Advanced Mathematics I
Volume II

By

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www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

The Mathematics Vision Project

"The most necessary task of civilization is to teach people how to think. It should be the primary purpose of our public schools . . . The trouble with our way of educating is that it does not give elasticity to the mind. It casts the brain into a mold. It insists that the child must accept. It does not encourage original thought or reasoning, and it lays more stress on memory than observation." Thomas A. Edison

The Mathematics Vision Project (MVP) was created as a resource to teachers who desire to implement the Common Core State Standards (CCSS) using a task-based approach that leads to skill and efficiency in mathematics by first developing understanding. The MVP approach is neither purely constructivist nor purely traditional. Rather, the approach takes seriously the Standards of Mathematical Practice and develops these practices through experiential learning in mathematics. Students engage in mathematical problem solving, guided by skilled teachers, with the desired outcome that students will achieve mathematics proficiency as defined in Adding It Up—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. (*Adding It Up*, p. 5) The authors have taken on the challenge made by the National Research Council to create a curriculum where students do not learn solely by either "internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own" (*Adding It Up*, p. 11) In this way, all the strands are developed in a balanced way and students achieve proficiency.

The Mathematics Vision Project is committed to helping educators implement the Common Core State Standards (CCSS) as part of a continuum of mathematics instruction addressing conceptual, procedural, and representational thinking; depth of knowledge; and assessment. The CCSS provide a coherent trajectory of mathematical content that students should be learning as they progress from kindergarten to 12th grade. This trajectory was developed from "research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time."(CCSS, p.4) The Standards are not just a checklist of sequential content that should be taught beginning in grade school and brought to a close in high school. In order to bring the vision of the Standards to life, instructional practice must change. The MVP method embraces a different way for teachers to organize instruction to deepen student learning of mathematics.

The ***MVP classroom experience*** begins by confronting students with an engaging problem and then allows them to grapple with solving it. As students' ideas emerge, take form, and are shared, the teacher orchestrates the student discussions and explorations towards a focused mathematical goal. As conjectures are made and explored, teachers use formative assessment to guide students as they embrace effective strategies for analyzing and solving problems. Students justify their own thinking while clarifying, describing, comparing, and questioning the thinking of others leading to refined thinking and mathematical fluency.

What begin as ideas become concepts which lead to formal, traditional mathematical definitions and properties. Strategies become algorithms that lead to procedures supporting efficiency and consistency. Representations become tools of communication which are formalized as mathematical models. This is how students learn mathematics. They learn by doing mathematics. They learn by verbalizing the way they see the mathematical ideas connect and by listening to how their peers perceived the problem. This process describes the Continuum of Mathematical Understanding and it informs how teaching should be conducted within the classroom.

Each module in the **MVP** educational program has been carefully designed and sequenced with rich mathematical tasks that have been formulated to generate and develop the mathematical concepts within the core. Careful attention has been placed upon the way mathematical knowledge emerges. Some tasks are developmental tasks while others are for solidifying or practicing the concepts. The tasks also encourage students to notice relationships and make connections between the concepts. In this way, students perceive mathematics as a coherent whole.

While the ***classroom experience*** begins by improving students' reasoning and sense-making skills, it does not conclude until mathematical understanding becomes procedural skill as evidenced through application. Hence, the ***Ready, Set, Go!*** homework assignments are focused on students practicing procedural skills and organizing principles to add structure to the ideas developed during the ***classroom experience.*** As in any discipline, practice is the refining element that brings fluency and agility to the skills of the participant. Together the ***classroom experience*** and the ***Ready, Set, Go!*** homework assignments present a balanced combination of procedure and understanding for the student practitioner.

The Mathematics Vision Project has produced the first high school textbook to outline the steps a practicing teacher can take to faithfully implement the **Integrated Pathway Secondary Mathematics 1** core standards. The modules have been carefully crafted and sequenced to allow the specific mathematical ideas identified in the core to surface and then flourish into rich mathematical knowledge and skill for all students. The textbook for **Integrated Pathway Secondary Mathematics 1** assumes that students enrolled in the course have been properly prepared. The **Getting Ready** module may be used in the classroom to review content that should have been mastered in previous course work but is also necessary for success with the new material. The ***Ready, Set, Go!*** homework assignments have been designed to continue to spiral a review of content. Combined, the ***classroom experience*** and the ***Ready, Set, Go!*** homework assignments offer a powerful blend of new learning and maintained proficiency.

For more information about the Learning Cycle follow the link.
http://edutech.csun.edu/trd/sites/edutech.csun.edu.rtcweb/files/CMI%20Article.doc.

For more information about the Mathematics Vision Project visit.
www.mathematicsvisionproject.com

Advanced Mathematics I
Module 5 Advanced Features of Functions

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Module 5 – Features of Functions

5.1 Getting Ready for a Pool Party
A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with three buckets	Took a break

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

Ready, Set, Go!

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Ready

Topic: Graphing linear and exponential functions

Graph each of the functions.

1. $f(x) = -2x + 5$

2. $g(x) = 4 - 3x$

3. $h(x) = 5(3)^x$

4. $k(x) = 4(2)^x$

5. $v(t) = 2.5t - 4$

6. $f(x) = 8(3)^x$

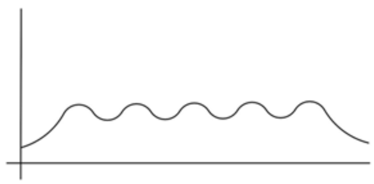
Set

Topic: Describing attributes of a function based on the graphical representation.

For each graph given match it to the contextual description that fits best. Then label the independent and dependent axis with the proper variables.

Graphs

____ 7.



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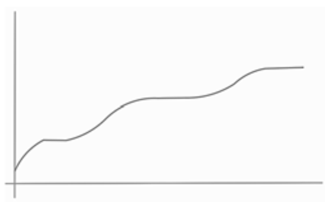

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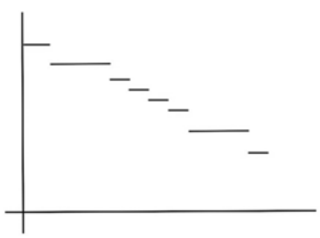

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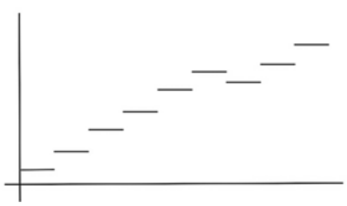

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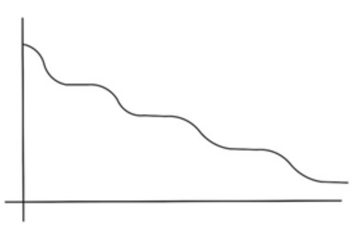


Contextual Descriptions

a. The amount of money in a savings account where regular deposits and some withdrawals are made.

b. The temperature of the oven on a day that mom bakes several batches of cookies.

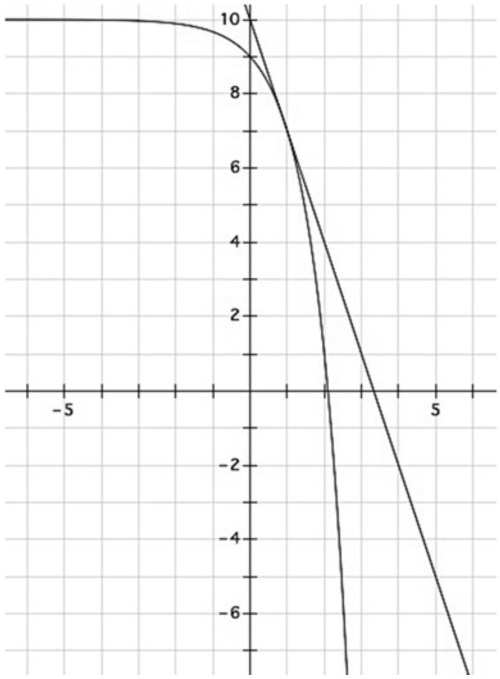
c. The amount of gasoline on hand at the gas station before a tanker truck delivers more.

d. The number of watermelons available for sale at the farmer's market on Thursday.

e. The amount of mileage recorded on the odometer of a delivery truck over a time period.

Given the pair of graphs on each coordinate grid, create a list of similarities the two graphs share and a list of differences. (Consider attributes like, continuous, discrete, increasing, decreasing, linear, exponential, restrictions on domain or range, etc.)

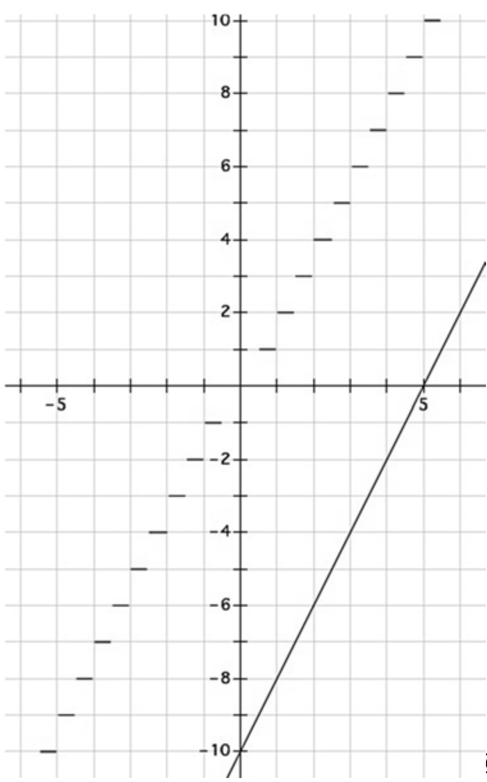
12.



Similarities:

Differences:

13.



Similarities:

Differences:

Go

Topic: Solving Equations

Find the value of *x* in each equation.

