# Minnesota Comprehensive Assessments - Series III (MCA-III)

# Mathematics Test Specifications for MCA-III, Grades 3–8 and 11

Based on the Minnesota K–12 Academic Standards in Mathematics (2007 version) Minor edits made to the document in 2019



September 2019

# MINNESOTA DEPARTMENT OF EDUCATION

# Mathematics Test Specifications for MCA-III, Grades 3–8 and 11

For a copy in an alternate format, contact Division of Statewide Testing Minnesota Department of Education 1500 Highway 36 West Roseville, MN 55113-4266 Phone (651) 582.8200 • Fax (651) 582.8874 <u>mde.testing@state.mn.us</u>

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The department thanks the Test Specifications Committee, as well as all of the panelists and teachers who reviewed this document in draft form, for their hard work and continued involvement.

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# **MCA-III MATHEMATICS TEST SPECIFICATIONS**

### Introduction

Mathematics test specifications for grades 3–8 and 11 of the Minnesota Comprehensive Assessments Series III (MCA-III) are presented in this document. The reader is encouraged to read the introductory information carefully because many important concepts are presented, including the purposes of the MCA-III in Mathematics, a description of the cognitive levels and other information about the format of the test specifications.

## **Purpose of the Minnesota Comprehensive Assessments**

The purpose of Minnesota assessments is to measure Minnesota students' achievement with regard to the Minnesota Academic Standards. Assessment results can be used to inform curriculum decisions at the district and school level, inform instruction at the classroom level and demonstrate student academic progress from year to year.

The passage of the No Child Left Behind Act (2000)<sup>1</sup> required that students be assessed in grades 3–8 and high school. The *Minnesota K–12 Academic Standards in Mathematics* were adopted in 2003; the Minnesota Comprehensive Assessments-Series II assessed these standards. The 2006 Minnesota Legislature approved the 2006 Omnibus Education Policy Act (see Minn. Stat. § 120B.023, subd. 2b). This legislation required the revision of the state's academic standards in mathematics in the 2006–2007 school year. The legislation also required that beginning in the 2010–2011 school year, state mathematics tests given in grades 3–8 align with the revised 2007 academic standards in mathematics. In addition, the legislation also required that beginning in the 2013–2014 school year, state mathematics tests given in grade 11 align with the revised 2007 academic standards in mathematics was significant enough that a new series of the MCA assessments was necessary. Thus, the Mathematics MCA-III are aligned with the 2007 *Minnesota K–12 Academic Standards in Mathematics*.

## **Purpose and Overview of the Test Specifications**

The primary purpose of test specifications is to help test developers build a test that is consistent over time. The *Mathematics Test Specifications for MCA-III, Grades 3–8 and 11* is also meant to serve as a source of information about the test design for teachers and the general public. Test specifications do not indicate what should be taught; the Minnesota academic standards do. Test specifications do not indicate how students should be

<sup>&</sup>lt;sup>1</sup> At the time of publication, the federal government was reviewing ESEA for changes and reauthorization. Information in this document is up to date based on time of publication. If changes are made to ESEA that affect this document, then this document will be revised accordingly in a timely manner.

taught; the classroom teacher does. Test specifications indicate which strands, standards and benchmarks will be assessed on the test and in what proportions. In addition, test specifications provide the types of items to be included, number of items and distribution of cognitive levels. Test specifications also clarify, define and/or limit how test items will be written.

As with any test, the MCA-III assess a sampling of student knowledge and do not test every standard or benchmark. There are standards and benchmarks that cannot be assessed with a standardized test. That does not mean that these skills should not be taught or assessed. Teachers need to instruct and assess their students on all of the academic standards. Standards and benchmarks that are not assessed on the MCA-III are indicated in this document with the phrase "Not assessed on the MCA-III." In addition, not all assessable benchmarks will be included on every assessment and some benchmarks are embedded within the assessment of other benchmarks.

Panels consisting of members of the Minnesota Academic Standards Committee and classroom teachers were convened to develop the Mathematics MCA-III Test Specifications. Many of the classroom teachers were recommended to the Minnesota Department of Education (MDE) by various education organizations, school districts and other stakeholder groups.

# **Item Specifications**

Item specifications are provided for each assessed benchmark. The item specifications provide restrictions of numbers, notation, scales, context and item limitations/requirements. The item specifications also list symbols and vocabulary that may be used in items. This list is cumulative in nature. For example, symbols and vocabulary listed at grade 3 are eligible for use in all of the grades that follow (i.e., grades 4 through 8 and 11).

# **Item Specification Considerations**

There are broad item-development issues addressed during the development of test items. Each of the following issues is considered for all of the items developed for the Mathematics MCA-III.

- 1. Each item will be written to measure primarily one benchmark; however, other benchmarks may also be reflected in the item content.
- 2. Items will be appropriate for students in terms of grade-level difficulty, expected knowledge of grade-level mathematical vocabulary, life experiences and reading level.
- 3. At a given grade, items will range in difficulty from easy to challenging for the intended population.
- 4. Items will not disadvantage or disrespect any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability or geographic region.
- 5. Items will be written to meet benchmark calculator requirements as specified in the academic standards and/or test specifications.
- 6. Each item will be written to clearly and unambiguously elicit the desired response.
- 7. For the Mathematics MCA-III, a reference sheet of appropriate formulas and conversions is provided to students in grades 5–11 for use during testing.

- 8. Items will be written according to the *MDE Guidelines for Test Construction*.
- 9. Advisory Panels will review items as specified in the MDE *Vendor Guide to Advisory Panels*.
- 10. Items will be written using principles of Universal Design (see the linguistic modification report from the US Department of Education LEP Partnership<sup>2</sup>). These principles include the following:
  - a. Use active voice rather than passive voice.
  - b. Avoid negation.
  - c. Avoid proper nouns.
  - d. Avoid using general language terms that have a special meaning in math contexts.
  - e. Reduce written context and be as universal as possible.
  - f. To the extent possible, write sentences that are simple and in standard word order.
- 11. A read-aloud of items is provided via built-in audio in the online assessment.

# **Cognitive Complexity**

Cognitive complexity refers to the cognitive demand associated with an item. The level of cognitive demand focuses on the type and level of thinking and reasoning required of the student on a particular item. Levels of cognitive complexity for MCA-III are based on Norman L. Webb's Depth of Knowledge<sup>3</sup> levels.

A Level 1 (recall) item requires the recall of information such as a fact, definition, term or simple procedure, as well as performing a simple algorithm or applying a formula. A well-defined and straight algorithmic procedure is considered to be at this level. A Level 1 item specifies the operation or method of solution and the student is required to carry it out.

A Level 2 (skill/concept) item calls for the engagement of some mental processing beyond a habitual response, with students required to make some decisions as to how to approach a problem or activity. Interpreting information from a simple graph and requiring reading information from the graph is a Level 2. An item that requires students to choose the operation or method of solution and then solve the problem is a Level 2. Level 2 items are often similar to examples used in textbooks.

Level 3 (strategic thinking) items require students to reason, plan or use evidence to solve the problem. In most instances, requiring students to explain their thinking is a Level 3. A Level 3 item may be solved using routine skills but the student is not cued or prompted as to which skills to use.

<sup>&</sup>lt;sup>2</sup> Abedi, J. & Sato, E. (2008). *Linguistic modification. Part I: Language factors in the assessment of English language learners: The theory and principles underlying the linguistic modification approach; Part II: A guide to linguistic modification: Increasing English language learner access to academic content.* Washington, DC: U.S. Department of Education: LEP Partnership. <u>View report online</u> (https://ncela.ed.gov/files/uploads/11/abedi\_sato.pdf).

<sup>&</sup>lt;sup>3</sup> Webb, N. L. *Alignment of science and mathematics standards and assessments in four states* (Research Monograph No. 18). Madison: University of Wisconsin – Madison, National Institute for Science Education, 1999.

Level 4 (extended thinking) items require complex reasoning, planning, developing and thinking, most likely over an extended period of time. Level 4 items are best assessed in the classroom, where the constraints of standardized testing are not a factor.

Using these cognitive complexity levels to categorize items ensures that the complexity of the test items matches the complexity of the content domain assessed. Based on the benchmarks included in the Mathematics MCA-III, Table 1 indicates the target proportion of test items at each cognitive level included in each test.

Grades	Level 1	Level 2	Level 3
3–8, 11	20%	30%	5%

# **Calculator Designation**

The Academic Standards committee identified several benchmarks across grades 3–8 that students should master without using a calculator. These benchmarks are identified in this document with the wording "using common algorithms." Since the MCA-III are aligned with the Minnesota academic standards, the items assessing benchmarks identified as non-calculator will be assessed within a non-calculator segment.

For Grade 11, there are no non-calculator benchmarks. A calculator will be available for student use on all items of the Grade 11 MCA-III.

# **Test Design by Grade Level**

A variety of item types will be used on the Mathematics MCA-III, including multiple-choice (MC) items and technology-enhanced (TE) items. Technology-enhanced items may consist of the following types of responses: type-in (student will type numerical answers in a box), graphing (student will plot data to complete various mathematical displays), drag-and-drop (students will formulate, rather than select, a response using drag-and-drop response options) and hot-spot (students will select multiple correct responses or will mark locations on mathematical graphics and displays). Table 2A indicates the number of operational items appearing on an online adaptive form of the Mathematics MCA-III for each grade level, as well as the range of items within a test form aligned to each strand for grades 3–8. Table 2B gives the same information for the paper accommodated Mathematics MCA-IIIs.

Grade	Number of Operational Items	Number & Operation	Algebra	Geometry & Measurement	Data Analysis (& Probability)
3	42	18–20	6–8	10–13	6–7
4	42	16–20	7–8	10–14	6–7
5	42	15–21	9–13	8–10	6–7
6	42	11–19	10–13	8–11	6–8
7	42	12–16	13–18	7–9	7–9
8	42	6–8	18–29	6–8	6–7
11	47	NA	21–24	13–15	8–13

# Table 2A.Range of Items per Strand for Online Adaptive Mathematics MCA-III<br/>Grades 3–8 and 11

# Table 2B.Range of Items per Strand for Paper Accommodated Mathematics MCA-III<br/>Grades 3–8 and 11

Grade	Number of Operational Items	Number & Operation	Algebra	Geometry & Measurement	Data Analysis (& Probability)
3	50	20–24	8–10	10–13	6–8
4	50	18–22	8–10	12–15	6–8
5	50	18–22	10–14	8–10	6–8
6	50	14–19	12–16	10–12	6–8
7	50	12–16	16–20	8–10	8–10
8	50	6–8	24–30	8–10	6–8
11	56	NA	25–29	16–18	10–15

Table 3A indicates the type and number of items for the online adaptive Mathematics MCA. Table 3B indicates the type and number of items for the paper accommodated Mathematics MCA.

