Executive Summary

Principles and Standards for School Mathematics

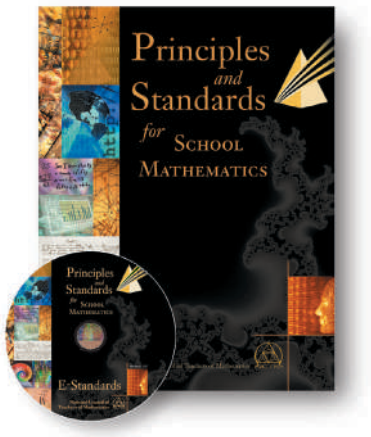
Overview

We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of doing and communicating mathematics continue to emerge and evolve. The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase.

In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. The National Council of Teachers of Mathematics (NCTM) challenges the notion that mathematics is for only the select few. On the contrary, everyone needs to understand mathematics. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

A Foundation for All Students

Principles and Standards for School Mathematics, published by NCTM in 2000, outlines the essential components of a high-quality school mathematics program. It calls for and presents a common foundation of mathematics to be learned by all students. It emphasizes the need for well-prepared and well-supported teachers and administrators. It acknowledges the importance of a carefully organized system for assessing students' learning and a program's effectiveness. It also underscores the need for all partners—students, teachers, administrators, community leaders, and parents—to contribute to building a high-quality program for all students.



What Is *Principles and Standards for School Mathematics?*

Principles and Standards for School Mathematics is a guide for focused, sustained efforts to improve students' school mathematics. It aims to do the following:

- **Set forth a comprehensive and coherent set of learning goals** for mathematics for all students from prekindergarten through grade 12 that will orient curricular, teaching, and assessment efforts during the next decades.
- **Serve as a resource for teachers,** education leaders, and policymakers to use in examining and improving the quality of mathematics instructional programs.
- **Guide the development** of curriculum frameworks, assessments, and instructional materials.
- **Stimulate ideas and ongoing conversations** at the national, state or provincial, and local levels about how best to help students gain a deep understanding of important mathematics.

Educational research shaped many of the proposals and claims made throughout *Principles and Standards.* The document contains references to research on what it is possible for students to learn about certain content areas, at certain levels, and under certain pedagogical conditions. The content and processes emphasized also reflect society's needs for mathematical literacy, past practice in mathematics education, and the values and expectations held by teachers, mathematics educators, mathematicians, and the general public.

Principles and Standards for School Mathematics is organized into four main parts:

- **Principles** for school mathematics
- **An Overview** of the Standards in prekindergarten through grade 12
- **Standards outlining in detail both the content and the processes of school mathematics,** accompanied by corresponding expectations, for four separate grade bands: prekindergarten through grade 2, grades 3–5, grades 6–8, and grades 9–12
- **A discussion** of steps needed to move toward the vision

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The **Principles** are statements reflecting basic precepts that are fundamental to a high-quality mathematics education. The document elaborates the underlying assumptions, values, and evidence on which these Principles are founded. The **Standards** are descriptions of what mathematics instruction should enable students to know and do. Together, the **Principles** and **Standards** constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems. The document includes, as an additional resource, an appendix, "Table of Standards and Expectations," that details the grade-band expectations for each Standard.

Six Principles for School Mathematics

Equity. ***Excellence in mathematics education requires equity—high expectations and strong support for all students.***

All students, regardless of their personal characteristics, backgrounds, or physical challenges, can learn mathematics when they have access to high-quality mathematics instruction. Equity does not mean that every student should receive identical instruction. Rather, it demands that reasonable and appropriate accommodations be made and appropriately challenging content be included to promote access and attainment for *all* students.

Curriculum. ***A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.***

In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepen and their ability to apply mathematics expands. An effective mathematics curriculum focuses on important mathematics that will prepare students for continued study and for solving problems in a variety of school, home, and work settings. A well-articulated curriculum challenges students to learn increasingly more sophisticated mathematical ideas as they continue their studies.

Teaching. ***Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.***

Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in doing mathematics are all shaped by the teaching they encounter in school. To be effective, teachers must understand and be committed to students as learners of mathematics. They must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. Teachers must be supported with ample opportunities and resources to enhance and refresh their knowledge.