14. $10^x = 100{,}000$

15. $3x + 7 = 5x - 21$

16. $-6x - 15 = 4x + 35$

17. $5x - 8 = 37$

18. $3^x = 81$

19. $3x - 12 = -4x + 23$

20. $10 = 2^x - 22$

21. $243 = 8x + 3$

22. $5^x - 7 = 118$

5.2 Floating Down the River

A Solidify Understanding Task

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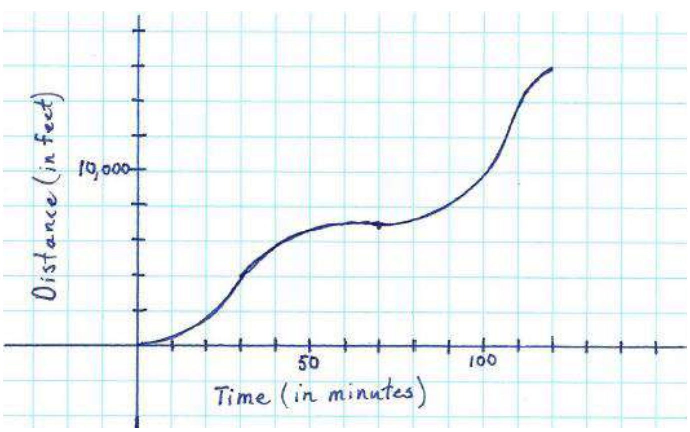
Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



2. Using the graph created by Maria, describe the key features of this relationship.

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.

- The depth of the water increases and decreases throughout the 120 minutes of floating down the river.
- The distance traveled is always increasing.
- The distance traveled is a function of time.
- The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.
- The domain of the distance/time graph is all real numbers.
- The y-intercept of the depth of water over time function is (0,0).
- The distance traveled increases and decreases over time.
- The water level is a function of time.
- The range of the distance/time graph is from [0, 15000].
- The domain of the depth of water with respect to time is from [0,120]
- The range of the depth of water over time is from [4,5].
- The distance/ time graph has no maximum value.
- The depth of water reached a maximum at 30 minutes.

Ready, Set, Go!

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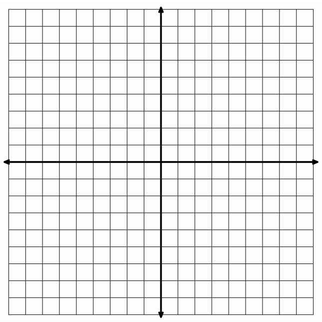
Ready

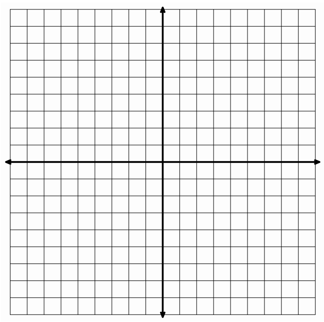
Topic: Solve systems by graphing

Graph each system of linear equations and find where $f(x) = g(x)$

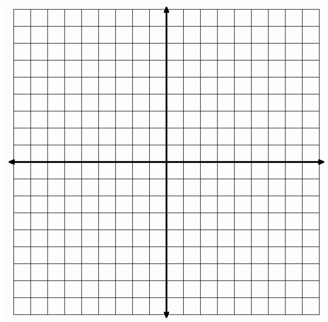
1. $\begin{cases} f(x) = 2x - 7 \\ g(x) = -4x + 5 \end{cases}$



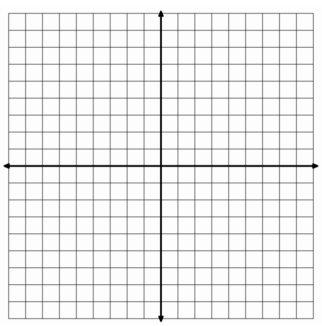
2. $\begin{cases} f(x) = -5x - 2 \\ g(x) = -2x + 1 \end{cases}$



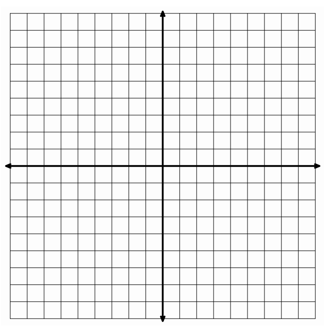
3. $\begin{cases} f(x) = -\frac{1}{2}x - 2 \\ g(x) = 2x + 8 \end{cases}$



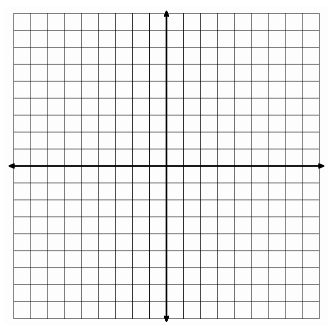
4. $\begin{cases} f(x) = \frac{2}{3}x - 5 \\ g(x) = -x \end{cases}$



5. $\begin{cases} f(x) = \frac{2}{3}x + 4 \\ g(x) = -\frac{1}{3}x + 1 \end{cases}$



6. $\begin{cases} f(x) = x \\ g(x) = -x - 3 \end{cases}$

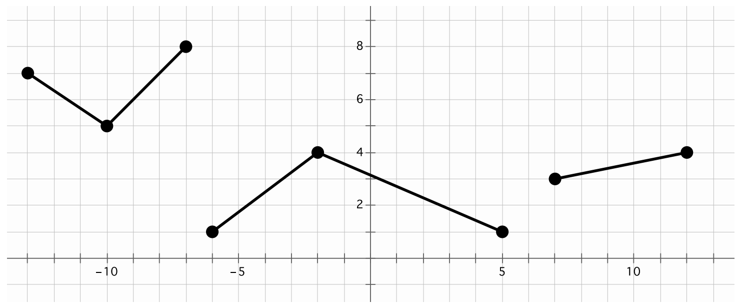


Set

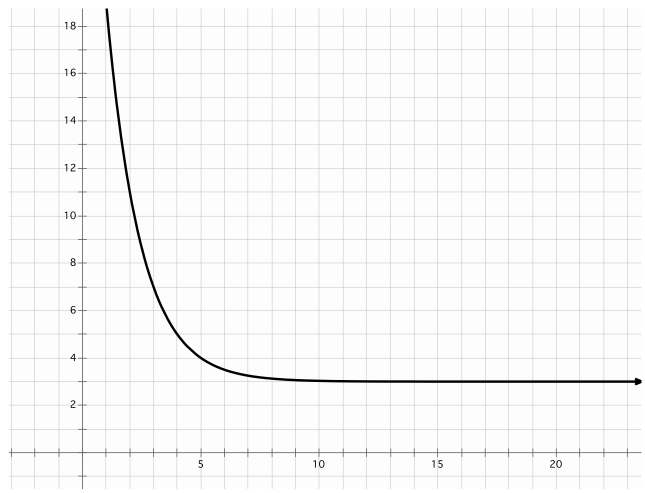
Topic: Describe features of a function from its graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

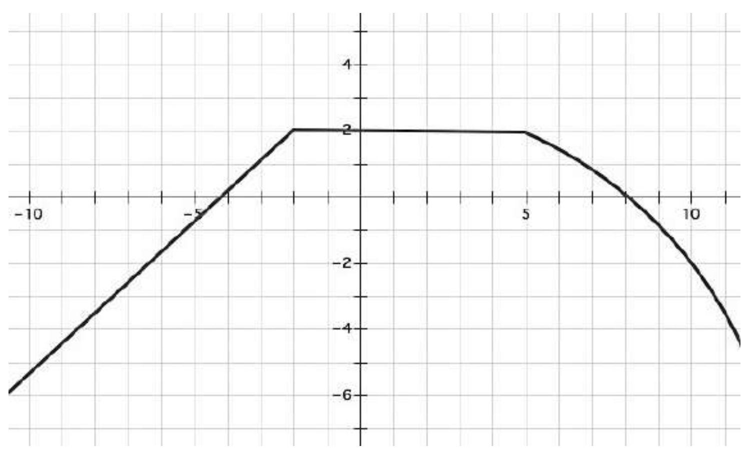
7. Description of function



8. Description of function



9. Description of function



Go

Topic: Create equations using both explicit and recursive notation.

Write equations for the given tables in both recursive and explicit form.

10.

n	$f(n)$
1	5
2	2
3	-1

Explicit:

Recursive:

11.

n	$f(n)$
1	6
2	12
3	24

Explicit:

Recursive:

12.

n	$f(n)$
0	-13
2	-5
3	-1

Explicit:

Recursive:

13.

n	$f(n)$
1	5
4	11
5	13

Explicit:

Recursive:

14.

n	$f(n)$
2	5
7	15,625
9	390,625

Explicit:

Recursive:

15.

n	$f(n)$
0	-4
1	-16
2	-64

Explicit:

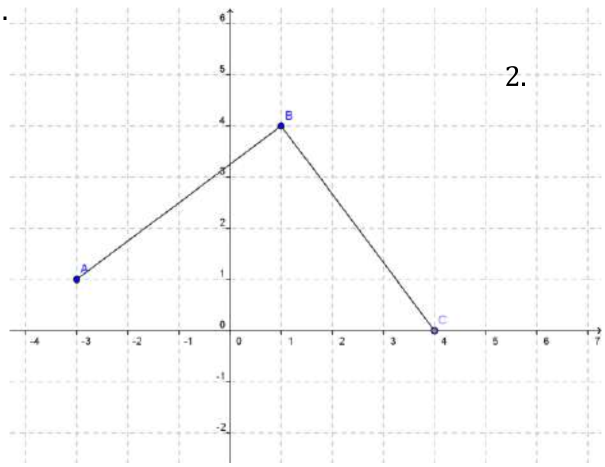
Recursive:

5.3 Features of Functions

A Practice Understanding Task

For each graph, determine if the relationship represents a function, and if so, state the key features of the function (intervals where the function is increasing or decreasing, the maximum or minimum value of the function, domain and range, x and y intercepts, etc.)

1.