Grade	MC Items (1 point)	Technology- Enhanced Items (1 point)	Total Items/Points
3	32–40	2–10	42
4	32–40	2–10	42
5	32–40	2–10	42
6	32–40	2–10	42
7	32–40	2–10	42
8	32–40	2–10	42
11	40–45	2–7	47

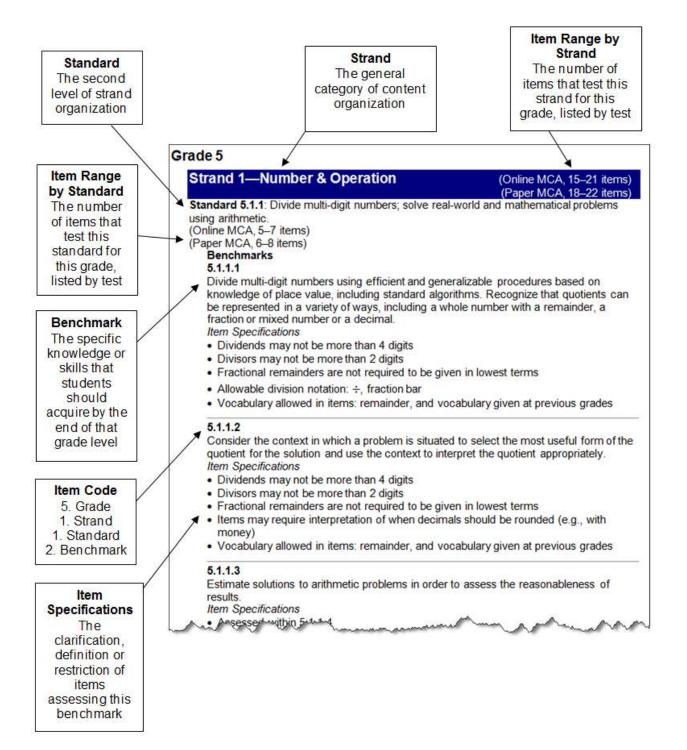
Table 3A.Type and Number of Items for Online Adaptive Mathematics MCA-IIIGrades 3–8 and 11

Table 3B.Type and Number of Items for Paper Accommodated Mathematics MCA-III<br/>Grades 3–8 and 11

Grade	MC Items (1 point)	Fill-In Items (1 point)	Total Items/Points
3	50	NA	50
4	50	NA	50
5	46–48	2–4	50
6	46–48	2–4	50
7	46–48	2–4	50
8	46–48	2–4	50
11	50–54	2–6	56

The minimum and maximum numbers of operational items by standard for the online adaptive and paper accommodated Mathematics MCA-III grades 3–8 and 11 are shown in Tables 5–11, located in Appendix A of this document.

# A Guide to Reading the Test Specifications



# An Explanation of Terms Related to the Grade-Level Tables

Strand: This is the most general categorization of content in the Minnesota Academic Standards.

**Standard:** Standards describe the expectations in mathematics that all students must satisfy to meet state requirements for credit.

**Benchmark:** The purpose of benchmarks is to provide details about "the academic knowledge and skills that schools must offer and students must achieve to satisfactorily complete" the standards (Minn. Stat. § 120B.023 (2006). Benchmarks are intended to "inform and guide parents, teachers, school districts and other interested persons and for use in developing tests consistent with the benchmarks" (Minn. Stat. § 120B.023 (2006)). Each standard is divided into several benchmarks.

**Item Code:** Test developers use this code to identify the strand, standard and benchmark to which a test item is aligned.

**Item Specifications:** These statements provide more specific clarifications, definitions or restrictions for the benchmark as it is assessed on the MCA.

**Item Range by Strand:** This range is the possible number of items that will be on the operational form from a specific strand.

**Item Range by Standard:** This range is the total number of items measuring the standard that could be on the test for the indicated strand. For example, for the paper Grade 3 Mathematics, 20–24 items are from Strand 1. Of those 20–24 Strand 1 items, 5–7 items are from Standard 1 (3.1.1).

# **Mathematics MCA-III Test Specifications Grade-Level Tables**

#### Grade 3

#### Strand 1—Number & Operation

(Online MCA, 18–20 items) (Paper MCA, 20–24 items)

**Standard 3.1.1:** Compare and represent whole numbers up to 100,000 with an emphasis on place value and equality.

(Online MCA, 4–6 items)

(Paper MCA, 5–7 items)

#### Benchmarks

#### 3.1.1.1

Read, write and represent whole numbers up to 100,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives such as bundles of sticks and base 10 blocks.

#### Item Specifications

• Vocabulary allowed in items: digits, value, plot, locate, point

#### 3.1.1.2

Use place value to describe whole numbers between 1000 and 100,000 in terms of ten thousands, thousands, hundreds, tens and ones.

Item Specifications

- Allowable expanded forms: 300 + 60 + 5, 3 hundreds + 6 tens + 5 ones
- Items may ask to identify a place a digit is in or the value of the digit in a place
- Vocabulary allowed in items: digits, value, equal

#### 3.1.1.3

Find 10,000 more or 10,000 less than a given five-digit number. Find 1000 more or 1000 less than a given four- or five-digit number. Find 100 more or 100 less than a given four- or five-digit number.

Item Specifications

• Vocabulary allowed in items: fewer, more, less, greater

#### 3.1.1.4

Round numbers to the nearest 10,000, 1000, 100 and 10. Round up and round down to estimate sums and differences.

#### Item Specifications

• Vocabulary allowed in items: estimate, round, nearest, closest

#### 3.1.1.5

Compare and order whole numbers up to 100,000.

- < and > symbols are not allowed
- Vocabulary allowed in items: least, greatest, compare, order, value

**Standard 3.1.2**: Add and subtract multi-digit whole numbers; represent multiplication and division in various ways; solve real-world and mathematical problems using arithmetic.

(Online MCA, 8–10 items) (Paper MCA, 8–10 items)

#### Benchmarks

#### 3.1.2.1

Add and subtract multi-digit numbers using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

#### Item Specifications

- Addition items may contain 3 whole number addends, at most
- Numbers used may contain 4 digits each, at most
- Items must not have context
- Vocabulary allowed in items: add, subtract, sum, difference, result

#### 3.1.2.2

Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology and the context of the problem, to assess the reasonableness of results.

#### Item Specifications

- Addition items may contain 3 whole number addends, at most
- Numbers used may contain 4 digits each, at most
- Addition and subtraction can be used in the same item
- Vocabulary allowed in items: add, subtract, sum, difference, result

#### 3.1.2.3

Represent multiplication facts using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting. Represent division facts by using a variety of approaches, such as repeated subtraction, equal sharing and forming equal groups. Recognize the relationship between multiplication and division.

#### Item Specifications

- Factors are limited to 1–12
- Variables are not used
- Vocabulary allowed in items: multiply, divide

#### 3.1.2.4

Solve real-world and mathematical problems involving multiplication and division, including both "how many in each group" and "how many groups" division problems.

- Factors are limited to 1–20; 1 factor must have only 1 digit
- Dividend is no greater than 100
- Vocabulary allowed in items: multiply, divide, product

#### 3.1.2.5

Use strategies and algorithms based on knowledge of place value, equality and properties of addition and multiplication to multiply a two- or three-digit number by a one-digit number. Strategies may include mental strategies, partial products, the standard algorithm and the commutative, associative and distributive properties.

#### Item Specifications

- Items must not have context
- The one-digit factor must be 2–6
- Vocabulary allowed in items: multiply, product

**Standard 3.1.3:** Understand meanings and uses of fractions in real-world and mathematical situations.

(Online MCA, 5–7 items) (Paper MCA, 5–7 items)

#### Benchmarks

#### 3.1.3.1

Read and write fractions with words and symbols. Recognize that fractions can be used to represent parts of a whole, parts of a set, points on a number line or distances on a number line.

#### Item Specifications

- Denominators are limited to 2, 3, 4, 6 and 8
- Sets may contain no more than 12 objects
- Vocabulary allowed in items: fraction, plot, locate, point

#### 3.1.3.2

Understand that the size of a fractional part is relative to the size of the whole.

Item Specifications

- Denominators are limited to 2, 3, 4, 6 and 8
- Sets may contain no more than 12 objects
- Vocabulary allowed in items: fraction

#### 3.1.3.3

Order and compare unit fractions and fractions with like denominators by using models and an understanding of the concept of numerator and denominator.

- Denominators are limited to 2, 3, 4, 6 and 8
- Sets may contain no more than 12 objects
- Vocabulary allowed in items: fraction, equal, least, greatest

#### Strand 2—Algebra

**Standard 3.2.1:** Use single-operation input-output rules to represent patterns and relationships and to solve real-world and mathematical problems.

(Online MCA, 2-3 items)

(Paper MCA, 3–4 items)

#### Benchmarks

#### 3.2.1.1

Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.

**Item Specifications** 

- At least 3 iterations of the pattern must be given
- Items may require identification of 3 or fewer terms beyond what is given
- Vocabulary allowed in items: rule, input, output, value

**Standard 3.2.2:** Use number sentences involving multiplication and division basic facts and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.

(Online MCA, 4–5 items) (Paper MCA, 5–6 items)

#### Benchmarks

#### 3.2.2.1

Understand how to interpret number sentences involving multiplication and division basic facts and unknowns. Create real-world situations to represent number sentences.

#### **Item Specifications**

- Variables, boxes or blanks may be used to represent unknown numbers
- Vocabulary allowed in items: number sentence, equation, value, represent

#### 3.2.2.2

Use multiplication and division basic facts to represent a given problem situation using a number sentence. Use number sense and multiplication and division basic facts to find values for the unknowns that make the number sentences true.

- Variables, boxes or blanks may be used to represent unknown numbers
- Vocabulary allowed in items: number sentence, equation, value, represent

Standard 3.3.1: Use geometric attributes to describe and create shapes in various contexts.

(Online MCA, 3–4 items) (Paper MCA, 3–4 items)

#### Benchmarks

#### 3.3.1.1

Identify parallel and perpendicular lines in various contexts, and use them to describe and create geometric shapes, such as right triangles, rectangles, parallelograms and trapezoids.

#### Item Specifications

- When identifying shapes by the attribute of parallel or perpendicular lines, shapes are limited to triangle, parallelogram, rectangle, rhombus, square and trapezoid
- Allowable notation: right angle symbol (square in corner)
- Items will not require students to identify right triangles by name
- Vocabulary allowed in items: parallel, perpendicular, right, figure

#### 3.3.1.2

Sketch polygons with a given number of sides or vertices (corners), such as pentagons, hexagons and octagons.

#### Item Specifications

- Allowable shapes: triangle, parallelogram, rectangle, rhombus, square, trapezoid, pentagon, hexagon, octagon
- Vocabulary allowed in items: sides, angles, vertices, figure

**Standard 3.3.2:** Understand perimeter as a measurable attribute of real-world and mathematical objects. Use various tools to measure distances.

#### (Online MCA, 3-4 items)

(Paper MCA, 3-4 items)

#### Benchmarks

#### 3.3.2.1

Use half units when measuring distances. Item Specifications

• Not assessed on the MCA-III

#### 3.3.2.2

Find the perimeter of a polygon by adding the lengths of the sides.

- Polygons may have 6 sides, at most
- Items may require finding the length of an unknown side given the lengths of the other sides and the perimeter
- Units are limited to inches, feet, yards, centimeters and meters
- Vocabulary allowed in items: perimeter, length, width, side, figure

#### 3.3.2.3

Measure distances around objects.

#### Item Specifications

- Items may require identification of appropriate tools or procedures for measuring distance
- Vocabulary allowed in items: tool, ruler, yardstick, meter stick, tape measure

**Standard 3.3.3:** Use time, money and temperature to solve real-world and mathematical problems.

(Online MCA, 4–6 items) (Paper MCA, 4–5 items)

#### Benchmarks

#### 3.3.3.1

Tell time to the minute using digital and analog clocks. Determine elapsed time to the minute.

Item Specifications

- Elapsed time must be within a two-hour span
- Vocabulary allowed in items: a.m., p.m.

#### 3.3.3.2

Know relationships among units of time.

Item Specifications

- Allowable conversions: minutes to hours, hours to minutes, hours to days, days to hours, days to weeks, weeks to days, months to years, years to months
- Items may require finding a conversion with mixed units in the answer (e.g., 12 days=1 week and 5 days)
- Vocabulary allowed in items: unit

#### 3.3.3.3

Make change up to 1 dollar in several different ways, including with as few coins as possible.

Item Specifications

- Allowable coins: penny, nickel, dime, quarter
- Allowable notation: \$5, \$0.75, 75¢
- When calculating change, the amount tendered is \$10, at most
- Vocabulary allowed in items: greatest, least, fewest, most, value

#### 3.3.3.4

Use an analog thermometer to determine temperature to the nearest degree in Fahrenheit and Celsius.

- Allowable notation: 15°F, 37°C
- Temperatures must be given in whole numbers
- Vocabulary allowed in items: thermometer, temperature, degrees, increase, decrease

#### Strand 4—Data Analysis

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

**Standard 3.4.1**: Collect, organize, display, and interpret data. Use labels and a variety of scales and units in displays.

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 3.4.1.1

Collect, display and interpret data using frequency tables, bar graphs, picture graphs and number line plots having a variety of scales. Use appropriate titles, labels and units.