Learning. ***Students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge.***

Research has solidly established the important role of conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, students can become effective learners. They will be able to recognize the importance of reflecting on their thinking and learning from their mistakes. Students become competent and confident in their ability to tackle difficult problems and willing to persevere when tasks are challenging.

Assessment. ***Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.***

When assessment is an integral part of mathematics instruction, it contributes significantly to students' mathematics learning. Assessment should inform and guide teachers as they make instructional decisions. The tasks teachers select for assessment convey a message to students about what kinds of mathematical knowledge and performance are valued. Feedback from assessment tasks helps students in setting goals, assuming responsibility for their own learning, and becoming more independent learners.

Technology. ***Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.***

Students can develop deeper understanding of mathematics with the appropriate use of technology. Technology can help support investigation by students in every area of mathematics and allow them to focus on decision making, reflection, reasoning, and problem solving. The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it.

Standards for Pre-K–12 Mathematics

What mathematical content and processes should students know and be able to use as they progress through school? *Principles and Standards for School Mathematics* presents an outline of the focus of school mathematics. High but attainable curriculum standards are required to produce a society that has both the capability to think and reason mathematically and a useful base of mathematical knowledge and skills needed in any walk of life.

The five **Content Standards** explicitly describe the five strands of content that students should learn, whereas the five **Process Standards** highlight ways of acquiring and applying content knowledge. The Standards, which span the entire range from prekindergarten through grade 12, are revisited at each of the four grade bands. Expectations for these grade bands are indicated, discussed, and illustrated with examples. A complete table of the Standards and Expectations by grade is included as an appendix in *Principles and Standards*.

Content Standards

Number and Operations. The Number and Operations Standard deals with understanding numbers, developing meanings of operations, and computing fluently. Young children focus on whole numbers with which they count, compare quantities, and develop an understanding of the structure of the base-ten number system. In higher grades, fractions and integers become more prominent. An understanding of numbers allows computational procedures to be learned and recalled with ease. Students should be able to perform computations in different ways. They should use mental methods and estimations in addition to doing paper-and-pencil calculations. Having computational fluency allows students to make good decisions about the use of calculators. Regardless of the method used to compute, students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate, and general.

Algebra. Algebraic symbols and procedures for working with them are a towering mathematical accomplishment in the history of mathematics and are critical in mathematical work. Algebra is best learned as a set of concepts and techniques tied to the representation of quantitative relations and as a style of mathematical thinking for formalizing patterns, functions, and generalizations. Although many adults think that algebra is an area of mathematics more suited to middle school or high school students, even young children can be encouraged to use algebraic reasoning as they study numbers and operations and as they investigate patterns and relations among sets of numbers. In the Algebra Standard, the connections of algebra to number and everyday situations are extended in the later grade bands to include geometric ideas.

Geometry. Geometry has long been regarded as the place in high school where students learn to prove geometric theorems. The Geometry Standard takes a broader view of the power of geometry by calling on students to analyze characteristics of geometric shapes and make mathematical arguments about the geometric relationship, as well as to use visualization, spatial reasoning, and geometric modeling to solve problems. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills.

Measurement. The study of measurement is crucial in the school mathematics curriculum because of its practicality and pervasiveness in so many aspects of life. The Measurement Standard includes understanding the attributes, units, systems, and processes of measurement as well as applying the techniques, tools, and formulas to determine measurements. Measurement can serve as a way to

integrate the different strands of mathematics because it offers opportunities to learn about and apply other areas of mathematics such as number, geometry, functions, and statistical ideas.

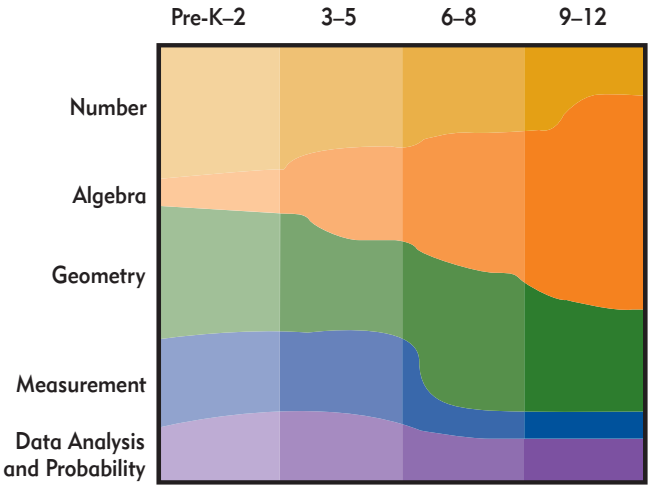
Data Analysis and Probability. Reasoning statistically is essential to being an informed citizen and consumer. The Data Analysis and Probability Standard calls for students to formulate questions and collect, organize, and display relevant data to answer these questions. Additionally, it emphasizes learning appropriate statistical methods to analyze data, making inferences and predictions based on data, and understanding and using the basic concepts of probability.