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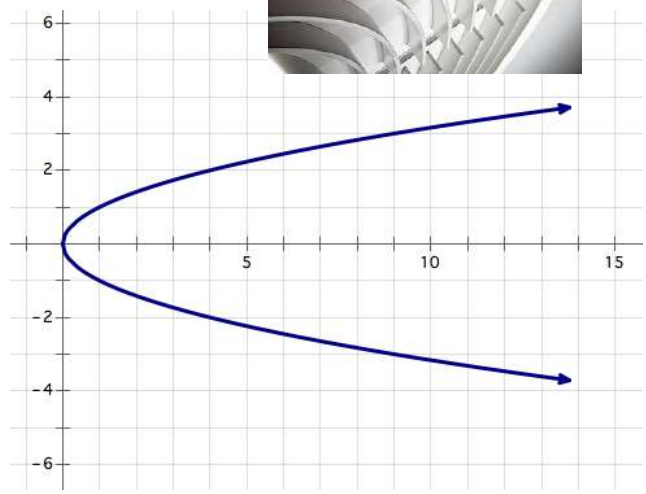

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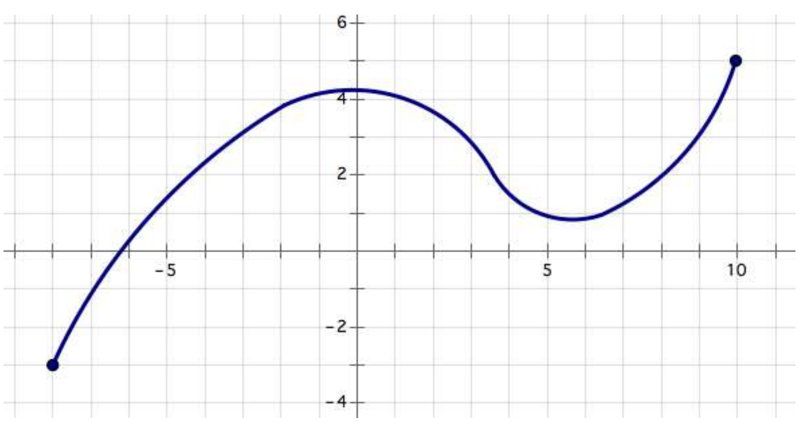

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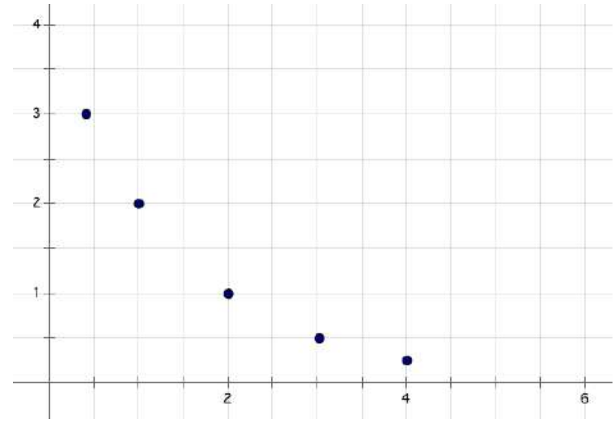

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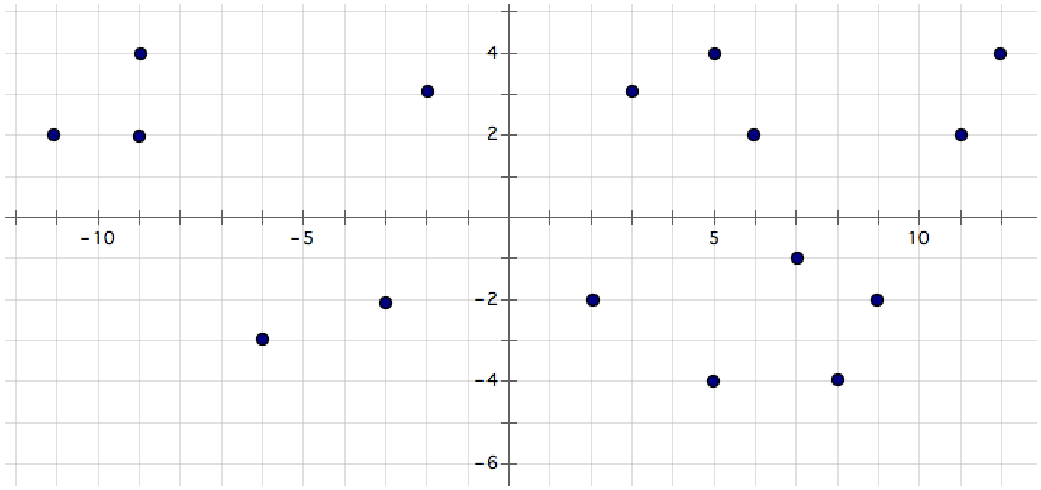

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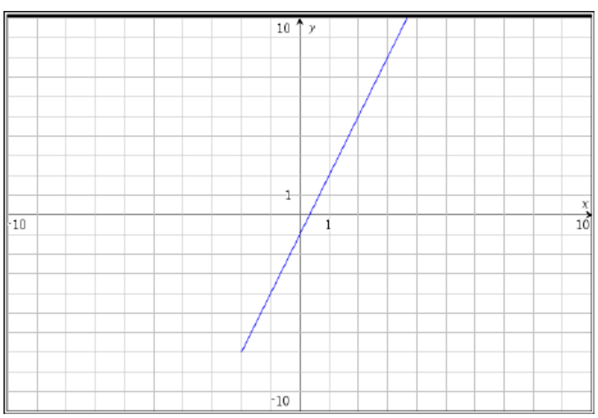
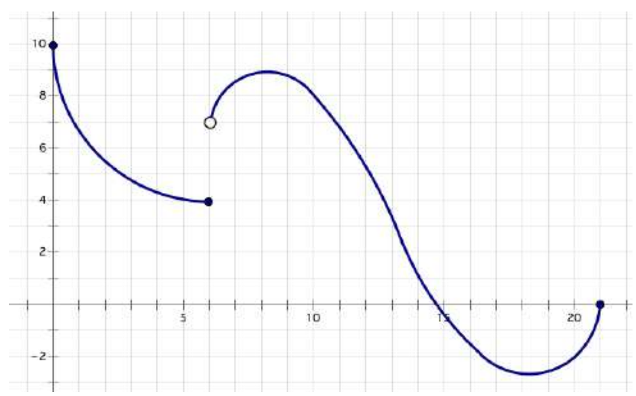


7.



The following represents a continuous function defined on the interval from [0, 6].

x	***f(x)***
0	2
1	-3
2	0
3	2
4	6
5	12
6	20

8. Determine the domain, range, x and y intercepts.
9. Based on the table, identify the minimum value and where it is located

The following represents a discrete function defined on the interval from [1,5].

x	***f(x)***
1	4
2	10
3	5
4	8
5	3

10. Determine the domain, range, x and y intercepts.
11. Based on the table, identify the minimum value and where it is located.

Describe the key features for each situation.

12. The amount of daylight dependent on the time of year.

13. The first term in a sequence is 36. Each consecutive term is exactly 1/2 of the previous term.

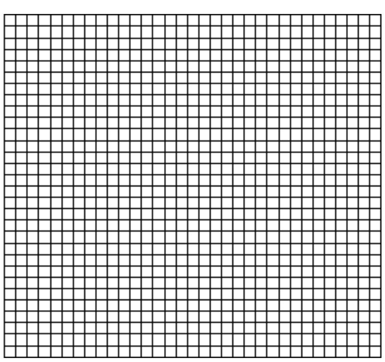
14. Marcus bought a $900 couch on a six months, interest free payment plan. He makes $50 payments to the loan each week.

15. The first term in a sequence is 36. Each consecutive term is 1/2 less than the previous term.

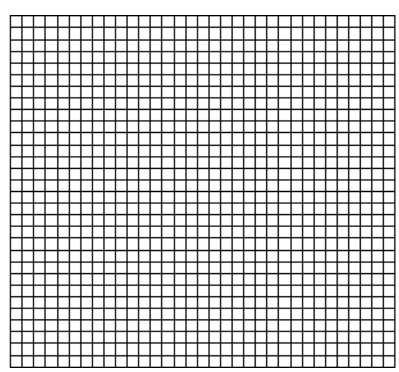
16. An empty 15 gallon tank is being filled with gasoline at a rate of 2 gallons per minute.

For each equation, sketch a graph and show key features of the graph.

17. $f(x) = -2x + 4, when\ \ x \geq 0$



18. $g(x) = 3^x$



Ready, Set, Go!

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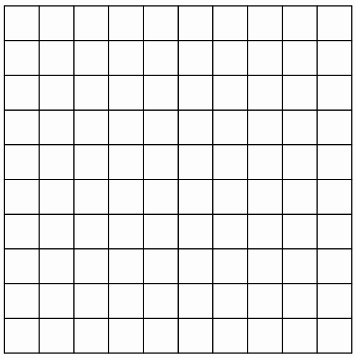
Ready

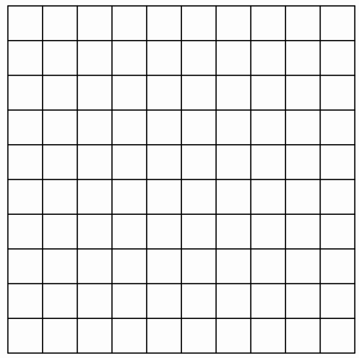
Topic: Creating graphical representations and naming the domain.

Sketch a graph to represent each function, then state the domain of the function.

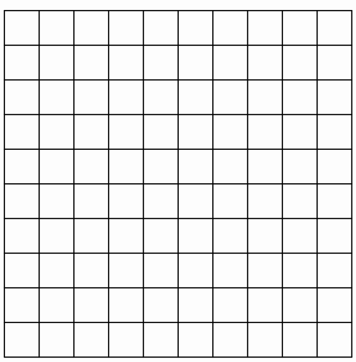
1. $y = 3x - 5$



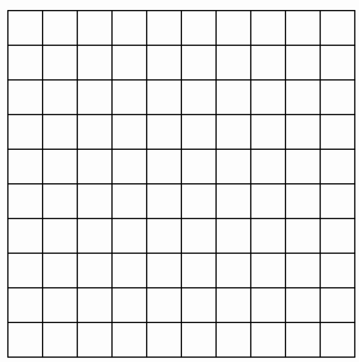
2. $f(x) = 3(4)^x$



3. A sequence of terms such that $f(0) = 1, f(n) = f(n - 1) - 7$



4. A sequence of terms such that $f(1) = 8, f(n) = \frac{1}{2} f(n-1)$



Set

Topic: Attributes of linear and exponential functions.