- Scale increments will not exceed 5
- Pictograph keys will not exceed 5
- Total number on graph or chart will not exceed 500
- Vocabulary allowed in items: pictograph, tally chart, bar graph, line plot, table, data, title, label, key, represent, scale

#### Grade 4

#### Strand 1—Number & Operation

(Online MCA, 16–20 items) (Paper MCA, 18–22 items)

**Standard 4.1.1:** Demonstrate mastery of multiplication and division basic facts; multiply multi-digit numbers; solve real-world and mathematical problems using arithmetic.

(Online MCA, 6–8 items) (Paper MCA, 8–10 items)

#### Benchmarks

#### 4.1.1.1

Demonstrate fluency with multiplication and division facts.

Item Specifications

- Factors are limited to 1–9
- Vocabulary allowed in items: quotient and vocabulary given at previous grades

#### 4.1.1.2

Use an understanding of place value to multiply a number by 10, 100 and 1000.

Item Specifications

- Numbers multiplied by 10, 100 and 1000 may contain at most, 2 digits
- Numbers must be whole numbers
- Vocabulary allowed in items: vocabulary given at previous grades

#### 4.1.1.3

Multiply multi-digit numbers using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

Item Specifications

- Items will contain multiplication of a one- or two-digit number by a two- or three-digit number
- Numbers must be whole numbers
- Items must not have context
- Vocabulary allowed in items: factor and vocabulary given at previous grades

#### 4.1.1.4

Estimate products and quotients of multi-digit whole numbers by using rounding, benchmarks and place value to assess the reasonableness of results.

**Item Specifications** 

• Assessed within 4.1.1.5

#### 4.1.1.5

Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction and multiplication of multi-digit whole numbers. Use various strategies, including the relationship between operations, the use of technology and the context of the problem to assess the reasonableness of results.

- Solutions must be less than 100,000
- Vocabulary allowed in items: operation, strategy, solve and vocabulary given at previous grades

#### 4.1.1.6

Use strategies and algorithms based on knowledge of place value, equality and properties of operations to divide multi-digit whole numbers by one- or two-digit numbers. Strategies may include mental strategies, partial quotients, the commutative, associative and distributive properties and repeated subtraction. *Item Specifications* 

#### nem specifications

- Dividend may contain at most, 3 digits
- Vocabulary allowed in items: quotient, divisor, dividend and vocabulary given at previous grades

**Standard 4.1.2:** Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities.

#### (Online MCA, 10-12 items)

(Paper MCA, 10–12 items)

#### Benchmarks

#### 4.1.2.1

Represent equivalent fractions using fraction models such as parts of a set, fraction circles, fraction strips, number lines and other manipulatives. Use the models to determine equivalent fractions.

#### Item Specifications

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Vocabulary allowed in items: equivalent, represent, numerator, denominator and vocabulary given at previous grades

#### 4.1.2.2

Locate fractions on a number line. Use models to order and compare whole numbers and fractions, including mixed numbers and improper fractions.

#### Item Specifications

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Vocabulary allowed in items: equivalent, numerator, denominator, improper fraction, mixed numbers, compare and vocabulary given at previous grades

#### 4.1.2.3

Use fraction models to add and subtract fractions with like denominators in real-world and mathematical situations. Develop a rule for addition and subtraction of fractions with like denominators.

Item Specifications

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Vocabulary allowed in items: numerator, denominator and vocabulary given at previous grades

#### 4.1.2.4

Read and write decimals with words and symbols; use place value to describe decimals in terms of thousands, hundreds, tens, ones, tenths, hundredths and thousandths.

Item Specifications

• Vocabulary allowed in items: decimal and vocabulary given at previous grades

#### 4.1.2.5

Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks.

Item Specifications

- Numbers used are from thousands to thousandths
- Allowable symbols: < and >
- Vocabulary allowed in items: decimal and vocabulary given at previous grades

#### 4.1.2.6

Read and write tenths and hundredths in decimal and fraction notations using words and symbols; know the fraction and decimal equivalents for halves and fourths.

Item Specifications

• Vocabulary allowed in items: decimal, equivalent and vocabulary given at previous grades

#### 4.1.2.7

Round decimals to the nearest tenth.

Item Specifications

- Numbers must be less than 500
- Decimals may be given up to thousandths
- Vocabulary allowed in items: decimal and vocabulary given at previous grades

Strand 2—Algebra	(Online MCA, 7–8 items)
	(Paper MCA, 8–10 items)

**Standard 4.2.1:** Use input-output rules, tables and charts to represent patterns and relationships and to solve real-world and mathematical problems.

(Online MCA, 3–4 items) (Paper MCA, 4–5 items)

#### Benchmarks

#### 4.2.1.1

Create and use input-output rules involving addition, subtraction, multiplication and division to solve problems in various contexts. Record the inputs and outputs in a chart or table.

- When creating a rule from pairs, 3 input-output pairs must be given; pairs are not required to be consecutive
- Output should not exceed 100
- Vocabulary allowed in items: vocabulary given at previous grades

**Standard 4.2.2:** Use number sentences involving multiplication, division and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.

(Online MCA, 3–4 items) (Paper MCA, 4–5 items)

#### Benchmarks

#### 4.2.2.1

Understand how to interpret number sentences involving multiplication, division and unknowns. Use realworld situations involving multiplication or division to represent number sentences.

**Item Specifications** 

- Numbers must be less than 100
- Variables, boxes or blanks may be used to represent unknown numbers
- Vocabulary allowed in items: variable and vocabulary given at previous grades

#### 4.2.2.2

Use multiplication, division and unknowns to represent a given problem situation using a number sentence. Use number sense, properties of multiplication and the relationship between multiplication and division to find values for the unknowns that make the number sentences true.

Item Specifications

- Numbers must be less than 100
- Variables, boxes or blanks may be used to represent unknown numbers
- Vocabulary allowed in items: variable and vocabulary given at previous grades

Strand 3—Geometry & Measurement	(Online MCA, 10–14 items) (Paper MCA, 12–15 items)

Standard 4.3.1: Name, describe, classify and sketch polygons.

(Online MCA, 3–4 items)

(Paper MCA, 4–5 items)

#### Benchmarks

#### 4.3.1.1

Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.

Item Specifications

- Naming of triangles is limited to equilateral, right, obtuse and acute
- Allowable notation: 90°
- Vocabulary allowed in items: vertex and vocabulary given at previous grades

#### 4.3.1.2

Describe, classify and draw quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Recognize quadrilaterals in various contexts.

- Naming of quadrilaterals is limited to quadrilateral, square, rectangle, trapezoid, rhombus, parallelogram and kite
- Allowable notation: 90°
- Vocabulary allowed in items: vertex, congruent, and vocabulary given at previous grades

**Standard 4.3.2:** Understand angle and area as measurable attributes of real-world and mathematical objects. Use various tools to measure angles and areas.

(Online MCA, 4–6 items) (Paper MCA, 5–7 items)

#### Benchmarks

#### 4.3.2.1

Measure angles in geometric figures and real-world objects with a protractor or angle ruler.

#### Item Specifications

• Not assessed on the MCA-III

#### 4.3.2.2

Compare angles according to size. Classify angles as acute, right and obtuse.

Item Specifications

- Allowable notation: 90°, angle arc
- Vocabulary allowed in items: vocabulary given at previous grades

#### 4.3.2.3

Understand that the area of a two-dimensional figure can be found by counting the total number of samesize square units that cover a shape without gaps or overlaps. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into  $1 \times 1$  unit squares and viewing these as grouped into rows and columns.

Item Specifications

• Vocabulary allowed in items: area, and vocabulary given at previous grades

#### 4.3.2.4

Find the areas of geometric figures and real-world objects that can be divided into rectangular shapes. Use square units to label area measurements.

Item Specifications

• Vocabulary allowed in items: area, and vocabulary given at previous grades

Standard 4.3.3: Use translations, reflections and rotations to establish congruency and understand symmetries.

#### (Online MCA, 3–4 items) (Paper MCA, 3–4 items)

#### Benchmarks

**4.3.3.1** Apply translations (slides) to figures.

#### Item Specifications

• Vocabulary allowed in items: translation, reflection, rotation, symmetry, congruent, transformation, image, and vocabulary given at previous grades

#### 4.3.3.2

Apply reflections (flips) to figures by reflecting over vertical or horizontal lines and relate reflections to lines of symmetry.

Item Specifications

• Vocabulary allowed in items: translation, reflection, rotation, symmetry, congruent, vertical, horizontal, transformation, image, and vocabulary given at previous grades

#### 4.3.3.3

Apply rotations (turns) of  $90^{\circ}$  clockwise or counterclockwise.

Item Specifications

• Vocabulary allowed in items: translation, reflection, rotation, symmetry, congruent, clockwise, counterclockwise, transformation, image, and vocabulary given at previous grades

#### 4.3.3.4

Recognize that translations, reflections and rotations preserve congruency and use them to show that 2 figures are congruent.

**Item Specifications** 

• Vocabulary allowed in items: translation, reflection, rotation, symmetry, congruent, transformation, image, and vocabulary given at previous grades

#### Strand 4—Data Analysis

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

**Standard 4.4.1:** Collect, organize, display and interpret data, including data collected over a period of time and data represented by fractions and decimals.

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 4.4.1.1

Use tables, bar graphs, timelines and Venn diagrams to display data sets. The data may include fractions or decimals. Understand that spreadsheet tables and graphs can be used to display data.

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Decimals are limited to hundredths
- When interpreting data, displays may include tables, bar graphs, timelines, Venn diagrams, line plots and pictographs
- Vocabulary allowed in items: timeline, Venn diagram, survey, and vocabulary given at previous grades

#### Strand 1—Number & Operation

Standard 5.1.1: Divide multi-digit numbers; solve real-world and mathematical problems using arithmetic.

(Online MCA, 5–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 5.1.1.1

Divide multi-digit numbers using efficient and generalizable procedures based on knowledge of place value, including standard algorithms. Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number or a decimal.

Item Specifications

- Dividends may not be more than 4 digits
- Divisors may not be more than 2 digits
- Fractional remainders are not required to be given in lowest terms
- Allowable division notation: ÷ , fraction bar
- Vocabulary allowed in items: remainder, and vocabulary given at previous grades

#### 5.1.1.2

Consider the context in which a problem is situated to select the most useful form of the quotient for the solution and use the context to interpret the quotient appropriately.

Item Specifications

- Dividends may not be more than 4 digits
- Divisors may not be more than 2 digits
- Fractional remainders are not required to be given in lowest terms
- Items may require interpretation of when decimals should be rounded (e.g., with money)
- Vocabulary allowed in items: remainder, and vocabulary given at previous grades

#### 5.1.1.3

Estimate solutions to arithmetic problems in order to assess the reasonableness of results.

Item Specifications

• Assessed within 5.1.1.4

#### 5.1.1.4

Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology and the context of the problem to assess the reasonableness of results.

- Solutions are less than 1,000,000
- Multiplication is limited to no more than three-digit numbers by no more than three-digit numbers
- Division is limited to no more than four-digit numbers by no more than two-digit numbers
- Fractional remainders are not required to be given in lowest terms
- Vocabulary allowed in items: vocabulary given at previous grades

**Standard 5.1.2:** Read, write, represent and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.

(Online MCA, 5–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 5.1.2.1

Read and write decimals using place value to describe decimals in terms of groups from millionths to millions.

#### Item Specifications

• Vocabulary allowed in items: place value, and vocabulary given at previous grades

#### 5.1.2.2

Find 0.1 more than a number and 0.1 less than a number. Find 0.01 more than a number and 0.01 less than a number. Find 0.001 more than a number and 0.001 less than a number.

#### Item Specifications

• Vocabulary allowed in items: place value, and vocabulary given at previous grades

#### 5.1.2.3

Order fractions and decimals, including mixed numbers and improper fractions, and locate on a number line. *Item Specifications* 

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 15, 16 and 20
- Mixed numbers are less than 10
- Vocabulary allowed in items: place value, and vocabulary given at previous grades

#### 5.1.2.4

Recognize and generate equivalent decimals, fractions, mixed numbers and improper fractions in various contexts.

