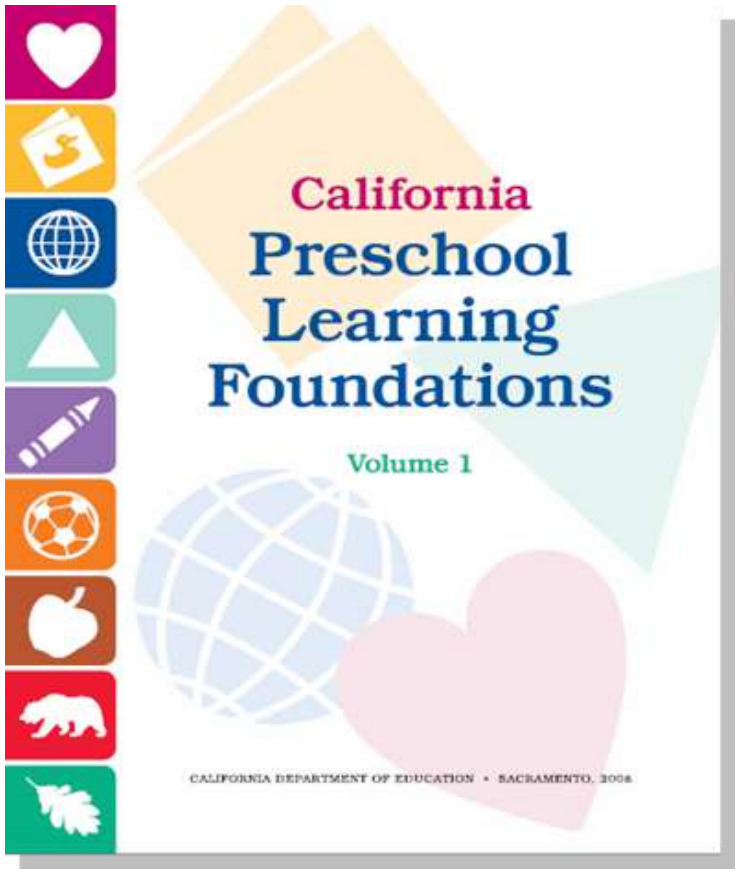
DRDP-R



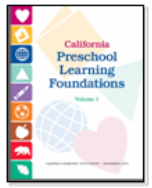
ECERS-R

California Preschool Learning Foundations

Handout 2A



PRESCHOOL FOUNDATIONS DEFINED



The purpose of the preschool foundations is to promote understanding of preschool children's learning and to guide instructional practice.

The preschool learning foundations are for all children, including children learning English and children with disabilities. They describe the knowledge and skills that young children typically exhibit:

- at around 48 and 60 months of age;
- as they complete their first or second year of preschool;
- with appropriate support; and
- when attending a high-quality preschool program.

The preschool foundations are used to:

- Understand children's developing knowledge and skills.
- Consider appropriate ways to support children's learning and development.

The preschool foundations are NOT

- **NOT** a checklist for assessment
- **NOT** a way of assessing children
- **NOT** a test bank of items
- **NOT** a description of how to teach children

California's Preschool Learning Foundations - Mathematics

- Number Sense
- Algebra and Functions (Classification and Patterning)
- Measurement
- Geometry
- Mathematical Reasoning