Determine if the statement is true or false, then justify why.

5. All linear functions are increasing.

6. Arithmetic sequences are an example of linear functions.

7. Exponential functions have a domain that includes all real numbers.

8. Geometric sequences have a domain that includes all integers.

9. The range for an exponential function includes all real numbers.

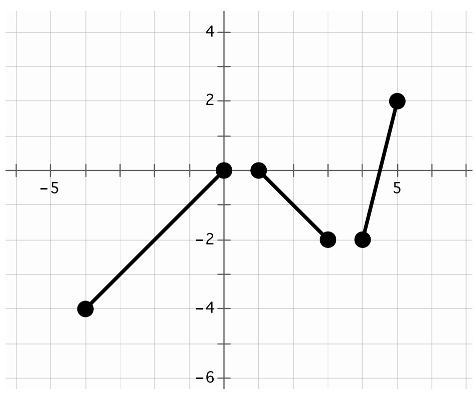
10. All linear relationships are functions with a domain and range containing all real numbers.

Go

Topic: Determine the domain of a function from the graphical representation.

For each graph determine the domain of the function.

11.



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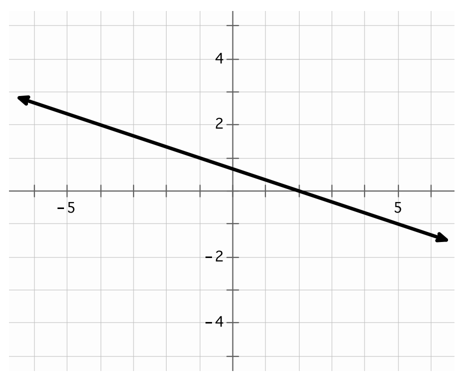

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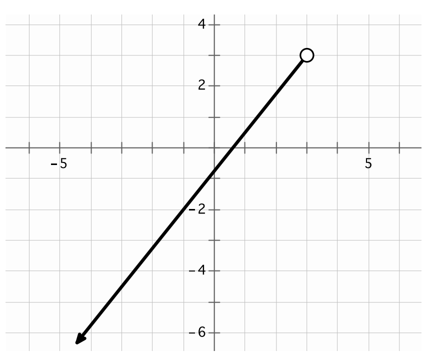

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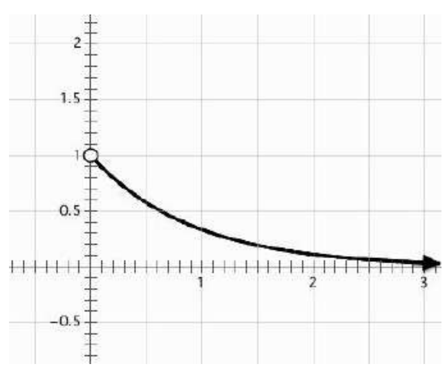

5.4 The Water Park

A Solidify Understanding Task

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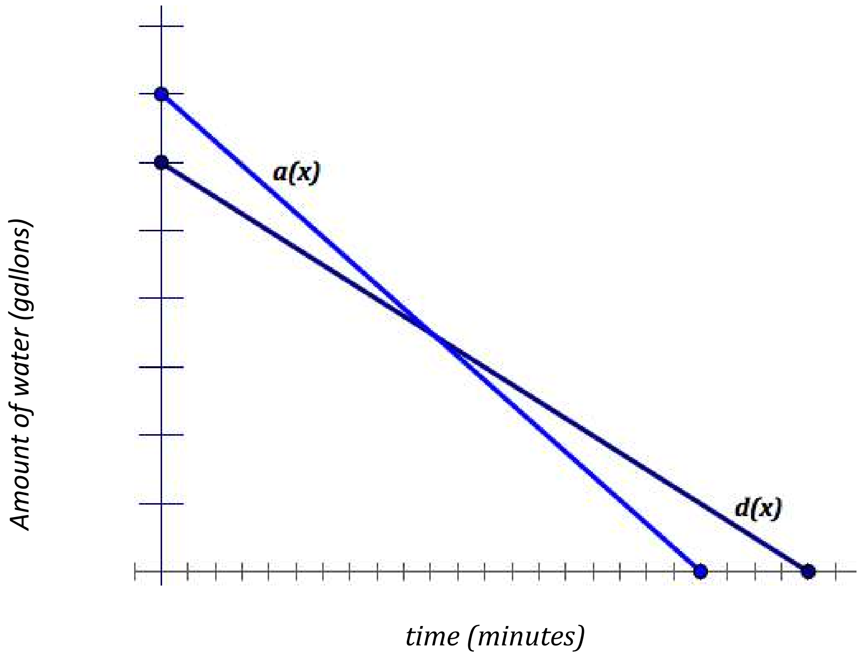
Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, *a(x),* and Dayne's pool, *d(x)*, over time.



Part I

1. Make as many observations as possible with the information given in the graph above.

Part II

Dayne figured out that the pump he uses drains water at a rate of 1000 gallons per minute and takes 24 minutes to drain.

2. Write the equation to represent the draining of Dayne's pool, *d(x)*. What does each part of the equation mean?

3. Based on this new information, correctly label the graph above.
4. For what values of x make sense in this situation? Use interval notation to write the domain of the situation.
5. Determine the range, or output values, that make sense in this situation.
6. Write the equation used to represent the draining of Aly's pool, $a(x)$. Using interval notation, state the domain and range for the function, $a(x)$ as well as the domain and range of the situation. Compare the two domains by describing the constraints made by the situation.

Part III

Based on the graph and corresponding equations for each pool, answer the following questions.

7. When is $a(x) = d(x)$? What does this mean?

8. Find $a(10)$. What does this mean?

9. If $d(x)=2000$, then $x=$ ____. What does this mean?

10. When is $a(x) > d(x)$? What does this mean?

Ready, Set, Go!

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Ready

Topic: Attributes of linear and exponential functions.

1. Write a well-developed paragraph comparing and contrasting linear and exponential functions. Be sure to include as many characteristics of each function as possible and be clear about the similarities and differences these functions have.

____

____

____

____

____

____

____

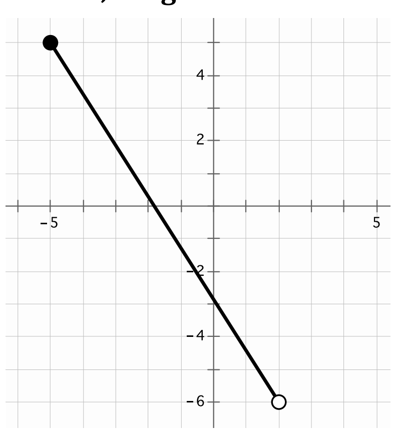
____

Set

Topic: Identifying attributes of a function from its graphical representation.

Based on the graph given in each problem below, identify attributes of the function such as the domain, range and whether or not the function is increasing or decreasing, etc.

2.



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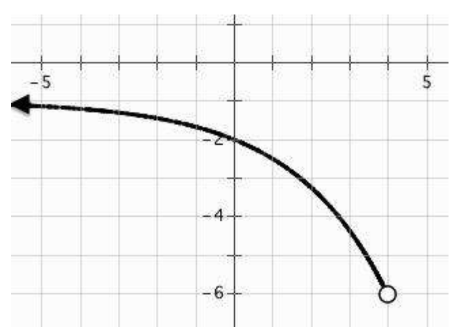

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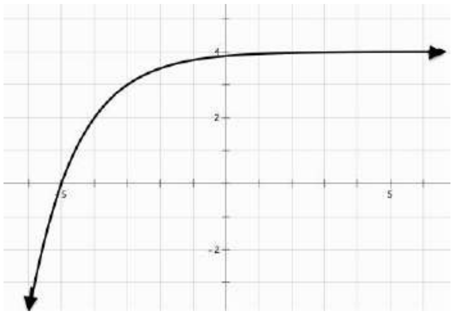

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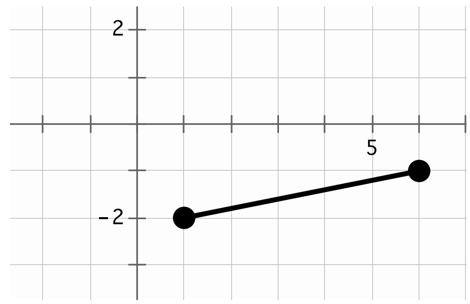

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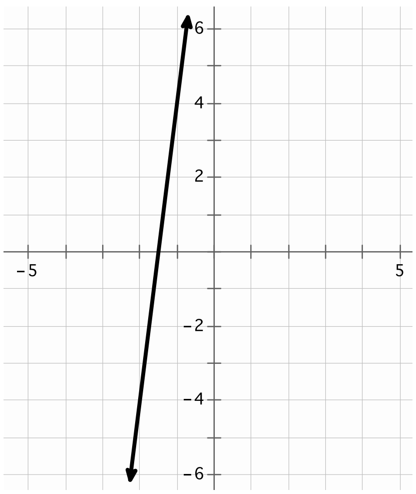

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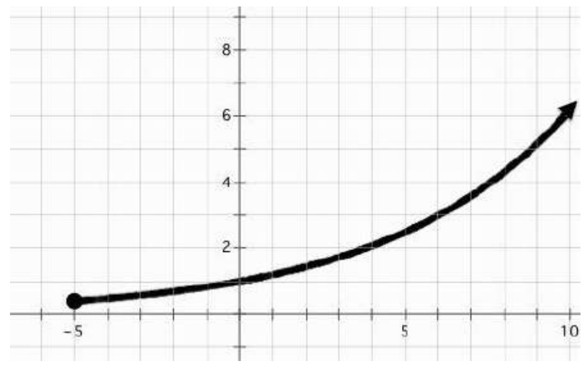