Item Specifications

- Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 25, 50 and 100
- Mixed numbers are less than 10
- Vocabulary allowed in items: place value, and vocabulary given at previous grades

#### 5.1.2.5

Round numbers to the nearest 0.1, 0.01 and 0.001.

- Numbers can be given up to millionths
- Vocabulary allowed in items: place value, and vocabulary given at previous grades

**Standard 5.1.3:** Add and subtract fractions, mixed numbers and decimals to solve real-world and mathematical problems.

(Online MCA, 5–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 5.1.3.1

Add and subtract decimals and fractions using efficient and generalizable procedures, including standard algorithms.

#### Item Specifications

- Addends, minuend and subtrahend denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Mixed numbers are less than 10
- Items do not require conversion between fractions and decimals
- Items must not have context
- Vocabulary allowed in items: vocabulary given at previous grades

#### 5.1.3.2

Model addition and subtraction of fractions and decimals using a variety of representations.

Item Specifications

- Addends, minuend and subtrahend denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Mixed numbers are less than 10
- Items do not require conversion between fractions and decimals
- Vocabulary allowed in items: vocabulary given at previous grades

#### 5.1.3.3

Estimate sums and differences of decimals and fractions to assess the reasonableness of results. *Item Specifications* 

• Assessed within 5.1.3.4

#### 5.1.3.4

Solve real-world and mathematical problems requiring addition and subtraction of decimals, fractions and mixed numbers, including those involving measurement, geometry and data.

- Addends, minuend and subtrahend denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12
- Mixed numbers are less than 10
- Fractions and decimals may be used within the same item
- Vocabulary allowed in items: vocabulary given at previous grades

#### Strand 2—Algebra

**Standard 5.2.1**: Recognize and represent patterns of change; use patterns, tables, graphs and rules to solve real-world and mathematical problems.

(Online MCA, 3–4 items) (Paper MCA, 4–6 items)

#### Benchmarks

#### 5.2.1.1

Create and use rules, tables, spreadsheets and graphs to describe patterns of change and solve problems. *Item Specifications* 

- In a growing pattern, 3 applications of the rule must be shown, though not necessarily consecutively
- In a table or graph, 3 input-output pairs must be given; pairs are not required to be consecutive
- Vocabulary allowed in items: vocabulary given at previous grades

#### 5.2.1.2

Use a rule or table to represent ordered pairs of positive integers and graph these ordered pairs on a coordinate system.

#### Item Specifications

- Scale increments on grids are limited to 1, 2 and 5
- Rules may be expressed using variables
- Vocabulary allowed in items: ordered pair, graph, and vocabulary given at previous grades

**Standard 5.2.2:** Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving whole numbers.

(Online MCA, 2–3 items) (Paper MCA, 2–3 items)

#### Benchmarks

#### 5.2.2.1

Apply the commutative, associative and distributive properties and order of operations to generate equivalent numerical expressions and to solve problems involving whole numbers.

- Expressions may not contain nested parentheses
- Items must not have context
- Vocabulary allowed in items: expression, and vocabulary given at previous grades

**Standard 5.2.3:** Understand and interpret equations and inequalities involving variables and whole numbers, and use them to represent and solve real-world and mathematical problems.

(Online MCA, 4–6 items)

(Paper MCA, 4–6 items)

#### Benchmarks

#### 5.2.3.1

Determine whether an equation or inequality involving a variable is true or false for a given value of the variable.

Item Specifications

- Allowable symbols: < and >
- Items must not have context
- Vocabulary allowed in items: inequality, and vocabulary given at previous grades

#### 5.2.3.2

Represent real-world situations using equations and inequalities involving variables. Create real-world situations corresponding to equations and inequalities.

Item Specifications

- < and > symbols are allowed
- Vocabulary allowed in items: inequality, and vocabulary given at previous grades

#### 5.2.3.3

Evaluate expressions and solve equations involving variables when values for the variables are given. *Item Specifications* 

- Items must not have context
- Vocabulary allowed in items: expression, and vocabulary given at previous grades

#### Strand 3—Geometry & Measurement

(Online MCA, 8–10 items) (Paper MCA, 8–10 items)

**Standard 5.3.1:** Describe, classify, and draw representations of three-dimensional figures.

(Online MCA, 3–4 items) (Paper MCA, 3–4 items)

#### Benchmarks

#### 5.3.1.1

Describe and classify three-dimensional figures including cubes, prisms and pyramids by the number of edges, faces or vertices as well as the types of faces.

- Prisms and pyramids are limited to triangular, rectangular, pentagonal, hexagonal and octagonal
- Vocabulary allowed in items: cube, prism, pyramid, cone, cylinder, edge, face, base, three-dimensional, triangular, rectangular, and vocabulary given at previous grades

#### 5.3.1.2

Recognize and draw a net for a three-dimensional figure. *Item Specifications* 

• Vocabulary allowed in items: net, cylinder, cube, prism, pyramid, edge, face, base, three-dimensional, triangular, rectangular, and vocabulary given at previous grades

**Standard 5.3.2:** Determine the area of triangles and quadrilaterals; determine the surface area and volume of rectangular prisms in various contexts.

#### (Online MCA, 5-6 items)

(Paper MCA, 5-6 items)

#### Benchmarks

#### 5.3.2.1

Develop and use formulas to determine the area of triangles, parallelograms and figures that can be decomposed into triangles.

#### Item Specifications

• Vocabulary allowed in items: formula, and vocabulary given at previous grades

#### 5.3.2.2

Use various tools and strategies to measure the volume and surface area of objects that are shaped like rectangular prisms.

Item Specifications

- When finding surface area, a graphic of the prism or net must be given
- When finding surface area, dimensions of figures are whole numbers
- Surface areas and volumes are no more than 360
- Vocabulary allowed in items: surface area, volume, net, and vocabulary given at previous grades

#### 5.3.2.3

Understand that the volume of a three-dimensional figure can be found by counting the total number of same-sized cubic units that fill a shape without gaps or overlaps. Use cubic units to label volume measurements.

Item Specifications

• Assessed within 5.3.2.2

#### 5.3.2.4

Develop and use the formulas V = lwh and V = Bh to determine the volume of rectangular prisms. Justify why base area B and height h are multiplied to find the volume of a rectangular prism by breaking the prism into layers of unit cubes.

- The definition of *B* as the area of the base must be given
- Vocabulary allowed in items: volume, base, height, and vocabulary given at previous grades

#### Strand 4—Data Analysis

#### **Standard 5.4.1:** Display and interpret data; determine mean, median and range.

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 5.4.1.1

Know and use the definitions of the mean, median and range of a set of data. Know how to use a spreadsheet to find the mean, median and range of a data set. Understand that the mean is a "leveling out" of data.

#### Item Specifications

- When finding mean, data sets contain, at most 10 numbers
- When finding median, data sets contain, at most 15 numbers
- Numbers are less than 300
- Vocabulary allowed in items: mean, median, range, minimum, maximum, and vocabulary given at previous grades

#### 5.4.1.2

Create and analyze double-bar graphs and line graphs by applying understanding of whole numbers, fractions and decimals. Know how to create spreadsheet tables and graphs to display data.

- Double-bar graphs have no more than 9 categories
- Line graphs have no more than 10 data points
- Scales are in increments of 1/2, 1, 2, 4, 5, 10, tenths if in decimal form or must be consistent with real world applications
- Vocabulary allowed in items: double-bar graph, line graph, and vocabulary given at previous grades

#### Strand 1—Number & Operation

**Standard 6.1.1:** Read, write, represent and compare positive rational numbers expressed as fractions, decimals, percents and ratios; write positive integers as products of factors; use these representations in real-world and mathematical situations.

(Online MCA, 5–7 items) (Paper MCA, 5–7 items)

#### Benchmarks

#### 6.1.1.1

Locate positive rational numbers on a number line and plot pairs of positive rational numbers on a coordinate grid.

#### Item Specifications

- Both axes must have the same scale
- Items may require locating points on either axis
- Vocabulary allowed in items: integer, x-axis, y-axis, horizontal axis, vertical axis, rational number, coordinate grid, and vocabulary given at previous grades

#### 6.1.1.2

Compare positive rational numbers represented in various forms. Use the symbols <, = and >.

Item Specifications

• Vocabulary allowed in items: is greater than, is less than, and vocabulary given at previous grades

#### 6.1.1.3

Understand that percent represents parts out of 100 and ratios to 100.

Item Specifications

- Allowable notation: 25%, ¼, 1:4
- Percents must be between 1 and 100, inclusive
- Vocabulary allowed in items: percent, ratio, and vocabulary given at previous grades

#### 6.1.1.4

Determine equivalences among fractions, decimals and percents; select among these representations to solve problems.

- Allowable notation: 50%, ¼, 0.95, 0. 25
- Percents must be between 1 and 100, inclusive
- Vocabulary allowed in items: vocabulary given at previous grades

#### 6.1.1.5

Factor whole numbers; express a whole number as a product of prime factors with exponents. *Item Specifications* 

- Prime factors are not greater than 13
- Numbers being factored are less than 1,000
- Vocabulary allowed in items: prime factor, prime factorization, exponent, power, base, and vocabulary given at previous grades

#### 6.1.1.6

Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

Item Specifications

 Vocabulary allowed in items: greatest common factor, least common multiple, and vocabulary given at previous grades

#### 6.1.1.7

Convert between equivalent representations of positive rational numbers.

Item Specifications

- Conversions are limited to within a representation (e.g., 7/4 = 1 <sup>3</sup>/<sub>4</sub> and  $3^2 = 3 \cdot 3$ , not 0.5 = 1/2)
- Vocabulary allowed in items: exponent, integer, and vocabulary given at previous grades

**Standard 6.1.2:** Understand the concept of ratio and its relationship to fractions and to the multiplication and division of whole numbers. Use ratios to solve real-world and mathematical problems.

### (Online MCA, 2–6 items)

(Paper MCA, 2-6 items)

#### Benchmarks

#### 6.1.2.1

Identify and use ratios to compare quantities; understand that comparing quantities using ratios is not the same as comparing quantities using subtraction.

Item Specifications

- Allowable ratio notation: ¼, 1 to 4, 1:4, 1 out of 4
- Vocabulary allowed in items: ratio, and vocabulary given at previous grades

#### 6.1.2.2

Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixtures and concentrations.

- Allowable ratio notation: 1/4, 1 to 4, 1:4, 1 out of 4, 25%
- Rates may be expressed using the word "per"
- Vocabulary allowed in items: ratio, percent, and vocabulary given at previous grades

#### 6.1.2.3

Determine the rate for ratios of quantities with different units. *Item Specifications* 

- Allowable ratio notation: <sup>1</sup>/<sub>4</sub>, 1 to 4, 1:4, 1 out of 4
- Rates may be expressed using the word "per"
- Vocabulary allowed in items: rate, ratio, unit rate, and vocabulary given at previous grades

#### 6.1.2.4

Use reasoning about multiplication and division to solve ratio and rate problems.

Item Specifications

- Allowable ratio notation: 1/4, 1 to 4, 1:4, 1 out of 4
- Rates may be expressed using the word "per"
- Vocabulary allowed in items: rate, ratio, and vocabulary given at previous grades

**Standard 6.1.3:** Multiply and divide decimals, fractions and mixed numbers; solve real-world and mathematical problems using arithmetic with positive rational numbers.

#### (Online MCA, 4-6 items)

(Paper MCA, 5–7 items)

#### Benchmarks

#### 6.1.3.1

Multiply and divide decimals and fractions using efficient and generalizable procedures, including standard algorithms.

#### Item Specifications

- Items must not have context
- Vocabulary allowed in items: reciprocal, and vocabulary given at previous grades

#### 6.1.3.2

Use the meanings of fractions, multiplication, division and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions.

Item Specifications

• Assessed within 6.1.3.1

#### 6.1.3.3

Calculate the percent of a number and determine what percent one number is of another number to solve problems in various contexts.

- Percents are not less than 1
- Percents over 100 are 110, 125, 150 and 200
- Vocabulary allowed in items: percent, and vocabulary given at previous grades

#### 6.1.3.4

Solve real-world and mathematical problems requiring arithmetic with decimals, fractions and mixed numbers.