Map of the Foundations Mathematics

Handout 2B

Strand

Domain

Age

Substrand

Foundations

Includes notes for children with disabilities

148

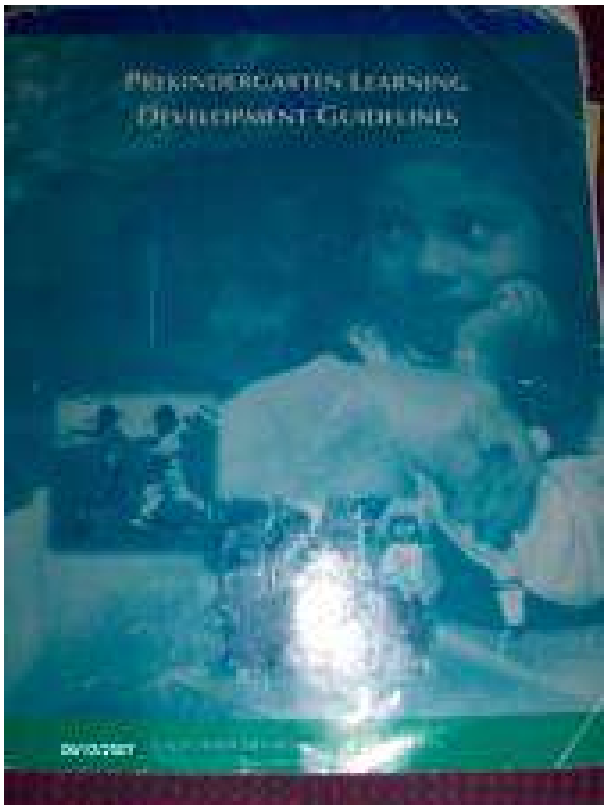
Number Sense*

At around 48 months of age	At around 60 months of age
1.0 Children begin to understand numbers and quantities in their everyday environment.	**1.0 Children expand their understanding of numbers and quantities in their everyday environment.**
1.1 Recite numbers in order to ten with increasing accuracy.†	**1.1** Recite numbers in order to twenty with increasing accuracy.†
Examples	Examples
• Recites one to ten incompletely or with errors while playing (e.g., "one, two, three, four, five, seven, ten"). • Recites one to ten while walking. • Recites one to ten while singing.	• Recites one to twenty incompletely or with errors (e.g., "one, two, three, four, five, . . . nine, ten, eleven, twelve, thirteen, fifteen , seventeen, eighteen, twenty"). • Chants one to twenty in order while swinging. • Recites one to twenty to show her friend how high she can count.
1.2 Begin to recognize and name a few written numerals.	**1.2** Recognize and know the name of some written numerals.
Examples	Examples
• Communicates, "That's a one," when playing with magnetic numerals. • Indicates or points to the numerals on a cube and names, "three, two, five." • Identifies the numeral 3 on the page of the *Five Little Speckled Frogs* book while sitting with a teacher.	• Names some numerals found in books or during a game. • Points to numerals in a number puzzle as the teacher names them.

* Throughout these mathematics foundations many examples describe the child manipulating objects. Children with motor impairments may need assistance from an adult or peer to manipulate objects in order to do things such as count, sort, compare, order, measure, create patterns, or solve problems. A child might also use adaptive materials (e.g., large manipulatives that are easy to grasp). Alternately, a child might demonstrate knowledge in these areas without directly manipulating objects. For example, a child might direct a peer or teacher to place several objects in order from smallest to largest. Children with visual impairments might be offered materials for counting, sorting, or problem solving that are easily distinguishable by touch. Their engagement is also facilitated by using containers, trays, and so forth that contain their materials and clearly define their work space.

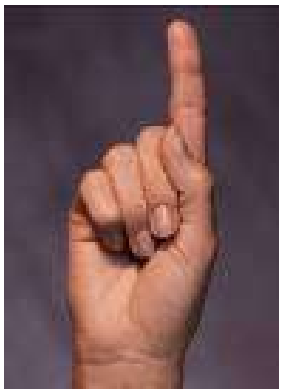
† Some children may not be able to count by either saying the numbers or signing them. Any means available to the child for demonstrating knowledge of numbers in order should be encouraged. For example, a child may indicate or touch number cards or might respond yes or no when an adult counts.

Prekindergarten Learning & Development Guidelines



Chapter 6 - Curriculum: Mathematics Learning and Development

Prekindergarten Learning & Development Guidelines for Mathematics



Program develops and builds on children's existing informal mathematical knowledge, recognizing that children enter preschool with different experiences in mathematics.

Teacher-guided and child-initiated activities are integrated in a mathematically rich learning environment, using multiple instructional approaches.

Prekindergarten Learning & Development Guidelines for Mathematics

The program implements a mathematics curriculum that lays the foundation for children's success in mathematics in elementary school.

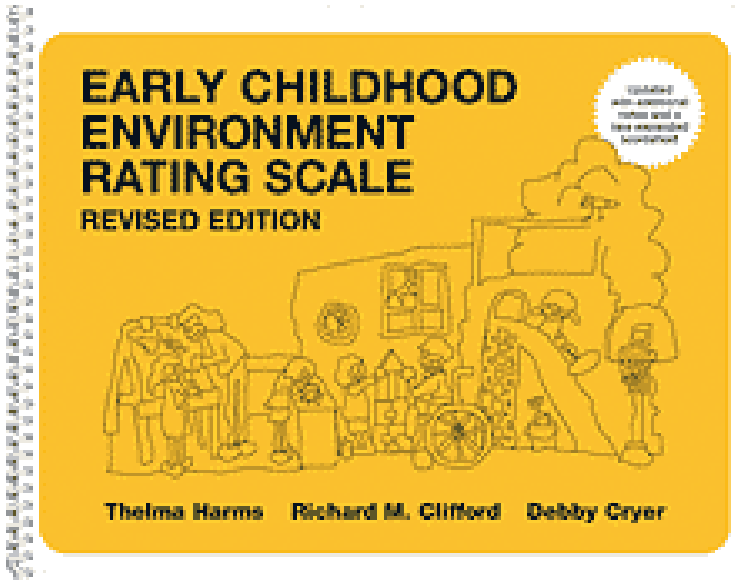
The program identifies clear, age appropriate goals for mathematics learning and development.