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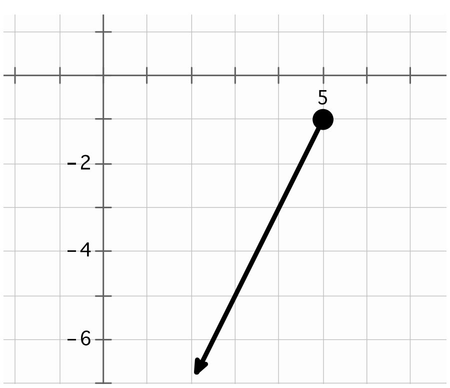

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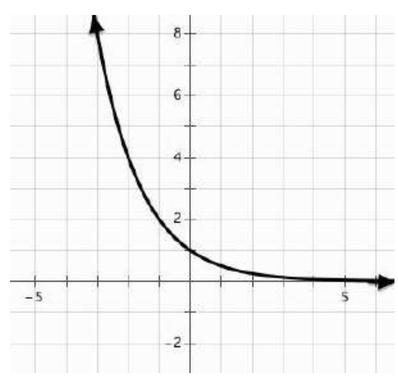

Go

Topic: Finding equations and rules for functions

Find both the explicit and the recursive equations for each table of values below.

10.

n	$f(n)$
1	3
2	5
3	7
4	9

Explicit:

Recursive:

11.

n	$f(n)$
2	4
3	8
4	16
5	32

Explicit:

Recursive:

12.

n	$f(n)$
6	23
7	19
8	15
9	11

Explicit:

Recursive:

13.

n	$f(n)$
1	1
2	3
3	9

Explicit:

Recursive:

14.

n	$f(n)$
3	8
4	4
5	2

Explicit:

Recursive:

15.

n	$f(n)$
6	7
9	13
12	19

Explicit:

Recursive:

16.

n	$f(n)$
2	40
4	32
8	16

Explicit:

Recursive:

17.

n	$f(n)$
2	16
3	4
4	1

Explicit:

Recursive:

18.

n	$f(n)$
17	5
20	10
26	20

Explicit:

Recursive:

5.5 Pooling It Together

A Solidify Understanding Task

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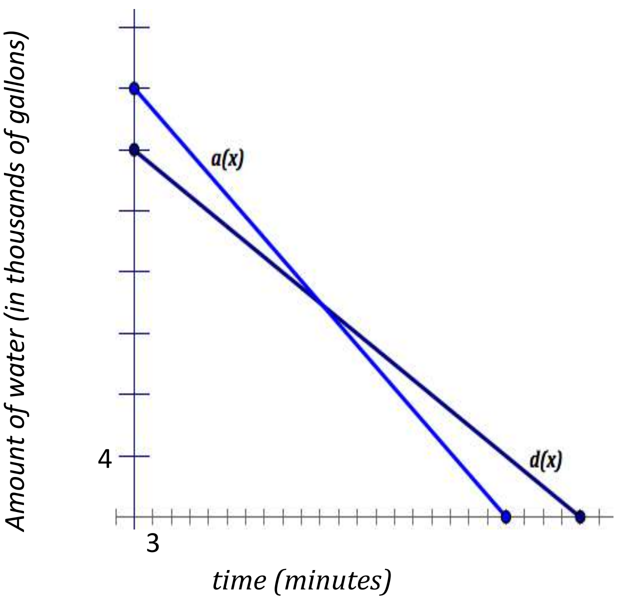
Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, *a(x)*, and Dayne's pool, *d(x)*, over time. In this scenario, they decided to work together to drain their pools and created the equation *g(x) = a(x) +d(x).* Using the graph below showing *a(x)* and *d(x)*, create a new set of axes and graph *g(x).* Identify *g(x)* and label (scale, axes).



Answer the following questions about *g(x)*.

1. What does *g(x)* represent?

2. Name the features of *g(x)* and explain what each means (each intercept, domain and range for this situation and for the equation, maxima and minima, whether or not *g(x)* is a function, etc.)

3. Write the equation for *g(x)* using the intercepts from the graph. Compare this equation to the sum of the equations created for *a(x)* and *d(x)* from "The Water Park" task. Should be they be equivalent? Why or why not?

When combining functions, a lot of connections can be made. Make at least three connections showing how the equations *a(x), d(x),* and *g(x)* relate to the graphs of *a(x), d(x),* and *g(x).*(hint: think about the key features of these functions).

For A Twist:

If Aly and Dayne's boss started to drain the water before they arrived and when they got there, there was already 5,000 less gallons of water to be drained, how would this impact the equation?

Write the new equation representing how long it will take them to drain the two pools.

Ready, Set, Go!

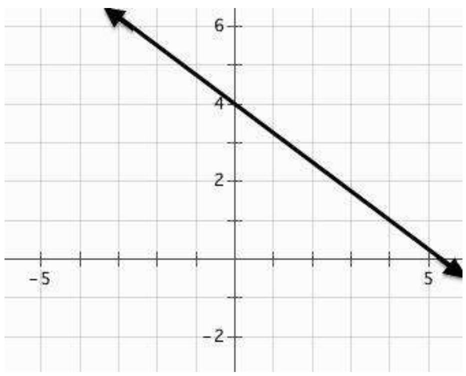
© eilt

Ready

Topic: Use a graphical representation to find solutions.

Use the graph of each function provided to find the values indicated.

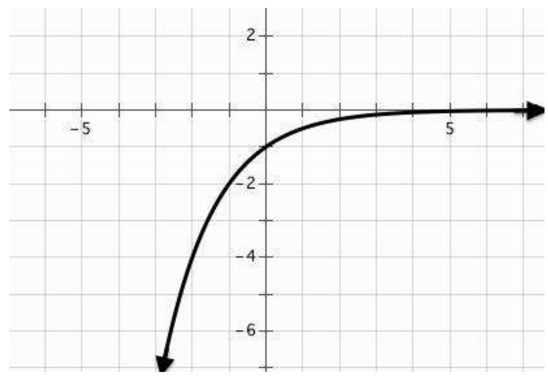
1. . $f(x)$



a. $f(4)$ = _______ b. $f(-4)$ = _________

c. $f(x) = 4$, x = ________ d. $f(x) = 7$, x = _______

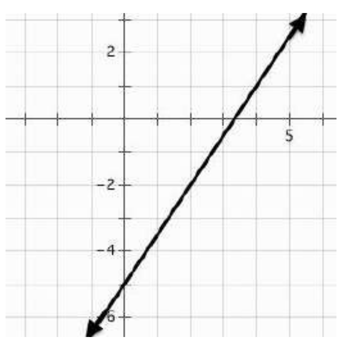
2. $g(x)$



a. $g(-1)$= _________ b. $g(-3)$ = __________

c. $g(x) = -4$ x = ______ d. $g(x) = -1$, x = ________

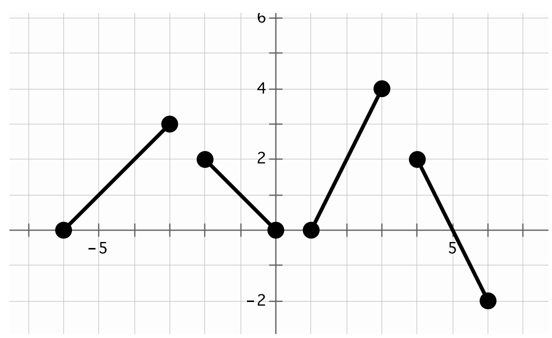
3. $h(x)$



a. $h(0)$ = _______ b. $h(3)$ = _________

c. $h(x) = 1$, x = ________ d. $h(x) = -2$, x = _____

4. $d(x)$



a. $d(-5)$ = _______ b. $d(4)$ = _________

c. $d(x) = 4$, x = ________ d. $d(x) = 0$, x = _____

Set

Topic: Given context of a function find solutions.

For each situation either create a function or use the given function to find and interpret solutions.

5. Fran collected data on the number of feet she could walk each second and wrote the following rule to model her walking rate $d(t) = 4t$.

a. What is Fran looking for if she writes "$d(12) =$ " ?

b. In this situation what does $d(t) = 100$ tell you?

c. How can the function rule be used to indicate a time of 16 seconds was walked?

d. How can the function rule be used to indicated that a distance of 200 feet was walked?

6. Ms. Callahan works hard to budget and predict her costs for each month. She is currently attempting to determine how much her cell phone company will likely charge her for the month. She is paying a flat fee of $80 a month for a plan that allows for unlimited calling but costs her an additional twenty cents per text message.

a. Write a function, c(t), for Ms. Callahan's current cell plan that will calculate the cost for the month based on the number of text messages she makes.

b. Find c(20)

d. Find c(t) = 100

c. Find c(45)

e. Find c(t) = 90

f. At what number of texts would $20 unlimited texting be less expensive then her current plan?

7. Mr. Multbank has developed a population growth model for the rodents in the field by his house. He believes that starting each spring the population can be modeled based on the number of weeks with the function $p(t) = 8(2^t)$.

a. Find $p(t) = 128$ b. Find $p(4)$ c. Find $p(10)$

d. Find the number of weeks it will take for the population to be over 20,000.

e. In a year with 16 weeks of summer, how many rodents would he expect by the end of the summer using Mr. Multbank's model? What are some factors that could change the actual result from your estimate?