Item Specifications

- Items are limited to no more than two operations
- Vocabulary allowed in items: reciprocal, and vocabulary given at previous grades

#### 6.1.3.5

Estimate solutions to problems with whole numbers, fractions and decimals and use the estimates to assess the reasonableness of results in the context of the problem.

Item Specifications

• Assessed within 6.1.3.4

Strand 2—Algebra	(Online MCA, 10–13 items)
	(Paper MCA, 12–16 items)

**Standard 6.2.1:** Recognize and represent relationships between varying quantities; translate from one representation to another; use patterns, tables, graphs and rules to solve real-world and mathematical problems.

(Online MCA, 3–4 items) (Paper MCA, 4–5 items)

#### Benchmarks

#### 6.2.1.1

Understand that a variable can be used to represent a quantity that can change, often in relation to another changing quantity. Use variables in various contexts.

**Item Specifications** 

- Allowable multiplication notation: 3x, xy,  $3 \cdot 4$ , 3(4)
- Equations will not contain exponents
- Vocabulary allowed in items: evaluate, and vocabulary given at previous grades

#### 6.2.1.2

Represent the relationship between two varying quantities with function rules, graphs and tables; translate between any two of these representations.

- Allowable multiplication notation: 3x, xy,  $3 \cdot 4$ , 3(4)
- Equations will not contain exponents
- Vocabulary allowed in items: translate, function, coordinate grid, and vocabulary given at previous grades

**Standard 6.2.2:** Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.

(Online MCA, 2–3 items) (Paper MCA, 2–3 items)

#### Benchmarks

## 6.2.2.1

Apply the associative, commutative and distributive properties and order of operations to generate equivalent expressions and to solve problems involving positive rational numbers.

Item Specifications

- Allowable multiplication notation: 3x, xy,  $3 \cdot 4$ , 3(4)
- Items must not have context
- Vocabulary allowed in items: order of operations, simplify, and vocabulary given at previous grades

**Standard 6.2.3:** Understand and interpret equations and inequalities involving variables and positive rational numbers. Use equations and inequalities to represent real-world and mathematical problems; use the idea of maintaining equality to solve equations. Interpret solutions in the original context.

## (Online MCA, 5–6 items)

## (Paper MCA, 6-8 items)

## Benchmarks

## 6.2.3.1

Represent real-world or mathematical situations using equations and inequalities involving variables and positive rational numbers.

#### Item Specifications

- Allowable multiplication notation: 3x, xy,  $3 \cdot 4$ , 3(4),  $x^2$
- <, > and = symbols are allowed
- Vocabulary allowed in items: vocabulary given at previous grades

## 6.2.3.2

Solve equations involving positive rational numbers using number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation. Interpret a solution in the original context and assess the reasonableness of results.

- Allowable multiplication notation: 3x, xy,  $3 \cdot 4$ , 3(4),  $x^2$
- Vocabulary allowed in items: reasonable, and vocabulary given at previous grades

#### Strand 3—Geometry & Measurement

(Online MCA, 8–11 items) (Paper MCA, 10–12 items)

**Standard 6.3.1:** Calculate perimeter, area, surface area and volume of two- and three-dimensional figures to solve real-world and mathematical problems.

(Online MCA, 3–5 items) (Paper MCA, 3–5 items)

#### Benchmarks

#### 6.3.1.1

Calculate the surface area and volume of prisms and use appropriate units, such as  $cm^2$  and  $cm^3$ . Justify the formulas used. Justification may involve decomposition, nets or other models.

Item Specifications

- Allowable notation: 3 square centimeters, 3 cm sq, 3 cm<sup>2</sup>
- Vocabulary allowed in items: vocabulary given at previous grades

#### 6.3.1.2

Calculate the area of quadrilaterals. Quadrilaterals include squares, rectangles, rhombuses, parallelograms, trapezoids and kites. When formulas are used, be able to explain why they are valid.

Item Specifications

- Congruent side marks (hash marks) may be used
- Allowable notation: 3 square centimeters, 3 cm sq, 3  $cm^2$
- Vocabulary allowed in items: vocabulary given at previous grades

#### 6.3.1.3

Estimate the perimeter and area of irregular figures on a grid when they cannot be decomposed into common figures and use correct units, such as cm and cm2.

Item Specifications

- Allowable notation: 3 square centimeters, 3 cm sq, 3  $cm^2$
- Vocabulary allowed in items: vocabulary given at previous grades

Standard 6.3.2: Understand and use relationships between angles in geometric figures.

## (Online MCA, 3–4 items) (Paper MCA, 3–5 items)

#### Benchmarks

#### 6.3.2.1

Solve problems using the relationships between the angles formed by intersecting lines.

- Allowable notation:  $\angle A$ , m $\angle A$ ,  $\triangle ABC$
- Vocabulary allowed in items: intersecting, vertical, adjacent, complementary, supplementary, straight, hypotenuse, leg, and vocabulary given at previous grades

## 6.3.2.2

Determine missing angle measures in a triangle using the fact that the sum of the interior angles of a triangle is 180°. Use models of triangles to illustrate this fact.

Item Specifications

- Allowable notation:  $\angle A$ , m $\angle A$ ,  $\triangle ABC$
- Vocabulary allowed in items: adjacent, complementary, supplementary, interior, exterior, hypotenuse, leg, and vocabulary given at previous grades

## 6.3.2.3

Develop and use formulas for the sums of the interior angles of polygons by decomposing them into triangles.

Item Specifications

- Allowable notation:  $\angle A$ , m $\angle A$ ,  $\triangle ABC$
- Vocabulary allowed in items: interior, diagonal, and vocabulary given at previous grades

**Standard 6.3.3**: Choose appropriate units of measurement and use ratios to convert within measurement systems to solve real-world and mathematical problems.

(Online MCA, 2–3 items) (Paper MCA, 2–3 items)

#### Benchmarks

#### 6.3.3.1

Solve problems in various contexts involving conversion of weights, capacities, geometric measurements and times within measurement systems using appropriate units.

Item Specifications

• Vocabulary allowed in items: customary, metric, capacity, and vocabulary given at previous grades

#### 6.3.3.2

Estimate weights, capacities and geometric measurements using benchmarks in measurement systems with appropriate units.

Item Specifications

• Vocabulary allowed in items: customary, metric, capacity, and vocabulary given at previous grades

### Strand 4—Data Analysis & Probability

(Online MCA, 6–8 items) (Paper MCA, 6–8 items)

**Standard 6.4.1:** Use probabilities to solve real-world and mathematical problems; represent probabilities using fractions, decimals and percents.

(Online MCA, 6–8 items) (Paper MCA, 6–8 items)

#### Benchmarks

#### 6.4.1.1

Determine the sample space (set of possible outcomes) for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by the use of tree diagrams, tables or pictorial representations.

Item Specifications

- Size of the sample space will not exceed 36
- Vocabulary allowed in items: probability, outcome, tree diagram, event, random, sample space, combinations, and vocabulary given at previous grades

#### 6.4.1.2

Determine the probability of an event using the ratio between the size of the event and the size of the sample space; represent probabilities as percents, fractions and decimals between 0 and 1 inclusive. Understand that probabilities measure likelihood.

Item Specifications

- Size of the sample space is no more than 100
- Vocabulary allowed in items: probability, outcome, event, likely, unlikely, certain, impossible, ratio, random, sample space, and vocabulary given at previous grades

#### 6.4.1.3

Perform experiments for situations in which the probabilities are known, and compare the resulting relative frequencies with the known probabilities; know that there may be differences.

Item Specifications

• Vocabulary allowed in items: probability, outcome, event, theoretical, frequency, relative frequency, random, and vocabulary given at previous grades

#### 6.4.1.4

Calculate experimental probabilities from experiments; represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown.

- Size of the sample space is no more than 100
- Vocabulary allowed in items: probability, outcome, event, experimental, frequency, predict, random, and vocabulary given at previous grades

## Strand 1—Number & Operation

**Standard 7.1.1:** Read, write, represent and compare positive and negative rational numbers, expressed as integers, fractions and decimals.

(Online MCA, 4-6 items)

(Paper MCA, 4-6 items)

#### Benchmarks

#### 7.1.1.1

Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal. Recognize that  $\pi$  is not rational, but that it can be approximated by rational numbers such as 22/7 and 3.14.

#### **Item Specifications**

- Allowable notation: . . . , *π* (written as a symbol, not as "pi")
- Vocabulary allowed in items: terminating, repeating, and vocabulary given at previous grades

#### 7.1.1.2

Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.

Item Specifications

• Vocabulary allowed in items: terminating, repeating, and vocabulary given at previous grades

#### 7.1.1.3

Locate positive and negative rational numbers on a number line, understand the concept of opposites, and plot pairs of positive and negative rational numbers on a coordinate grid.

#### Item Specifications

• Vocabulary allowed in items: opposite, coordinate, origin, and vocabulary given at previous grades

#### 7.1.1.4

Compare positive and negative rational numbers expressed in various forms using the symbols <, >, =,  $\leq$ ,

and ≥.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 7.1.1.5

Recognize and generate equivalent representations of positive and negative rational numbers, including equivalent fractions.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

**Standard 7.1.2**: Calculate with positive and negative rational numbers, and rational numbers with whole number exponents, to solve real-world and mathematical problems.

(Online MCA, 8–10 items) (Paper MCA, 8–10 items)

#### Benchmarks

## 7.1.2.1

Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

Item Specifications

- Items must not have context
- Vocabulary allowed in items: vocabulary given at previous grades

## 7.1.2.2

Use real-world contexts and the inverse relationship between addition and subtraction to explain why the procedures of arithmetic with negative rational numbers make sense.

#### Item Specifications

• Vocabulary allowed in items: inverse and vocabulary given at previous grades

## 7.1.2.3

Understand that calculators and other computing technologies often truncate or round numbers.

Item Specifications

• Assessed within 7.1.2.4

#### 7.1.2.4

Solve problems in various contexts involving calculations with positive and negative rational numbers and positive integer exponents, including computing simple and compound interest.

Item Specifications

 Vocabulary allowed in items: simple interest, compound interest, and vocabulary given at previous grades

#### 7.1.2.5

Use proportional reasoning to solve problems involving ratios in various contexts.

#### Item Specifications

• Vocabulary allowed in items: proportion and vocabulary given at previous grades

#### 7.1.2.6

Demonstrate an understanding of the relationship between the absolute value of a rational number and distance on a number line. Use the symbol for absolute value.

#### Item Specifications

• Vocabulary allowed in items: absolute value and vocabulary given at previous grades

## Strand 2—Algebra

**Standard 7.2.1:** Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.

(Online MCA, 1–2 items) (Paper MCA, 1–2 items)

#### Benchmarks

## 7.2.1.1

Understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form y/x = k or y = kx. Distinguish proportional relationships from other relationships, including inversely proportional relationships (xy = k or y = k/x).

Item Specifications

• Vocabulary allowed in items: proportional, inversely, and vocabulary given at previous grades

## 7.2.1.2

Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (constant of proportionality). Know how to use graphing technology to examine what happens to a line when the unit rate is changed.

## Item Specifications

• Vocabulary allowed in items: proportional, origin, slope, and vocabulary given at previous grades

**Standard 7.2.2:** Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original context.

(Online MCA, 5–7 items) (Paper MCA, 6–8 items)

## Benchmarks

## 7.2.2.1

Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these representations.

#### Item Specifications

• Vocabulary allowed in items: proportional, origin, slope, and vocabulary given at previous grades

## 7.2.2.2

Solve multi-step problems involving proportional relationships in numerous contexts.

- Contexts may include (but are not limited to) discounts, tax, and percent of change
- Vocabulary allowed in items: proportional and vocabulary given at previous grades

## 7.2.2.3

Use knowledge of proportions to assess the reasonableness of solutions. *Item Specifications* 

• Assessed within 7.2.2.1 and 7.2.2.2

## 7.2.2.4

Represent real-world or mathematical situations using equations and inequalities involving variables and positive and negative rational numbers.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

**Standard 7.2.3:** Apply understanding of order of operations and algebraic properties to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers and grouping symbols; evaluate such expressions.