The program establishes a partnership with parents and other caregivers in preparing children for mathematics learning.

Prekindergarten Learning & Development Guidelines

Handout 3

Early Childhood Environmental Rating Scales Revised (ECERS-R)

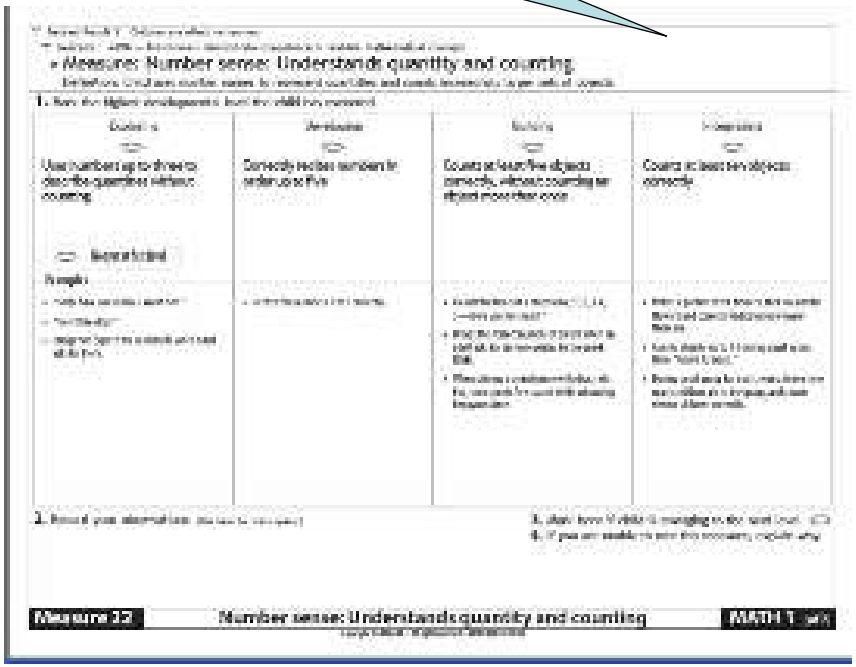


Item 26 - Math/Number:

Different types of materials for math/number help children to experience counting, measuring, comparing quantities, recognizing shapes, and become familiar with the written number.

Desired Results Developmental Profile-Revised (DRDP-R)

Measures 22 & 23



DRDP-R

- An observational assessment instrument
- Developmental continuum
- For all children, including English-language learners and children with disabilities

Desired Result 2:
Children are effective learners

Indicator: MATH-*Preschoolers demonstrate competence in real-life mathematical concepts*