Process Standards

Problem Solving. Solving problems is not only a goal of learning mathematics but also a major means of doing so. It is an integral part of mathematics, not an isolated piece of the mathematics program. Students require frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort. They are to be encouraged to reflect on their thinking during the problem-solving process so that they can apply and adapt the strategies they develop to other problems and in other contexts. By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom.

Reasoning and Proof. Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. People who reason and think analytically tend to note patterns, structure, or regularities in both real-world and mathematical situations. They ask if those patterns are accidental or if they occur for a reason. They make and investigate mathematical conjectures. They develop and evaluate mathematical arguments and proofs, which are formal ways of expressing particular kinds of reasoning and justification. By exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense.

Communication. Mathematical communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language.



The Content Standards should receive different emphases across the grade bands.

Explanations should include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections.

Connections. Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. When students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a coherent whole. They see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students learn not only mathematics but also about the utility of mathematics.

Representations. Mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, number and letter symbols, spreadsheet displays, and so on. The ways in which mathematical ideas are represented is fundamental to how people understand and use those ideas. Many of the representations we now take for granted are the result of a process of cultural refinement that took place over many years. When students gain access to mathematical representations and the ideas they express and when they can create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena.

Ensuring a High-Quality Mathematics Education for All Students

Principles and Standards provides a catalyst for the continued improvement of mathematics education. It represents the best current understanding of mathematics teaching and learning and the contextual factors that shape it. *Principles and Standards* articulates principles to guide decisions about school mathematics and high, but attainable, standards.

Realizing the vision of mathematics education that is described in *Principles and Standards* requires the continued creation of high-quality instructional materials and technology. It requires enhanced preparation for teachers and increased opportunities for professional growth. It requires the creation of assessments aligned with curricular goals. Realizing the vision depends on the active participation of teachers, students, school administrators, teacher-leaders, policymakers, parents and other caregivers, mathematicians, mathematics educators, and the local community. It will require that the vision be shared and understood and that everyone concerned be committed to improving the future of all children.

Learning with Understanding. Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need them and challenges for those who stand to benefit from them. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment.

Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

Resources

The National Council of Teachers of Mathematics (NCTM) has also produced supporting resources for *Principles and Standards for School Mathematics*. Among them are a set of Frequently Asked Questions about *Principles and Standards,* a Quick Reference Guide that outlines Standards and Expectations by grade band, and an outreach CD to assist those making presentations about *Principles and Standards* or interested in exploring the ten Standards and learning more about them. The Navigations book series translates the *Principles and Standards* into action in the classroom and highlights major mathematics content areas in grade-band-specific volumes. Most recently, the Council published the *Administrator's Guide: How to Support and Improve Mathematics Education in Your School* for school administrators and others responsible for implementing standards, as well as *A Research Companion to "Principles and Standards for School Mathematics."* All are available for sale from NCTM Customer Service (800-235-7566) or the NCTM Web site (www.nctm.org/catalog).

National Council of Teachers of Mathematics

Since 1920, the National Council of Teachers of Mathematics has been dedicated to the improvement of school mathematics at all levels. Through its publications, conferences, Web site offerings, and other services, this professional organization of more than 90,000 members provides a forum for discussing new developments, sharing innovative classroom experiences, and evaluating trends in the teaching of mathematics. NCTM and its 250 Affiliates in the United States and Canada form a network that serves as a clearinghouse of information and resources for all topics related to mathematics education.

A searchable electronic version of *Principles and Standards,* as well as print copies, can be purchased through the NCTM Web site, www.nctm.org, or by contacting

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