(Online MCA, 3–4 items)

(Paper MCA, 4–6 items)

#### Benchmarks

#### 7.2.3.1

Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.

Item Specifications

- Items must not have context
- Vocabulary allowed in items: simplify and vocabulary given at previous grades

#### 7.2.3.2

Evaluate algebraic expressions containing rational numbers and whole number exponents at specified values of their variables.

**Item Specifications** 

- Expressions contain no more than 3 variables
- Vocabulary allowed in items: evaluate, substitute, and vocabulary given at previous grades

#### 7.2.3.3

Apply understanding of order of operations and grouping symbols when using calculators and other technologies.

#### Item Specifications

• Assessed within 7.2.3.1 and 7.2.3.2

**Standard 7.2.4:** Represent real-world and mathematical situations using equations with variables. Solve equations symbolically, using the properties of equality. Also solve equations graphically and numerically. Interpret solutions in the original context.

(Online MCA, 4–5 items) (Paper MCA, 4–6 items)

## Benchmarks

## 7.2.4.1

Represent relationships in various contexts with equations involving variables and positive and negative rational numbers. Use the properties of equality to solve for the value of a variable. Interpret the solution in the original context.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 7.2.4.2

Solve equations resulting from proportional relationships in various contexts. *Item Specifications* 

• Vocabulary allowed in items: vocabulary given at previous grades

## Strand 3—Geometry & Measurement

(Online MCA, 7–9 items) (Paper MCA, 8–10 items)

**Standard 7.3.1:** Use reasoning with proportions and ratios to determine measurements, justify formulas and solve real-world and mathematical problems involving circles and related geometric figures.

(Online MCA, 3–4 items) (Paper MCA, 4–5 items)

#### Benchmarks

## 7.3.1.1

Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is  $\pi$ . Calculate the circumference and area of circles to solve problems in various contexts.

#### Item Specifications

- Allowable notation:  $\pi$  (written as a symbol, not as "pi")
- Items may assess finding the area and arc length of a sector
- Items do not assess finding the perimeter of a sector
- Vocabulary allowed in items: radius, diameter, circumference, and vocabulary given at previous grades

## 7.3.1.2

Calculate the volume and surface area of cylinders and justify the formulas used.

- Units must be consistent throughout an item; conversions are not allowed
- Vocabulary allowed in items: radius, diameter, circumference, cylinder, lateral area, and vocabulary given at previous grades

# **Standard 7.3.2:** Analyze the effect of change of scale, translations and reflections on the attributes of two-dimensional figures.

## (Online MCA, 4–5 items)

(Paper MCA, 4–5 items)

## Benchmarks

## 7.3.2.1

Describe the properties of similarity, compare geometric figures for similarity and determine scale factors. *Item Specifications* 

- Allowable notation: ~ (similar),  $\cong$  (congruent),  $\overline{FG}$  (segment FG), FG (length of segment FG)
- Vocabulary allowed in items: similar, corresponding, scale factor, and vocabulary given at previous grades

## 7.3.2.2

Apply scale factors, length ratios and area ratios to determine side lengths and areas of similar geometric figures.

## Item Specifications

- Allowable notation: ~ (similar),  $\cong$  (congruent),  $\overline{FG}$  (segment FG), FG (length of segment FG)
- Vocabulary allowed in items: similar, corresponding, scale factor, and vocabulary given at previous grades

## 7.3.2.3

Use proportions and ratios to solve problems involving scale drawings and conversions of measurement units.

#### Item Specifications

- Conversions are limited to no more than 2 per item
- Vocabulary allowed in items: similar, corresponding, scale drawing, conversion, and vocabulary given at previous grades

## 7.3.2.4

Graph and describe translations and reflections of figures on a coordinate grid, and determine the coordinates of the vertices of the figure after the transformation.

- Allowable notation: J and J' (labels for points before and after transformation)
- Allowable translation notation:  $(x, y) \rightarrow (x + 3, y 2)$
- Images may be reflected over vertical lines, horizontal lines and the lines y = x and y = -x
- Vocabulary allowed in items: vocabulary given at previous grades

#### Strand 4—Data Analysis & Probability

**Standard 7.4.1:** Use mean, median and range to draw conclusions about data and make predictions.

(Online MCA, 3–4 items) (Paper MCA, 3–5 items)

#### Benchmarks

## 7.4.1.1

Design simple experiments, and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets and make predictions.

Item Specifications

- Data displays are limited to no more than 10 categories
- Data displays from previous grades may be used
- Vocabulary allowed in items: stem-and-leaf plot, and vocabulary given at previous grades

## 7.4.1.2

Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet to examine this impact.

Item Specifications

- Data sets are limited to no more than 10 data points
- Vocabulary allowed in items: outlier and vocabulary given at previous grades

Standard 7.4.2: Display and interpret data in a variety of ways, including circle graphs and histograms.

(Online MCA, 1–2 items) (Paper MCA, 1–2 items)

#### Benchmarks

#### 7.4.2.1

Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.

- Circle graphs have no more than 6 sectors
- Histograms have no more than 5 intervals
- Vocabulary allowed in items: circle graph, histogram, frequency table, and vocabulary given at previous grades

**Standard 7.4.3:** Calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.

(Online MCA, 3–4 items) (Paper MCA, 3–5 items)

#### Benchmarks

#### 7.4.3.1

Use random numbers generated by a calculator or a spreadsheet or taken from a table to simulate situations involving randomness, make a histogram to display the results and compare the results to known probabilities.

Item Specifications

• Not assessed on the MCA-III

## 7.4.3.2

Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 7.4.3.3

Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## Strand 1—Number & Operation

(Online MCA, 6–8 items) (Paper MCA, 6–8 items)

**Standard 8.1.1:** Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.

(Online MCA, 6–8 items)

(Paper MCA, 6–8 items)

#### Benchmarks

#### 8.1.1.1

Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational.

#### **Item Specifications**

- Allowable notation:  $\sqrt{18}$
- Vocabulary allowed in items: irrational, real, square root, radical, and vocabulary given at previous grades

## 8.1.1.2

Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers. *Item Specifications* 

- Allowable notation:  $\sqrt{18}$
- Vocabulary allowed in items: square root, radical, consecutive, and vocabulary given at previous grades

#### 8.1.1.3

Determine rational approximations for solutions to problems involving real numbers.

Item Specifications

- Allowable notation:  $\sqrt{18}$
- Vocabulary allowed in items: square root, radical, consecutive, and vocabulary given at previous grades

#### 8.1.1.4

Know and apply the properties of positive and negative integer exponents to generate equivalent numerical expressions.

- Allowable notation:  $-x^2$ ,  $(-x)^2$ ,  $-3^2$ ,  $(-3)^2$
- Expressions may be numeric or algebraic
- Vocabulary allowed in items: vocabulary given at previous grades

## 8.1.1.5

Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, and express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved.

**Item Specifications** 

• Vocabulary allowed in items: scientific notation, significant digits, standard form, and vocabulary given at previous grades

## Strand 2—Algebra

(Online MCA, 18–29 items) (Paper MCA, 24–30 items)

**Standard 8.2.1:** Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.

(Online MCA, 4–5 items) (Paper MCA, 4–5 items)

#### Benchmarks

#### 8.2.1.1

Understand that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as f(x), to represent such relationships.

Item Specifications

 Vocabulary allowed in items: independent, dependent, constant, coefficient, and vocabulary given at previous grades

## 8.2.1.2

Use linear functions to represent relationships in which changing the input variable by some amount leads to a change in the output variable that is a constant times that amount.

Item Specifications

 Vocabulary allowed in items: independent, dependent, constant, coefficient, and vocabulary given at previous grades

#### 8.2.1.3

Understand that a function is linear if it can be expressed in the form f(x) = mx + b or if its graph is a straight line.

#### Item Specifications

• Vocabulary allowed in items: linear, constant, coefficient, and vocabulary given at previous grades

## 8.2.1.4

Understand that an arithmetic sequence is a linear function that can be expressed in the form f(x) = mx + b, where x = 0, 1, 2, 3, ...

Item Specifications

- Vocabulary allowed in items: nth term, arithmetic sequence, geometric sequence, linear function, non-linear function, progression, common difference, and vocabulary given at previous grades
- Allowable notation: items must specify the domain as x = 0,1,2,3,... or x = 1,2,3,4,...

## 8.2.1.5

Understand that a geometric sequence is a non-linear function that can be expressed in the form

 $f(x) = ab^x$ , where x = 0,1,2,3,...

Item Specifications

- Vocabulary allowed in items: n<sup>th</sup> term, arithmetic sequence, geometric sequence, linear function, non-linear function, exponential, progression, common ratio, and vocabulary given at previous grades
- Allowable notation: items must specify the domain as  $x = 0,1,2,3, \dots$  or  $x = 1,2,3,4, \dots$

**Standard 8.2.2:** Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions and explain results in the original context.

(Online MCA, 4–6 items) (Paper MCA, 4–6 items)

#### Benchmarks

#### 8.2.2.1

Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate one representation to another.

#### Item Specifications

• Vocabulary allowed in items: linear function, and vocabulary given at previous grades

## 8.2.2.2

Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change and that the y-intercept is zero when the function represents a proportional relationship. *Item Specifications* 

- Coordinates used for determining slope must contain integer values
- Vocabulary allowed in items: linear function, intercept, and vocabulary given at previous grades

#### 8.2.2.3

Identify how coefficient changes in the equation f(x) = mx + b affect the graphs of linear functions. Know how to use graphing technology to examine these effects.

Item Specifications

• Vocabulary allowed in items: linear function, intercept, coefficient, constant, and vocabulary given at previous grades

## 8.2.2.4

Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

Item Specifications

• Vocabulary allowed in items: nth term, arithmetic sequence, geometric sequence, linear function, nonlinear function, progression, and vocabulary given at previous grades

## 8.2.2.5

Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

Item Specifications

• Vocabulary allowed in items: n<sup>th</sup> term, arithmetic sequence, geometric sequence, linear function, nonlinear function, progression, and vocabulary given at previous grades

**Standard 8.2.3:** Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.

(Online MCA, 2–4 items) (Paper MCA, 3–5 items)

## Benchmarks

## 8.2.3.1

Evaluate algebraic expressions, including expressions containing radicals and absolute values, at specified values of their variables.

Item Specifications

- Items must not have context
- Directives may include: simplify, evaluate
- Vocabulary allowed in items: vocabulary given at previous grades

#### 8.2.3.2

Justify steps in generating equivalent expressions by identifying the properties used, including the properties of algebra. Properties include the associative, commutative and distributive laws and the order of operations, including grouping symbols.

- Items must not have context
- Vocabulary allowed in items: associative, commutative, distributive, identity, property, order of operations, and vocabulary given at previous grades

**Standard 8.2.4:** Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.

(Online MCA, 8–14 items) (Paper MCA, 10–15 items)

## Benchmarks

## 8.2.4.1

Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 8.2.4.2

Solve multi-step equations in one variable. Solve for one variable in a multi-variable equation in terms of the other variables. Justify the steps by identifying the properties of equalities used.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 8.2.4.3

Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line.

Item Specifications

- Items must not have context
- Vocabulary allowed in items: slope-intercept form, point-slope form, standard form, and vocabulary given at previous grades

## 8.2.4.4

Use linear inequalities to represent relationships in various contexts.

Item Specifications

- Inequalities contain no more than 1 variable
- Vocabulary allowed in items: vocabulary given at previous grades

## 8.2.4.5

Solve linear inequalities using properties of inequalities. Graph the solutions on a number line.

## Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 8.2.4.6

Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities, and graph the solutions on a number line.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 8.2.4.7

Represent relationships in various contexts using systems of linear equations. Solve systems of linear equations in two variables symbolically, graphically and numerically.

Item Specifications

• Vocabulary allowed in items: system of equations, undefined, infinite, intersecting, identical, and vocabulary given at previous grades

#### 8.2.4.8

Understand that a system of linear equations may have no solution, one solution or an infinite number of solutions. Relate the number of solutions to pairs of lines that are intersecting, parallel or identical. Check whether a pair of numbers satisfies a system of two linear equations in two unknowns by substituting the numbers in both equations.