Measure 22: Number sense: Understands quantity and counting

Measure 23: Number sense: Math operations

Measure 24: Shapes

Measure 25: Time

Measure 26: Classification

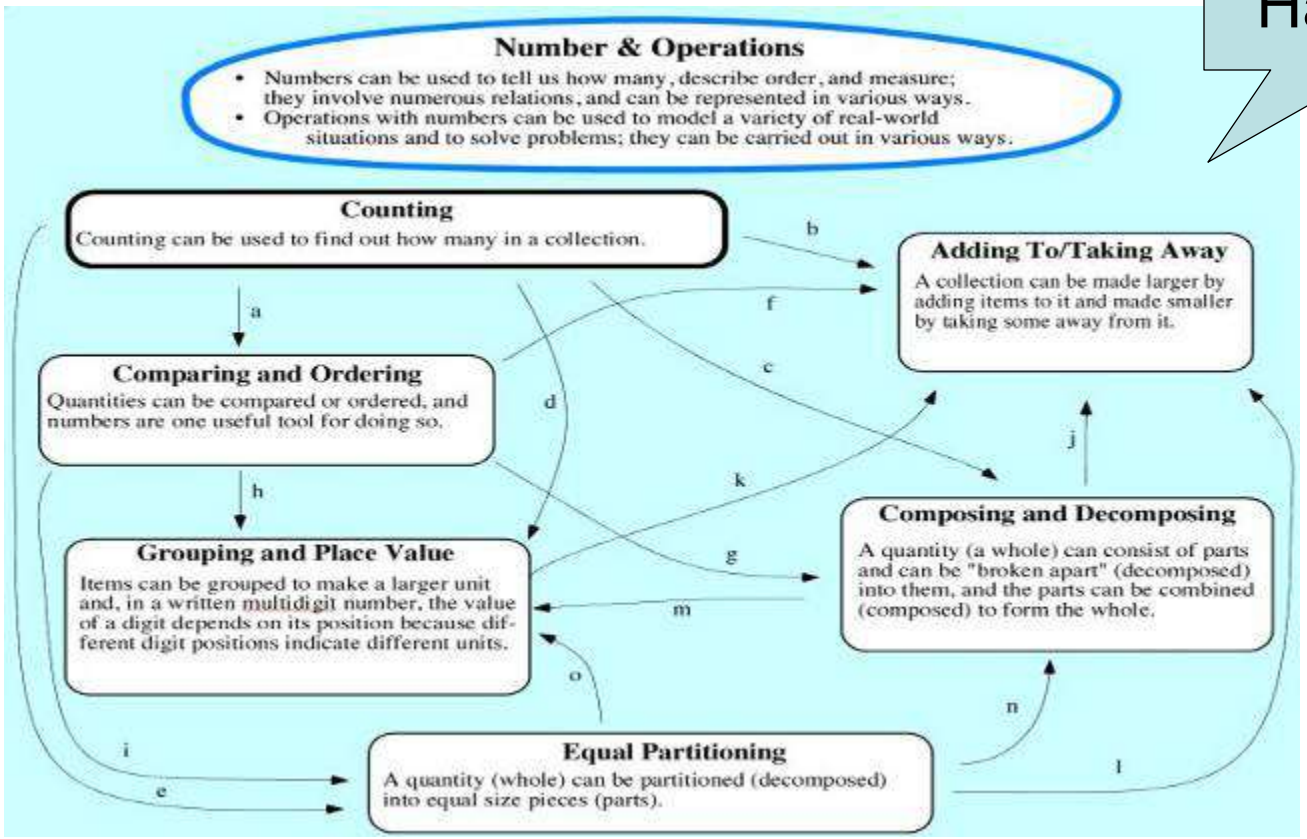
Measure 27: Measurement

Measure 28: Patterning

Number and Operations

Number and Operations

Handout 4



Clements, D.H., *Engaging Young Children In Mathematics*, Lawrence Erlbaum Associates, 2004.

Number and Operations in Play

Counting:

- Recognizing quantities by sight without actually counting objects
- Reading numbers
- Putting objects together or taking objects apart
- Adding or taking away numbers

Seo, K.-H. “What Children’s Play Tells Us about Teaching Mathematics,” *Young Children*, January, 2003.

Activities for Multiple Goals

- Make and imagine small collections of items nonverbally
- Count by ones to ten
- Know the last counting words tells “how many”
- Count out (produce a collection)
- Subitize - quickly “see” and label with a number
- Identify whether collections are the “same” number or which is “more” visually

Clements, D. H., & Sarama, J., *Building Blocks Real Math PreK Curriculum,* 2007.

Recognition of Small Number

- Early number is not counting, but recognition of number
- Recognition of number is not numerals but the understanding of discreet quantities

Clements, D.H., *Engaging Young Children In Mathematics*, 2004, pg. 17.

Clements, D.H., *Engaging Young Children in Mathematics,* 2004, pg. 17.

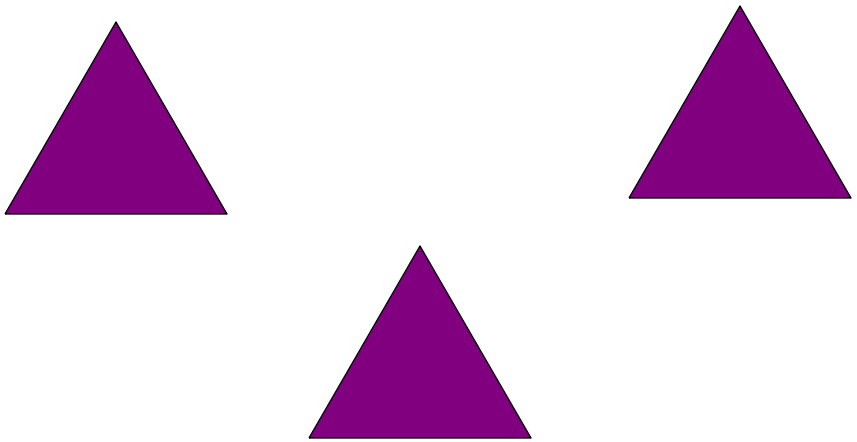
Recognition of Small Number

- Simple, but continuous, intervention makes a big difference - Hannula's 3-year-olds
- Large differences in spontaneous recognition of number

Recognition of Small Number

Teachers can make huge differences by providing thousands of spontaneous experiences!