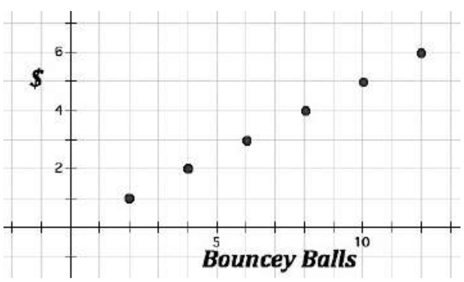
Go

Topic: Discrete and continuous

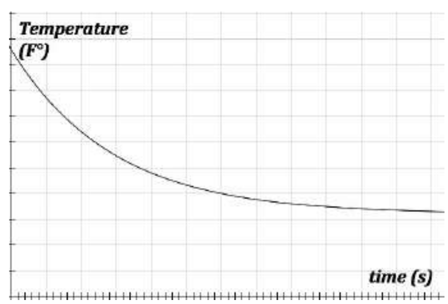
For each context or representation determine whether it is discrete or continuous or could be modeled best in a discrete or continuous way and state why.

8. Susan has a savings plan where she places $5 a week in her piggy bank.

9.



10.



11. Marshal tracks the number of hits he gets each baseball game and is recording his total number of hits for the season in a table.

12. The distance you have traveled since the day began.

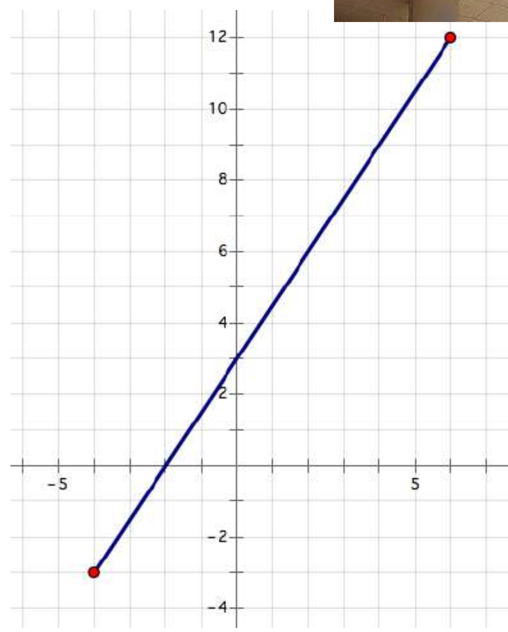
13.

Number of Gum Balls	Cost
5	1
10	2
15	3
20	4

5.6 Interpreting Functions
A Practice Understanding Task

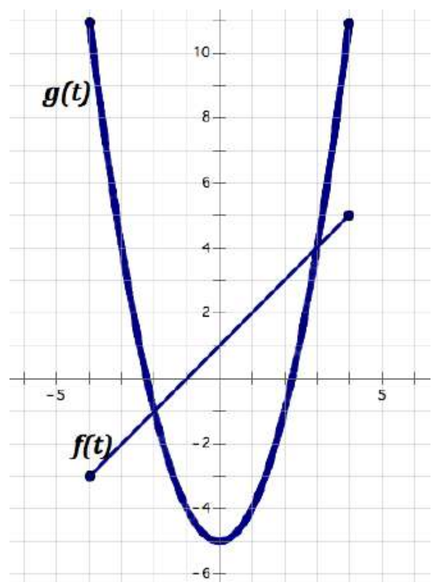
Given the graph of *f(x),* answer the following questions. Unless otherwise specified, restrict the domain of the function to what you see in the graph below. Approximations are appropriate answers.

1. What is *f*(2)?
2. For what values, if any, does *f(x)* = 3?
3. What is the x-intercept?
4. What is the domain of *f(x)*?
5. On what intervals is *f(x)* > 0?
6. On what intervals is *f(x)* increasing?
7. On what intervals is *f(x)* decreasing?
8. For what values, if any, is *f(x)* > 3?



Consider the linear graph of *f(t)* and the nonlinear graph of *g(t*) to answer questions 9-14. Approximations are appropriate answers.

9. Where is *f(t)* = *g(t)*?
10. Where is *f(t)* > *g(t)* ?
11. What is *f(0) + g(0)* ?
12. What is *f(-1) + g(-1)* ?
13. Which is greater: *f(0) or g(-3)*?
14. Graph: *f(t)* + *g(t)* from [-1, 3]



The following table of values represents two continuous functions, *f(x) and g(x)*. Use the table to answer the following questions:

x	*f(x)*	*g(x)*
-5	42	-13
-4	30	-9
-3	20	-5
-2	12	-1
-1	6	3
0	2	7
1	0	11
2	0	15
3	2	19
4	6	23
5	12	27
6	20	31

15. What is *g*(-3)?

16. For what value(s) is *f(x)* = 0?

17. For what values is *f(x)* increasing?

18. On what interval is *g(x)* > *f(x)*

19. Which function is changing faster in the interval [-5, 0]? Why?

Use the following relationships to answer the questions below.

$h(x) = 2^x$ $\quad f(x) = 3x - 2$ $\quad g(x) = 5$ $\quad x = 4$ $\quad y = 5x + 1$

20. Which of the above relations are functions? Explain.

21. Find *f(2), g(2), and h(2).*

22. Write the equation for *g(x) + h(x).*

23. Where is *g(x)< h(x)*?

24. Where is *f(x)* increasing?

25. Which of the above functions has the fastest growth rate?

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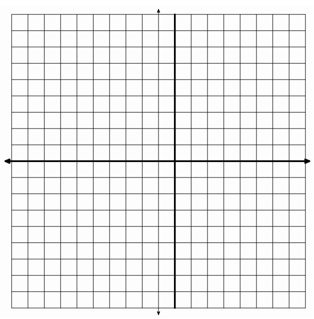
Ready, Set, Go!

Ready

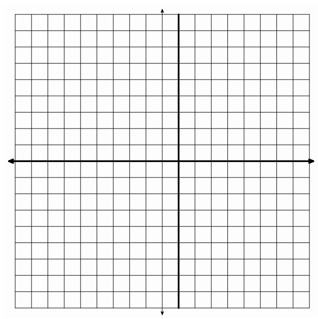
Topic: Solve systems of equations

Solve each system of equations either by graphing, substitution, elimination, or matrix row reduction. *Use each method at least once.*

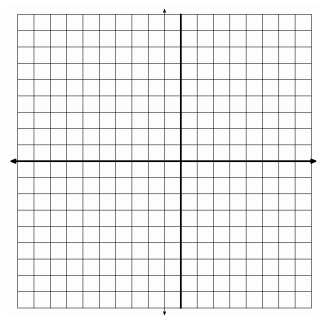
1. $\begin{cases} 3x+4 \\ 4x+1 \end{cases}$



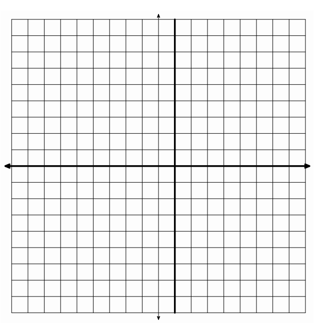
2. $\begin{cases} -5x+12 \\ -2x-3 \end{cases}$



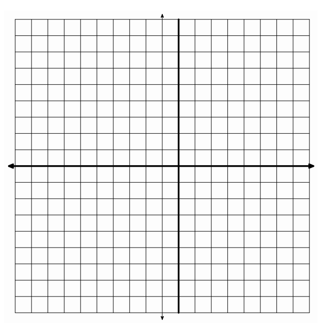
3. $\begin{cases} \frac{1}{2}\,x+2 \\ 2x-7 \end{cases}$



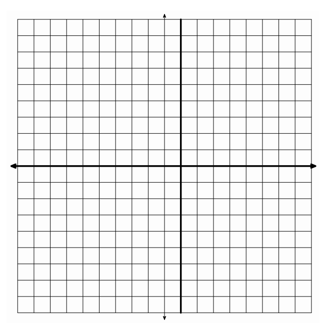
4. $\begin{cases} -\frac{2}{3}\,x+5 \\ -x+7 \end{cases}$



5. $\begin{cases} x+5 \\ -x-3 \end{cases}$



6. $\begin{cases} x-6 \\ -x-6 \end{cases}$

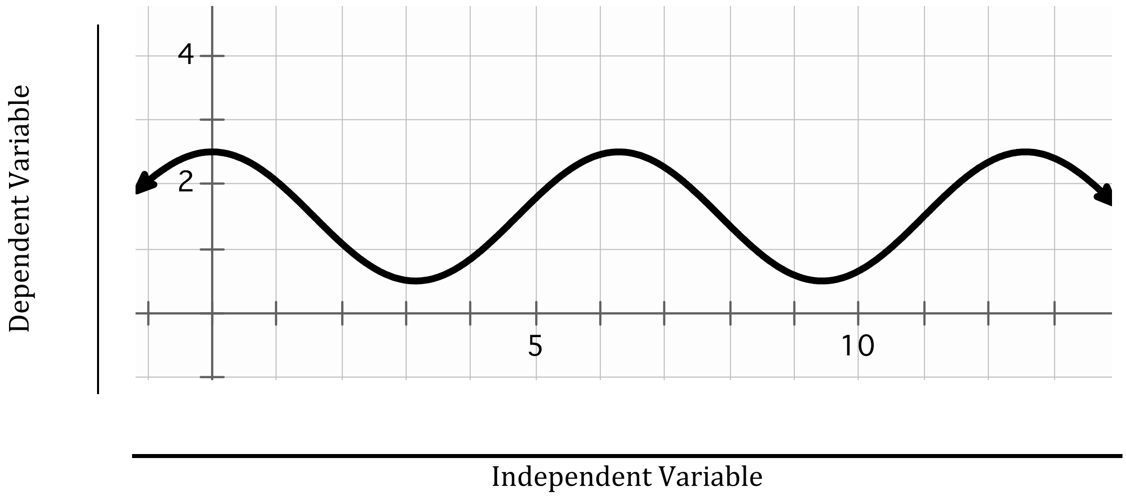


Set

Topic: Connecting context to graphical representations

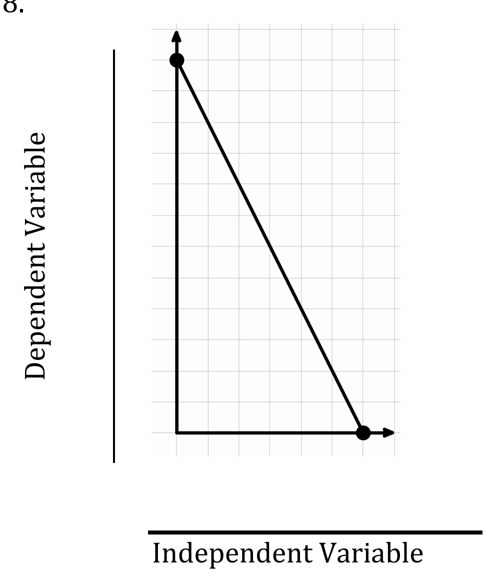
For each graph create a context, provide independent and dependent variables that will fit the context you choose. Then create a story that describes what is happening on the graph.