Item Specifications

• Assessed within 8.2.4.7

#### 8.2.4.9

Use the relationship between square roots and squares of a number to solve problems.

Item Specifications

- Allowable notation: ±3
- Items may assess the interpretation of square roots based on the context of the item
- Vocabulary allowed in items: square root and vocabulary given at previous grades

Strand 3—Geometry & Measurement	(Online MCA, 6–8 items)
	(Paper MCA, 8–10 items)

Standard 8.3.1: Solve problems involving right triangles using the Pythagorean Theorem and its converse.

(Online MCA, 3–4 items) (Paper MCA, 3–5 items)

Benchmarks

8.3.1.1

Use the Pythagorean Theorem to solve problems involving right triangles.

- Congruent angle marks may be used
- Vocabulary allowed in items: Pythagorean Theorem and vocabulary given at previous grades

## 8.3.1.2

Determine the distance between two points on a horizontal or vertical line in a coordinate system. Use the Pythagorean Theorem to find the distance between any two points in a coordinate system.

Item Specifications

- Graphs are not provided when finding horizontal or vertical distance
- Vocabulary allowed in items: Pythagorean Theorem and vocabulary given at previous grades

#### 8.3.1.3

Informally justify the Pythagorean Theorem by using measurements, diagrams and computer software. *Item Specifications* 

• Not assessed on the MCA-III

Standard 8.3.2: Solve problems involving parallel and perpendicular lines on a coordinate system.

(Online MCA, 3–4 items) (Paper MCA, 3–5 items)

## Benchmarks

## 8.3.2.1

Understand and apply the relationships between the slopes of parallel lines and between the slopes of perpendicular lines. Dynamic graphing software may be used to examine these relationships.

## Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

## 8.3.2.2

Analyze polygons on a coordinate system by determining the slopes of their sides.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 8.3.2.3

Given a line on a coordinate system and the coordinates of a point not on the line, find lines through that point that are parallel and perpendicular to the given line symbolically and graphically.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### Strand 4—Data Analysis & Probability

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

**Standard 8.4.1:** Interpret data using scatterplots and approximate lines of best fit. Use lines of best fit to draw conclusions about data.

(Online MCA, 6–7 items) (Paper MCA, 6–8 items)

#### Benchmarks

## 8.4.1.1

Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit and determine an equation for the line. Use appropriate titles, labels and units. Know how to use graphing technology to display scatterplots and corresponding lines of best fit.

Item Specifications

- Data sets are limited to no more than 30 data points
- Vocabulary allowed in items: scatterplot, line of best fit, correlation and vocabulary given at previous grades

#### 8.4.1.2

Use a line of best fit to make statements about approximate rate of change and to make predictions about values not in the original data set.

Item Specifications

• Vocabulary allowed in items: scatterplot, line of best fit, and vocabulary given at previous grades

#### 8.4.1.3

Assess the reasonableness of predictions using scatterplots by interpreting them in the original context. *Item Specifications* 

• Vocabulary allowed in items: scatterplot, line of best fit, and vocabulary given at previous grades

## Grade 11

## Strand 2—Algebra

**Standard 9.2.1:** Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.

(Online MCA, 4–6 items) (Paper MCA, 5–7 items)

#### Benchmarks

## 9.2.1.1

Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain.

Item Specifications

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

## 9.2.1.2

Distinguish between functions and other relations defined symbolically, graphically or in tabular form. *Item Specifications* 

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

## 9.2.1.3

Find the domain of a function defined symbolically, graphically or in a real-world context.

Item Specifications

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

#### 9.2.1.4

Obtain information and draw conclusions from graphs of functions and other relations.

Item Specifications

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

#### 9.2.1.5

Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form  $f(x) = ax^2 + bx + c$ , in the form  $f(x) = a(x - h)^2 + k$ , or in factored form.

#### Item Specifications

• Vocabulary allowed in items: line of symmetry, parabola, quadratic, vertex and vocabulary given at previous grades.

## 9.2.1.6

Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.

#### Item Specifications

 Vocabulary allowed in items: maximum, minimum, interval, zeros and vocabulary given at previous grades

#### 9.2.1.7

Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.

Item Specifications

• Vocabulary allowed in items: asymptote and vocabulary given at previous grades

#### 9.2.1.8

Make qualitative statements about the rate of change of a function, based on its graph or table of values. *Item Specifications* 

• Vocabulary allowed in items: vocabulary given at previous grades

#### 9.2.1.9

Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

**Standard 9.2.2:** Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.

(Online MCA, 5-10 items)

(Paper MCA, 6–12 items)

#### Benchmarks

#### 9.2.2.1

Represent and solve problems in various contexts using linear and quadratic functions.

#### Item Specifications

• Vocabulary allowed in items: quadratic and vocabulary given at previous grades

#### 9.2.2.2

Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.

**Item Specifications** 

• Vocabulary allowed in items: growth factor, decay, exponential and vocabulary given at previous grades

## 9.2.2.3

Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, exponential and vocabulary given at previous grades

## 9.2.2.4

Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively.

Item Specifications

 Vocabulary allowed in items: recursive, geometric series, common ratio and vocabulary given at previous grades

## 9.2.2.5

Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.2.2.6

Sketch the graphs of common non-linear functions such as  $f(x) = \sqrt{x}$ , f(x) = |x|, f(x) = 1/x,  $f(x) = x^3$  and translations of these functions, such as  $f(x) = \sqrt{x-2} + 4$ . Know how to use graphing technology to graph these functions.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: vocabulary given at previous grades

**Standard 9.2.3:** Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.

(Online MCA, 3–6 items)

(Paper MCA, 4–7 items)

#### Benchmarks

#### 9.2.3.1

Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.

Item Specifications

• Vocabulary allowed in items: polynomial and vocabulary given at previous grades

## 9.2.3.2

Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree. *Item Specifications* 

• Vocabulary allowed in items: polynomial, degree of a polynomial and vocabulary given at previous grades

#### 9.2.3.3

Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares.

Item Specifications

• Vocabulary allowed in items: polynomial, monomial and vocabulary given at previous grades

#### 9.2.3.4

Add, subtract, multiply, divide and simplify algebraic fractions.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 9.2.3.5

Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers.

Item Specifications

• Vocabulary allowed in items: complex number and vocabulary given at previous grades

#### 9.2.3.6

Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving  $n^{th}$  roots.

Item Specifications

• Vocabulary allowed in items:  $n^{th}$  root and vocabulary given at previous grades

#### 9.2.3.7

Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

**Standard 9.2.4:** Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and  $n^{th}$  root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.

(Online MCA, 3-7 items)

(Paper MCA, 4-8 items)

## Benchmarks

## 9.2.4.1

Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, *n*<sup>th</sup> root and vocabulary given at previous grades

## 9.2.4.2

Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: exponential and vocabulary given at previous grades

#### 9.2.4.3

Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.

#### Item Specifications

• Vocabulary allowed in items: quadratic, complex, non-real and vocabulary given at previous grades

#### 9.2.4.4

Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.

#### Item Specifications

• Vocabulary allowed in items: boundary and vocabulary given at previous grades

## 9.2.4.5

Solve linear programming problems in two variables using graphical methods.

#### Item Specifications

 Vocabulary allowed in items: constraint, boundary, feasible region and vocabulary given at previous grades

#### 9.2.4.6

Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 9.2.4.7

Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods.

**Item Specifications** 

• Vocabulary allowed in items: extraneous and vocabulary given at previous grades

#### 9.2.4.8

Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

Strand 3—Geometry & Measurement	(Online MCA, 13–15 items)
	(Paper MCA, 16–18 items)

**Standard 9.3.1:** Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.

(Online MCA, 3–4 items) (Paper MCA, 3–5 items)

#### Benchmarks

#### 9.3.1.1

Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.

#### Item Specifications

• Vocabulary allowed in items: sphere and vocabulary given at previous grades

## 9.3.1.2

Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.

Item Specifications

• Vocabulary allowed in items: regular polygon, sphere, compose, decompose and vocabulary given at previous grades

#### 9.3.1.3

Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### 9.3.1.4

Understand and apply the fact that the effect of a scale factor k on length, area and volume is to multiply each by k,  $k^2$  and  $k^3$ , respectively.

Item Specifications

• Vocabulary allowed in items: scale factor, magnitude and vocabulary given at previous grades

#### 9.3.1.5

Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.

Item Specifications

• Assessed within 9.3.1.1 through 9.3.1.4

**Standard 9.3.2:** Construct logical arguments, based on axioms, definitions and theorems, to prove theorems and other results in geometry.

#### (Online MCA, 0-2 items)

(Paper MCA, 0-2 items)

#### Benchmarks

#### 9.3.2.1

Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.

Item Specifications

• Assessed within 9.3.2.2 and 9.3.2.4

#### 9.3.2.2

Accurately interpret and use words and phrases such as "if...then," "if and only if," "all," and "not." Recognize the logical relationships between an "if...then" statement and its inverse, converse and contrapositive.

- Allowable notation:  $p \rightarrow q$  (if p then q)
- Vocabulary allowed in items: inverse, converse, contrapositive, negation and vocabulary given at previous grades

## 9.3.2.3

Assess the validity of a logical argument and give counterexamples to disprove a statement. *Item Specifications* 

• Assessed within 9.3.2.4

## 9.3.2.4

Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations.

Item Specifications

- Allowable notation: CPCTC (Corresponding Parts of Congruent Triangles are Congruent), CPCFC (Corresponding Parts of Congruent Figures are Congruent), SAS (Side-Angle-Side), SSS (Side-Side-Side), ASA (Angle-Side-Angle)
- Vocabulary allowed in items: contradiction and vocabulary given at previous grades

#### 9.3.2.5

Use technology tools to examine theorems, make and test conjectures, perform constructions and develop mathematical reasoning skills in multi-step problems. The tools may include compass and straight edge, dynamic geometry software, design software or Internet applets.

#### Item Specifications

• Vocabulary allowed in items: angle bisector, perpendicular bisector, midpoint of a segment and vocabulary given at previous grades

**Standard 9.3.3:** Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.

(Online MCA, 3–7 items) (Paper MCA, 4–8 items)

#### Benchmarks

#### 9.3.3.1

Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results.

Item Specifications

- Allowable notation:  $\perp$  (perpendicular),  $\parallel$  (parallel)
- Vocabulary allowed in items: transversal, interior, exterior, corresponding, alternate and vocabulary given at previous grades

#### 9.3.3.2

Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results.

Item Specifications

• Vocabulary allowed in items: transversal, interior, exterior, corresponding, alternate, vertical and vocabulary given at previous grades

## 9.3.3.3

Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.

## Item Specifications

• Vocabulary allowed in items: equilateral, isosceles, scalene and vocabulary given at previous grades

#### 9.3.3.4

Apply the Pythagorean Theorem and its converse to solve problems and logically justify results. *Item Specifications* 

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.3.3.5

Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.3.3.6

Know and apply properties of congruent and similar figures to solve problems and logically justify results. *Item Specifications* 

Vocabulary allowed in items: vocabulary given at previous grades

#### 9.3.3.7

Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.

#### Item Specifications

• Vocabulary allowed in items: regular polygon, isosceles and vocabulary given at previous grades

#### 9.3.3.8

Know and apply properties of a circle to solve problems and logically justify results.

Item Specifications

• Vocabulary allowed in items: arc, central angle, inscribed, circumscribed, tangent, chord and vocabulary given at previous grades

Standard 9.3.4: Solve real-world and mathematical geometric problems using algebraic methods.

(Online MCA, 4–6 items) (Paper MCA, 5–7 items)

#### Benchmarks

## 9.3.4.1

Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.

## Item Specifications

- Items do not include context.
- Vocabulary allowed in items: trigonometric ratios, sine, cosine, tangent and vocabulary given at previous grades

## 9.3.4.2

Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.

#### Item Specifications

 Vocabulary allowed in items: trigonometric ratios, sine, cosine, tangent and vocabulary given at previous grades

## 9.3.4.3

Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.