How many triangles did you see?



That's Subitizing!

Learning Trajectory for Subitizing

- Small collection namer
- Maker of small collections
- Perceptual subitizer to 4
- Perceptual subitizer to 5

Simple, but Continuous, Intervention Makes a Difference!

Developmental Sequence of Counting

Group Activity

Please Close Folders and Notes

- Take cards from envelope
- As a group, place cards in a developmental sequence

Developmental Sequence of Counting

- Saying number words in sequence. May omit some numbers when reciting the number words
- Counts a small set of objects (five or six) but may not have one-to-one correspondence
- May count correctly a larger set of objects (about ten) by keeping track of counted and uncounted objects
- Understands that the number name of the last objects counted represents the total number of objects in the group

Resource: California DRAFT Preschool Mathematics Framework

Developmental Sequence of Counting

- Knows to say the number words one to ten in the correct order, but is still learning the sequence between ten and twenty
- Creates set with a certain number of objects
- Knows to say the number words up to twenty correctly

Resource: California DRAFT Preschool Mathematics Framework

Math Operations

Levels of Math Operations

- **Comparing & Ordering -** Can tell which group has more or less
- **Adding to/Taking away -** Solve problems with three to five objects
- **Composing and Decomposing -** Can develop the ability to recognize that three and two are “hiding inside” five
- **Grouping** - Can count groups of objects

Clements, D.H., *Engaging Young Children in Mathematics,* 2004, pg 20-23.

Meet Jamey


Desired Results
333 N. Lantana, Suite 277
Camarillo, CA 93010
Getting To Know You
Through Observation

Handout #5

LET'S PLAY Worksheet

As you play your game, think about what: 1) the Number Sense **Foundation** the game stresses, 2) **DRDP-R Measure** and the continuum of development the game stresses, 3) the developmental **sequence** the game stresses, and 4) the **goals** you would have for a child in your class.

_____

Name of Game

Foundation (Number Sense) Circle the foundation and check the age level the game stresses	**DRDP-R Measure**	**Counting/Subitizing Math Operations**	**Write a goal that would be appropriate for a child in your class**
Circle **√ 48 mos.** **60 mos.** 1.1 ____ ____ 1.2 ____ ____ 1.3 ____ ____ 1.5 ____ ____ 2.1 ____ ____ 2.2 ____ ____ 2.3 ____ ____ 2.4 ____ ____	**Measure #:** ____ **√ Check:** Exploring Developing Building Integrating		

Handouts 5 & Measures 22 & 23

Let's Play

- Follow the directions to the game.
- After playing the game, talk about the foundations, developmental sequence, DRDP-R measures involved in the game. Use the handout to structure the discussion.
- Write a goal for a child in your class. For example, "Maria will count to five using one-to-one correspondence."
- Later participants will be asked to present the activity to the group. Be ready to share the math concepts in the activity.

Teachers are the **key!**

The Importance of Questions

Mathematics is about thinking, not just doing something with manipulatives.

- How did you know?
- Why did you do it that way?
- How did you figure that out?
- What else would work like this?
- What would happen if?

Handout 6

The Power of Asking “Why?” and “How did you know?”

“You present problems, and they figure out what to do. Then you ask what process they used. I’m amazed they learned to! They’ll use this knowledge to answer science questions. They really do critical thinking. Asking ‘How do you know?’ Starting at Prek is very powerful!”

- Anne, preschool teacher in the TRIAD research project

Clements & Sarama - U.S. Dept. of Education's IES-funded TRIAD research project

In Classrooms that Promote Number Sense, Teachers Will…

What Teachers Do to Support Number Sense in the Classroom…

- Move to the assigned chart.
- Take five minutes to record ideas.
- Move to next chart when signaled.
- Add ideas to next chart and continue on at the signal.
- At the last chart, circle the top three ideas.
- Share top three ideas with group.

Activities for Multiple Goals

- Ability to recognize and duplicate a small collection nonverbally
- Count by ones to ten
- Know the last counting words tells “how many”
- Count out (produce a collection)
- Subitize - quickly “see” and label with a number
- Identify whether collections are the “same” number or which is “more” visually

Clements, D. H., & Sarama, J., *Building Blocks Real Math PreK Curriculum, 2003.*

Tips for Supporting ALL Learners

Handout 7

Tips for Supporting ALL Learners

- Assess what the child knows – scaffold.
- Slow down! Emphasize accuracy with counting.
- Guide the child's hand while counting, if they are working on one-to-one correspondence.
- Repeated practice.
- Provide "wait" time.