7.



Description of context and a story for the graph:

8.



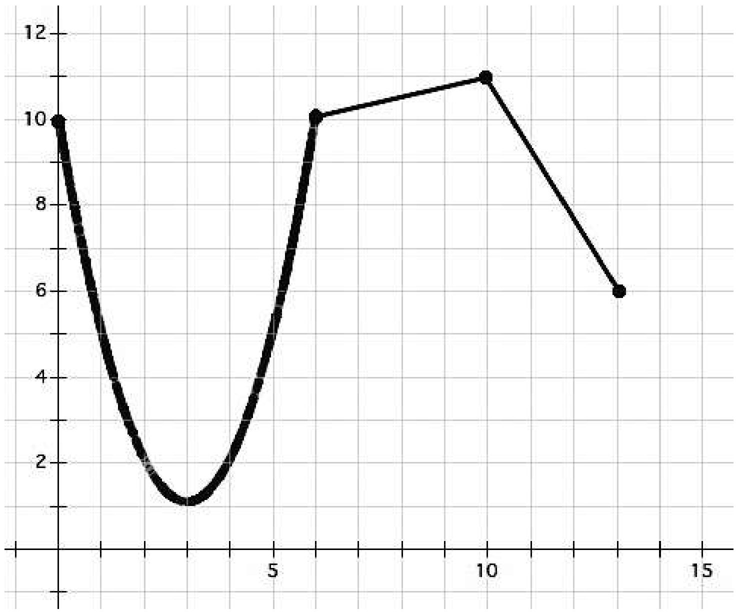
Description of context and a story for the graph:

Go

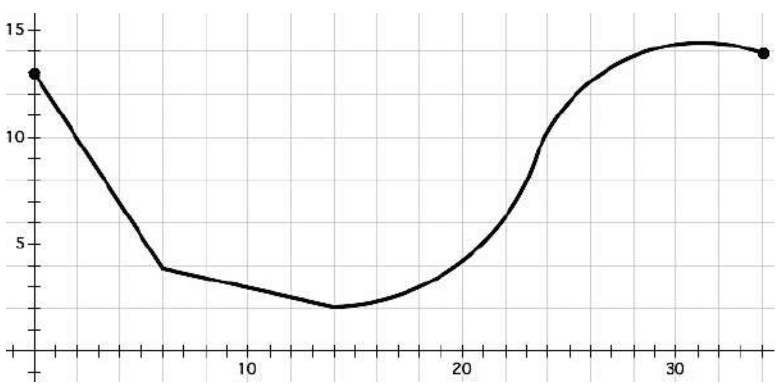
Topic: Describe features of a function from its graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

9. Description of function:

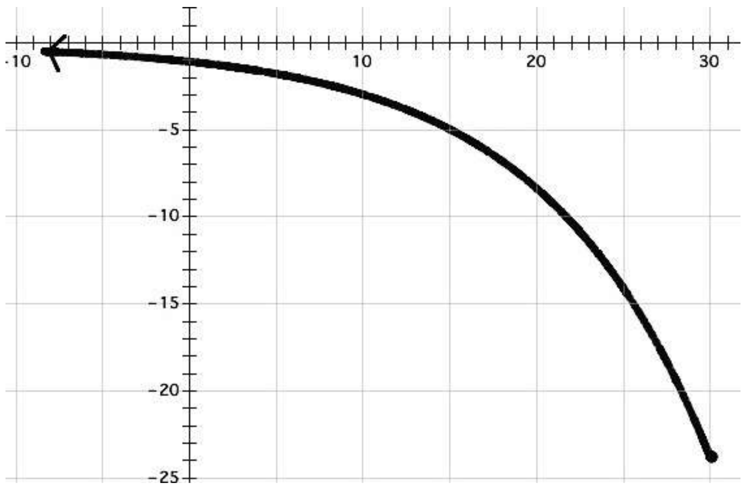


10. Description of function:



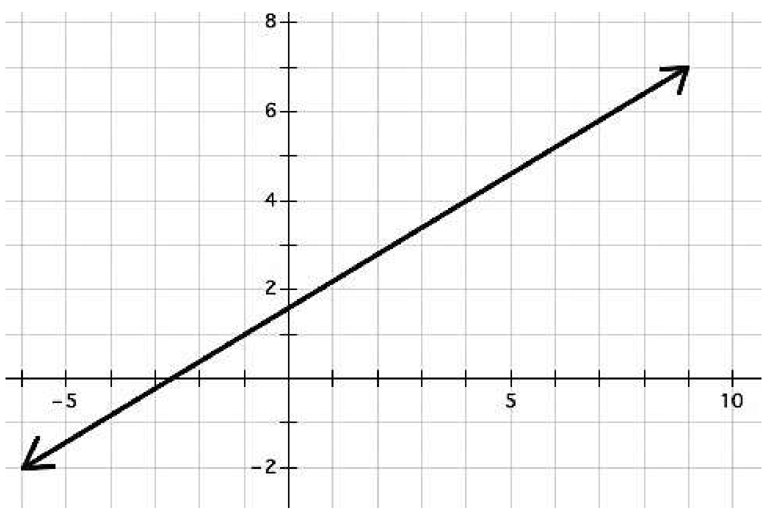
11.

Description of function:



12.

Description of function:



5.7 A Water Function
A Develop Understanding Task

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Andrew walked around the water park taking photos of his family with his phone. Later, he discovered his phone was missing. So that others could help him look for his lost phone, he drew a picture that 'retraced his steps' showing where he had walked.

If we wanted to determine Andrew's location in the park with respect to time, would his location be a function of time?

Why or why not? Explain.

1. Situation A: Sketch a graph of the total distance Andrew walked if he walked at a constant rate the entire time.

2. Situation B: Sketch a graph of Andrew's distance from the entrance (his starting point) as a function of time.

3. How would the graph of each situation change if Andrew stopped at the slide for a period of time? Would this change whether or not this situation is a function?

Ready, Set, Go!

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Ready

Topic: Mathematical comparisons

Use the given comparison statements to answer the questions.

1. 3 out of 5 students prefer playing football to playing basketball.

    a. What percent of students prefer playing football?

    b. What percent of students prefer playing basketball?

2. The ratio of student wearing yellow to students not wearing yellow is 3 to 7.

    a. What fraction of students have on yellow?

    b. What percent of students don't have on yellow?

3. Of the students at school, 40% attended the basketball game.

    a. What fraction of the students attended the basketball game?

    b. How many times more students did not attend the basketball game?

4. 1000 students ride buses to school while 600 walk or carpool.

    a. What fraction of students ride the bus?

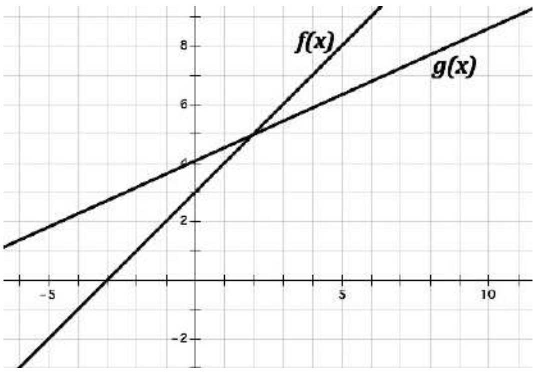
    b. How many more students ride the bus than walk or carpool?

    c. What percent of students walk or carpool?

Set

Topic: Comparing functions from different representations

Use the given representation of the functions to answer the questions.

5. 

a. Where does $f(x) = g(x)$?

b. What is $f(4) + g(4)$?

c. What is g(-2) – f(-2) ?

d. On what interval is $g(x) > f(x)$?

e. Sketch f(x) + g(x) on the graph provided.

6. The functions $a(x)$ and $b(x)$ are defined in the table below. Each function is a set of exactly five ordered pairs.

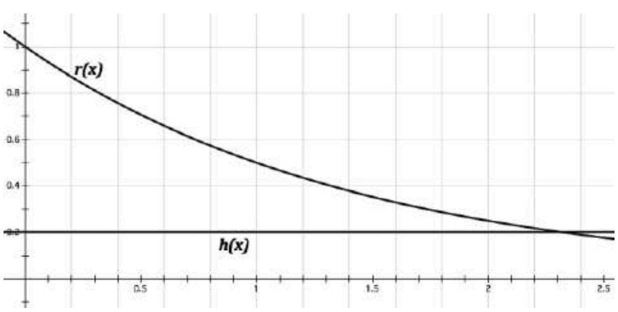
x	***a*(*x*)**	***b*(*x*)**
-3	1	-1
-1	7	-5
0	3	-10
2	8	2
7	3	3

a. What is $a(-3)+b(-3)$?

b. What is $a(-1) - b(-1)$?

c. What is $a(0) + b(0)$?

d. Add two columns to the table and provided a(x) + b(x) in one and a(x) – b(x) in the other.

7. 