#### Item Specifications

• Assessed within 9.3.4.1 and 9.3.4.2

#### 9.3.4.4

Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.

Item Specifications

• Vocabulary allowed in items: midpoint and vocabulary given at previous grades

#### 9.3.4.5

Know the equation for the graph of a circle with radius r and center (h, k),  $(x - h)^2 + (y - k)^2 = r^2$ , and justify this equation using the Pythagorean Theorem and properties of translations.

#### Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.3.4.6

Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90°, to solve problems involving figures on a coordinate grid.

Item Specifications

- Allowable notation: *P*′ (*P* prime)
- Vocabulary allowed in items: pre-image, image, isometry and vocabulary given at previous grades

#### 9.3.4.7

Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

#### Strand 4—Data Analysis & Probability

(Online MCA, 8–13 items) (Paper MCA, 10–15 items)

**Standard 9.4.1:** Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships.

(Online MCA, 3–5 items)

(Paper MCA, 3–6 items)

#### Benchmarks

#### 9.4.1.1

Describe a data set using data displays, including box-and-whisker plots; describe and compare data sets using summary statistics, including measures of center, location and spread. Measures of center and location include mean, median, quartile and percentile. Measures of spread include standard deviation, range and inter-quartile range. Know how to use calculators, spreadsheets or other technology to display data and calculate summary statistics.

#### Item Specifications

• Vocabulary allowed in items: box-and-whisker plot, quartile, percentile, inter-quartile range, standard deviation, central tendency and vocabulary given at previous grades.

#### 9.4.1.2

Analyze the effects on summary statistics of changes in data sets.

Item Specifications

• Vocabulary allowed in items: quartile, percentile, inter-quartile range, standard deviation, central tendency and vocabulary given at previous grades

## 9.4.1.3

Use scatterplots to analyze patterns and describe relationships between two variables. Using technology, determine regression lines (line of best fit) and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.

Item Specifications

• Vocabulary allowed in items: regression line, correlation coefficient and vocabulary given at previous grades

### 9.4.1.4

Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Item Specifications

- Allowable notation:  $\bar{x}$  (mean of a sample)
- Vocabulary allowed in items: standard deviation, normal distribution, normal curve and vocabulary given at previous grades

**Standard 9.4.2:** Explain the uses of data and statistical thinking to draw inferences, make predictions and justify conclusions.

(Online MCA, 1–2 items) (Paper MCA, 1–2 items)

#### Benchmarks

#### 9.4.2.1

Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Show how graphs and data can be distorted to support different points of view. Know how to use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays.

Item Specifications

• Not assessed on the MCA-III

## 9.4.2.2

Identify and explain misleading uses of data; recognize when arguments based on data confuse correlation and causation.

Item Specifications

• Vocabulary allowed in items: causation and vocabulary given at previous grades

#### 9.4.2.3

Design simple experiments and explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.

- Items do not require students to design experiments.
- Vocabulary allowed in items: vocabulary given at previous grades

**Standard 9.4.3:** Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.

(Online MCA, 4–7 items) (Paper MCA, 5–8 items)

## Benchmarks

## 9.4.3.1

Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.4.3.2

Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

Item Specifications

• Vocabulary allowed in items: simulation and vocabulary given at previous grades

#### 9.4.3.3

Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.

Item Specifications

• Vocabulary allowed in items: simulation and vocabulary given at previous grades

#### 9.4.3.4

Use random numbers generated by a calculator or a spreadsheet, or taken from a table, to perform probability simulations and to introduce fairness into decision making.

Item Specifications

- Items do not require students to generate random numbers
- Vocabulary allowed in items: simulation and vocabulary given at previous grades

#### 9.4.3.5

Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems.

#### Item Specifications

• Vocabulary allowed in items: intersections, unions, complements of events, conditional and vocabulary given at previous grades

## 9.4.3.6

Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.

Item Specifications

- Allowable notation:  $\cup$  (union),  $\cap$  (intersection)
- Vocabulary allowed in items: intersections, unions, complements and vocabulary given at previous grades

## 9.4.3.7

Understand and use simple probability formulas involving intersections, unions and complements of events. *Item Specifications* 

• Vocabulary allowed in items: intersections, unions and complements of events and vocabulary given at previous grades

#### 9.4.3.8

Apply probability concepts to real-world situations to make informed decisions.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

## 9.4.3.9

Use the relationship between conditional probabilities and relative frequencies in contingency tables. *Item Specifications* 

• Vocabulary allowed in items: conditional and vocabulary given at previous grades

# Appendix A

The minimum and maximum numbers of operational items by standard for the online adaptive Mathematics MCA-III grades 3–8 and 11 and the paper accommodated Mathematics MCA-III grades 3–8 and 11 are shown in Tables 4–10.

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		3.1.1	5	4–6
1 – Number & Operation	18–20	3.1.2	5	8–10
		3.1.3	3	5–7
2 – Algebra	6–8	3.2.1	1	2–3
		3.2.2	2	4–5
3 – Geometry & Measurement	10–13	3.3.1	2	3–4
		3.3.2	3	3–4
		3.3.3	4	4–6
4 – Data Analysis	6–7	3.4.1	1	6–7

Table 4B. Paper Accommodated MCA-III Grade 3 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		3.1.1	5	5–7
1 – Number & Operation	20–24	3.1.2	5	8–10
		3.1.3	3	5–7
2 – Algebra	8–10	3.2.1	1	3–4
		3.2.2	2	5–6
3 – Geometry & Measurement	10–13	3.3.1	2	3–4
		3.3.2	3	3–4
		3.3.3	4	4–5
4 – Data Analysis	6–8	3.4.1	1	6–8

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 Number & Operation	16–20	4.1.1	6	6–8
1 – Number & Operation	10-20	4.1.2	7	10–12
2 – Algebra	7–8	4.2.1	1	3–4
		4.2.2	2	3–4
3 – Geometry & Measurement	10–14	4.3.1	2	3–4
		4.3.2	4	4–6
		4.3.3	4	3–4
4 – Data Analysis	6–7	4.4.1	1	6–7

 Table 5A.
 Online Adaptive MCA-III Grade 4 Minimum and Maximum Item Counts by Standard

Table 5B. Paper Accommodated MCA-III Grade 4 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 Number & Operation	18–22	4.1.1	6	8–10
1 – Number & Operation	10-22	4.1.2	7	10–12
2 – Algebra	8–10	4.2.1	1	4–5
		4.2.2	2	4–5
	12–15	4.3.1	2	4–5
3 – Geometry & Measurement		4.3.2	4	5–7
Wedstreffent		4.3.3	4	3–4
4 – Data Analysis	6–8	4.4.1	1	6–8

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		5.1.1	4	5–7
1 – Number & Operation	15–21	5.1.2	5	5–7
		5.1.3	4	5–7
2 – Algebra	9–13	5.2.1	2	3–4
		5.2.2	1	2–3
		5.2.3	3	4–6
3 – Geometry & Measurement	8–10	5.3.1	2	3–4
		5.3.2	4	5–6
4 – Data Analysis	6–7	5.4.1	2	6–7

 Table 6A.
 Online Adaptive MCA-III Grade 5 Minimum and Maximum Item Counts by Standard

 Table 6B. Paper Accommodated MCA-III Grade 5 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		5.1.1	4	6–8
1 – Number & Operation	18–22	5.1.2	5	6–8
		5.1.3	4	6–8
2 – Algebra	10–14	5.2.1	2	4–6
		5.2.2	1	2–3
		5.2.3	3	4–6
3 – Geometry & Measurement	8–10	5.3.1	2	3–4
		5.3.2	4	5–6
4 – Data Analysis	6–8	5.4.1	2	6–8

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		6.1.1	7	5–7
1 – Number & Operation	11–19	6.1.2	4	2–6
		6.1.3	5	4–6
	10–13	6.2.1	2	3–4
2 – Algebra		6.2.2	1	2–3
		6.2.3	2	5–6
3 – Geometry & 8 Measurement 8-		6.3.1	3	3–5
	8–11	6.3.2	3	3–4
		6.3.3	2	2–3
4 – Data Analysis & Probability	6–8	6.4.1	4	6–8

Table 7A. Online Adaptive MCA-III Grade 6 Minimum and Maximum Item Counts by Standard

Table 7B. Paper Accommodated MCA-III Grade 6 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
		6.1.1	7	5–7
1 – Number & Operation	14–19	6.1.2	4	2–6
		6.1.3	5	5–7
2 – Algebra	12–16	6.2.1	2	4–5
		6.2.2	1	2–3
		6.2.3	2	6–8
		6.3.1	3	3–5
3 – Geometry & Measurement	10–12	6.3.2	3	3–5
Wedstrement		6.3.3	2	2–3
4 – Data Analysis & Probability	6–8	6.4.1	4	6–8

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 – Number & Operation	12–16	7.1.1	5	4–6
	12-10	7.1.2	6	8–10
		7.2.1	2	1–2
2 Alashas	13–18	7.2.2	4	5–7
2 – Algebra		7.2.3	3	3–4
		7.2.4	2	4–5
3 – Geometry &	7.0	7.3.1	2	3–4
Measurement	7–9	7.3.2	4	4–5
4 – Data Analysis & Probability	7–9	7.4.1	2	3–4
		7.4.2	1	1–2
		7.4.3	3	3–4

Table 8A. Online Adaptive MCA-III Grade 7 Minimum and Maximum Item Counts by Standard

Table 8B. Paper Accommodated MCA-III Grade 7 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 Number 9 Operation	12–16	7.1.1	5	4–6
1 – Number & Operation		7.1.2	6	8–10
	16–20	7.2.1	2	1–2
2 – Algebra		7.2.2	4	6–8
		7.2.3	3	4–6
		7.2.4	2	4–6
3 – Geometry & Measurement	8–10	7.3.1	2	4–5
		7.3.2	4	4–5
4 – Data Analysis & Probability	8–10	7.4.1	2	3–5
		7.4.2	1	1–2
		7.4.3	3	3–5

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 – Number & Operation	6–8	8.1.1	5	6–8
2 – Algebra	18–29	8.2.1	5	4–5
		8.2.2	5	4–6
		8.2.3	2	2–4
		8.2.4	9	8–14
3 – Geometry & Measurement	6–8	8.3.1	3	3–4
		8.3.2	3	3–4
4 – Data Analysis & Probability	6–7	8.4.1	3	6–7

Table 9A. Online Adaptive MCA-III Grade 8 Minimum and Maximum Item Counts by Standard

Table 9B. Paper Accommodated MCA-III Grade 8 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
1 – Number & Operation	6–8	8.1.1	5	6–8
2 – Algebra	24–30	8.2.1	5	4–5
		8.2.2	5	4–6
		8.2.3	2	3–5
		8.2.4	9	10–15
3 – Geometry & Measurement	8–10	8.3.1	3	3–5
		8.3.2	3	3–5
4 – Data Analysis & Probability	6–8	8.4.1	3	6–8

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
2 – Algebra	21–24	9.2.1	9	4–6
		9.2.2	6	5–10
		9.2.3	7	3–6
		9.2.4	8	3–7
3 – Geometry & Measurement	13–15	9.3.1	5	3–4
		9.3.2	5	0–2
		9.3.3	8	3–7
		9.3.4	7	4–6
4 – Data Analysis & Probability	8–13	9.4.1	4	3–5
		9.4.2	3	1–2
		9.4.3	9	4–7

Table 10A. Online Adaptive MCA-III Grade 11 Minimum and Maximum Item Counts by Standard

## **Table 10B.** Paper Accommodated MCA-III Grade 11 Minimum and Maximum Item Counts by Standard

Strand	Range of Items per Strand	Standard	Number of Benchmarks per Standard	Range of Items per Standard
2 – Algebra	25–29	9.2.1	9	5–7
		9.2.2	6	6–12
		9.2.3	7	4–7
		9.2.4	8	4–8
3 – Geometry & Measurement	16–18	9.3.1	5	3–5
		9.3.2	5	0–2
		9.3.3	8	4–8
		9.3.4	7	5–7
4 – Data Analysis & Probability	10–15	9.4.1	4	3–6
		9.4.2	3	1–2
		9.4.3	9	5–8