Tips for Supporting ALL Learners

- Make it concrete! Count “real” objects.
- Ask the child to make a verbal plan. “Let’s count them by starting at the top.”
- Move items as they are counted.
- Involve the child’s whole body as much as possible.
- Simplify vocabulary and directions

Tips for Supporting English Learners

- Use concise language - speak clearly
- Use oral descriptions when talking about concepts
- Model and act out - Total Physical Response (TPR)
- Consider stages of language development

Let's Play - Part 2

Now revisit your game and consider the following:

- How might you document children's growth and development?
- How might you adapt this game for children with disabilities?
- What are some strategies for English Learners?

Refer to the back side of your Let's Play worksheet to facilitate your discussion and record your ideas.

Handout 5

Handout #5

LET'S PLAY Worksheet - Part 2

Name of the Game: _____

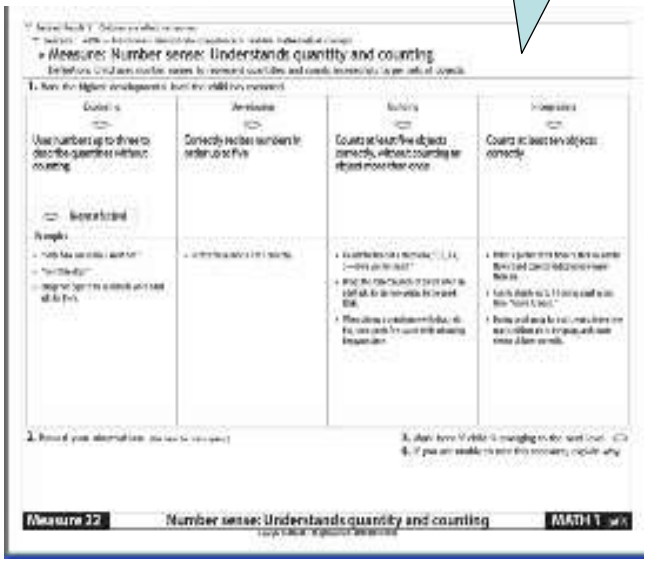
How might you document children's growth and development?	
How might you adapt this game for children with disabilities?	
What are some strategies for English Learners?	

In Classrooms That Promote Number Sense, You Will See Children…

DRDP-R Measures 22 and 23

- Review the measures at the table.
- Brainstorm what children might do at each developmental level.
- Record ideas on the measure pages provided.
- How might you document children's learning?

Handout 8



Taking It Back to the Classroom

Consider:

- The research
- The activities
- The games played and displays
- Conversations
- Handouts
- Ideas that were new or important

Handout 9

Five Little Seashells

Handout 10

Five little seashells sleeping on the shore.

(Hold up 5 fingers, bending down one for each verse)

Swish! went a big wave, and then there were four.

(Move arms, palms up, to make the waves)

Four little seashells quiet as can be.

Swish! went a big wave, and then there were three.

Three little seashells pearly and new.

Swish! went a big wave, and then there were two.

Two little seashells having great fun.

Swish! went a big wave, and then there was one.

One little seashell lying in the sun.

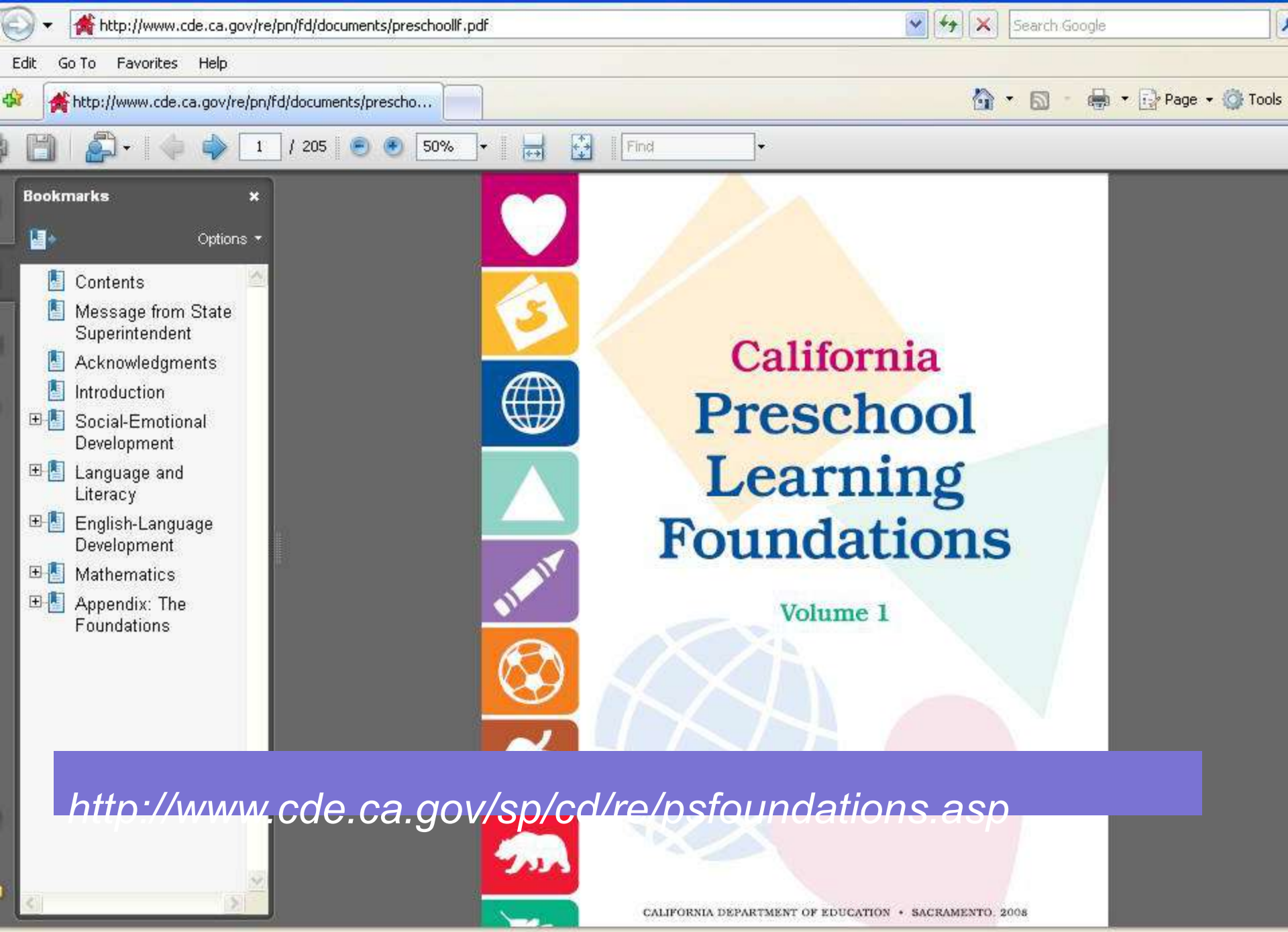
Swish! went a big wave, and then there were none.

Five little seashells gone out to sea.

(Point out to sea)

CDE Web Site

- At the Web address, the underlined Preschool Learning Foundations link leads to the 192 page publication providing easy access to the chapters and sections.
- The Appendix, on pages 173-192, provides a summary list of the foundations.
- Frequently Asked Questions (FAQ) are posted on the Web site. Questions can be sent to psfoundations@cde.ca.gov.


http://www.cde.ca.gov/re/pn/fd/documents/preschoollf.pdf
Search Google
Edit Go To Favorites Help
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Page Tools
1 / 205
50%
Find
Bookmarks
Options
Contents
Message from State Superintendent
Acknowledgments
Introduction
Social-Emotional Development
Language and Literacy
English-Language Development
Mathematics
Appendix: The Foundations
California
Preschool Learning Foundations
Volume 1
CALIFORNIA DEPARTMENT OF EDUCATION • SACRAMENTO, 2008
http://www.cde.ca.gov/sp/cd/re/psfoundations.asp

To Purchase the Preschool Learning Foundations Book

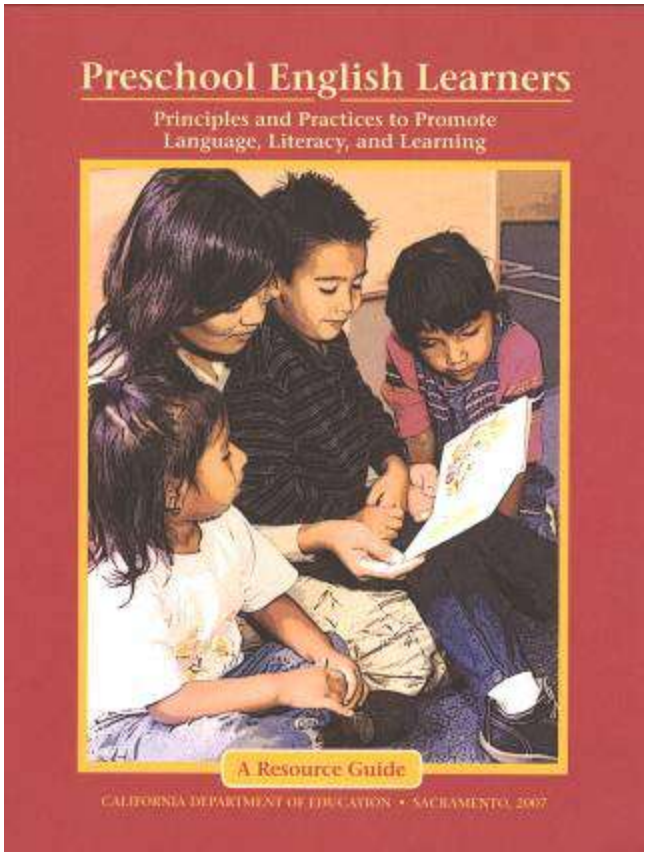
- The Preschool Learning Foundations publication is available for purchase from the CDE Press for $19.95 plus shipping and handling.

- Ordering information can be found at the CDE Web site www.cde.ca.gov/re/pn/rc or by calling 1- 800-995-4099

To Purchase PEL Guide

NOW in Spanish!

- *Preschool English Learners: Principles and Practices to Promote Language, Literacy and Learning* publication is available for purchase from the CDE Press for $15.95.
- Ordering information can be found at the CDE Web site www.cde.ca.gov/re/pn/rc or by calling 1-800-995-4099.
- *Appendix A* is translated into Chinese, Hmong, Korean, Spanish, Tagalog, and Vietnamese, and is available on the CDE Web site.



Take a Stand

Did you like math when you were in school?

CPIN - Region
Contact info added
here.

You can announce
your next event or ???

Thank You for Coming!

Please take a minute and complete the evaluation before leaving.

Optional Icebreakers

Grade Yourself in Math

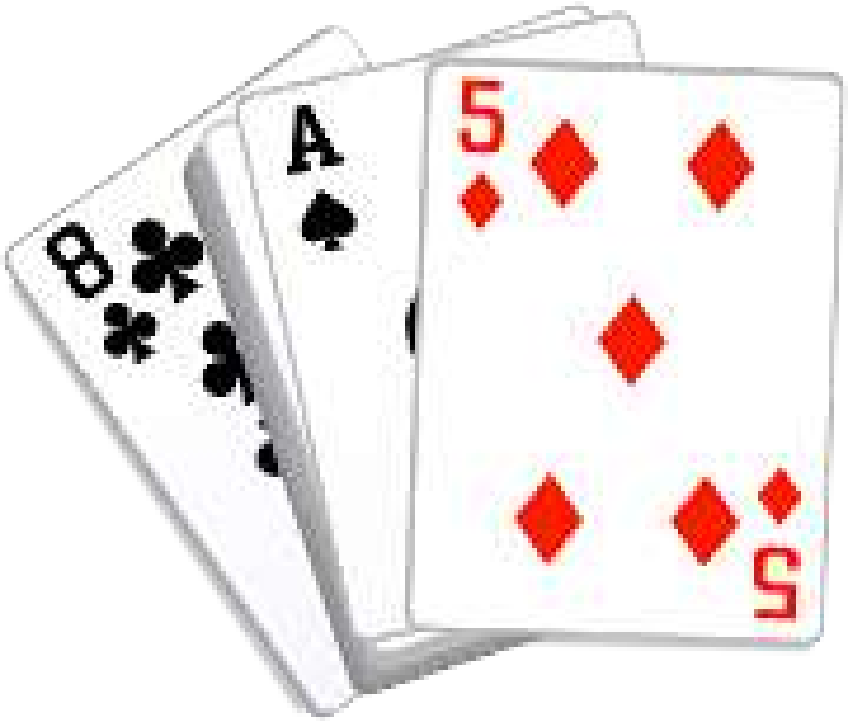
- On a scale of “1” (A+) to “5” (F), how did you perform in math in school?
- In your math group, talk about experiences with Math in school.
- Be ready to share with the whole group.

The following slides are for the debrief of activities and the Make and Take directions. Place them where you think they fit best.

How Did the Activity Support Mathematical Thinking and the Foundations?

- Share your experience with the activity & materials used.
- What foundations did the activity address?
- Are there any changes or additions to make?

Math Make and Take



How to Play “Peace”

- Each player has 5 shuffled cards.
- Place cards face down in a pile in front of you.
- Turn one card over and partner turns one over.
- Whoever has the card with the most dots takes the pair.
- Continue playing until someone has all the cards.