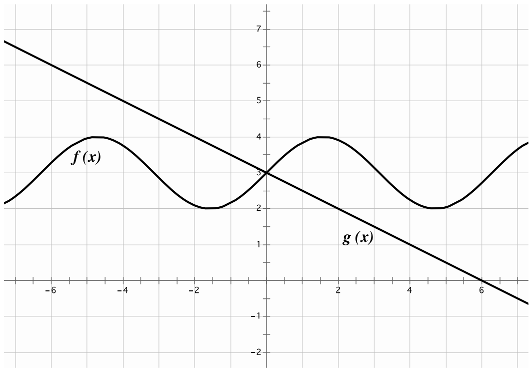
a. Where is $r(x) > h(x)$?

b. What is $r(1) - h(1)$?

c. What is $r(0) + h(0)$?

d. Create an explicit rule for $r(x)$ and for $h(x)$.

e. Sketch $r(x) - h(x)$ on the graph.

8. 

a. Where does f(x) = g(x)?

b. What is f(4) + g(4) ?

c. What is g(-2) – f(-2) ?

d. On what interval is g(x) > f(x) ?

e. Sketch f(x) - g(x) on the graph provided.

Go

Topic: Solving equations for a specified variable. Literal equations.

Rewrite each equation in slope-intercept form (y = mx+b).

9. $12x + 3y = 6$

10. $8x + y = 5$

11. $y - 5 = -3(x + 2)$

12. $9x - y = 7$

13. $y - 9x = 4(x - 2)$

14. $16x = 20 + 8y$

Write an explicit function for the linear function that goes through the given point with the given slope simplified into slope-intercept form.

15. m = 3, (-1, 2)

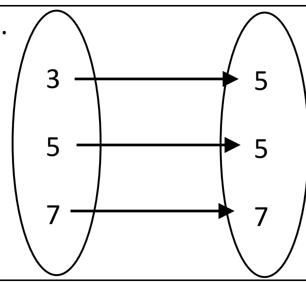
16. m = -5, (3, 4)

17. m = $\frac{3}{4}$, (-4, 2)

5.8 To Function or Not To Function

A Practice Understanding Task

Determine if the following relationships are functions and then justify your reasoning.

<table>
<tr>
<td>1.A person's name versus their social security number.</td>
<td>2. A person's social security number versus their name.</td>
<td>3. The cost of gas versus the amount of gas pumped.</td>
</tr>
<tr>
<td>4.
{ (3,6), (4, 10), (8,12), (4, 10)}</td>
<td>5. The temperature in degrees Fahrenheit with respect to the time of day.</td>
<td>6.
<table><tr><td>distance</td><td>days</td></tr><tr><td>6</td><td>2</td></tr><tr><td>10</td><td>4</td></tr><tr><td>6</td><td>5</td></tr><tr><td>9</td><td>8</td></tr></table></td>
</tr>
<tr>
<td>7. The area of a circle as it relates to the radius.</td>
<td>8.
</td>
<td>9. The radius of a cylinder is dependent on the volume.</td>
</tr>
<tr>
<td>10. The size of the radius of a circle dependent on the area.</td>
<td>11. Students letter grade dependent on the percent earned.</td>
<td>12. The length of fence needed with respect to the amount of area to be enclosed.</td>
</tr>
<tr>
<td>13. The explicit formula for the recursive situation below:
$f(1) = 3$ and
$f(n + 1) = f(n) + 4$</td>
<td>14.
If x is a rational number, then $f(x) = 1$
If x is an irrational number, then $f(x) = 0$</td>
<td>15. The national debt with respect to time.</td>
</tr>
</table>

Ready, Set, Go!

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Ready

Topic: Determine domain and range, and whether a relation is a function or not a function.

Determine if each set of ordered pairs is a function or not then state the domain and range.

1. { (-7, 2), (3, 5), (8, 4), (-6, 5), (-2, 3)} Function: Yes / No

Domain: Range:

2. { (9, 2), (0, 4), (4, 0), (5, 3), (2, 7) (0, -3), (3, -1)} Function: Yes / No

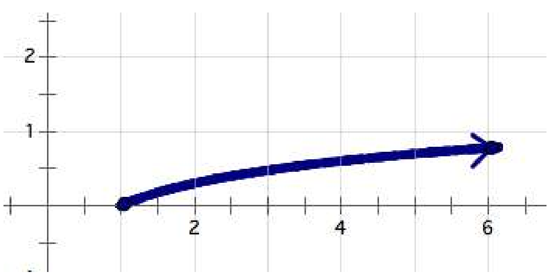
Domain: Range:

3. { (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9)} Function: Yes / No

Domain: Range:

For the representation of the function given determine the domain and range.

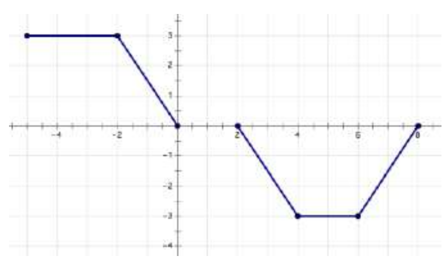
4.



Domain:

Range:

5.



Domain:

Range:

6. $f(x) = -2x + 7$

Domain:

Range:

7. $g(x) = 3\,(5)^x$

Domain:

Range:

8. The elements in the table define the entirety of the function.

Domain:

Range:

x	h(x)
1	9
2	98
3	987
4	9876

Set

Topic: Determine whether or not the relationship is a function.

Determine if the relationship presented is a function or not and provide a justification.

9. The distance a person is from the ground related to time as they ride a Ferris Wheel.

10. The amount of daylight during a day throughout the calendar year.

11. The value of a Volkswagen Bug convertible from time of first purchase in 1978 to now.

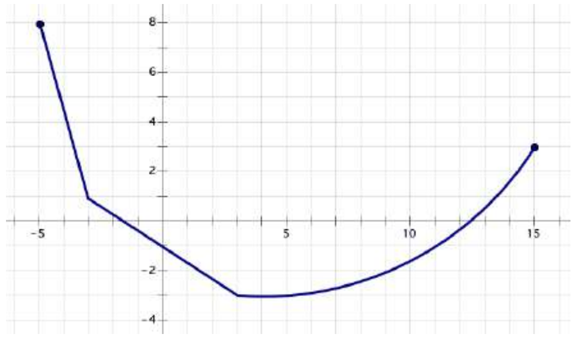
12. A person's name and their phone number.

13. The stadium in which a football player is playing related to the outcome of the game.

Go

Topic: Determining features of functions and finding solutions using functions.

14. For the graph given below provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.



15. For the given situation use the given function to find and interpret solutions.

Hope has been tracking the progress of her family as they travel across the country during their vacation and she has created a function, d(t)= 78t to model the progress they are making.

a. What would Hope be attempting to find if she writes "d(4)= 78(4)" ?

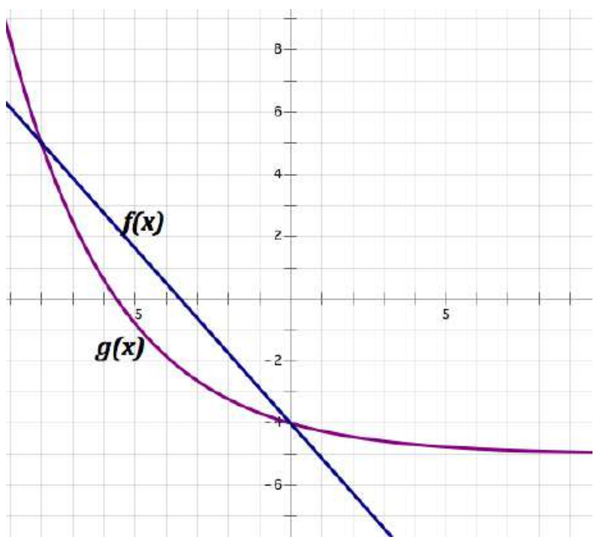
b. What would d(t)= 450 mean in this situation?

c. What would d(3.5) mean in this situation?

d. How could Hope use the function to find the time it would take to travel 800 miles?

Use the given representation of the functions to answer the questions.

16.



a. Where does $f(x) = g(x)$?

b. What is $g(0) + f(0)$?

c. On what interval(s) is $g(x) > f(x)$?

d. What is $g(-8) + f(-8)$?

5.9 Match That Function

A Practice Understanding Task

Welcome to Match That Function! To play, sort the deck of cards into sets by grouping cards together that describe a specific relationship. Each set is supposed to have four cards; however, one card is missing from every set. After you have sorted the cards into ten sets, create a fourth card for each set that would complete the set. Be sure to use a different representation than what is provided.

Ready, Set, Go!

Ready

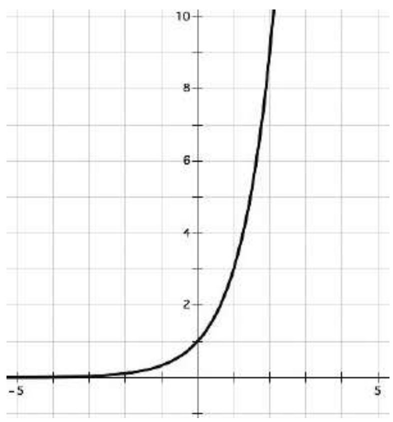
Topic: Find the output or input based on what is given.

For each function find the desired solutions.

1. $h(t) = 2t - 5$

a. $h(-4) =$ ____ b. $h(t) = 23, t =$ ____ c. $h(13) =$ ____ d. $h(t) = -33, t =$ _____

2. $g(x)$



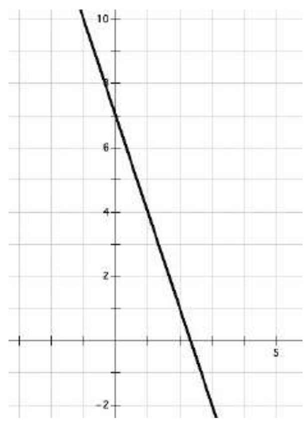
a. $g(2) =$ _____

b. $g(x) = 3,\ x =$ _____

c. $g(0) =$ ______

d. What is the explicit rule for $g(x)$

3. $r(x)$



a. $r(-1) =$ _____

b. $r(x) = 4, x =$ _____

c. $r(2) =$ _____

d. What is the explicit rule for $r(x)$

Set

Topic: Describing the key features of functions and creating a representation of a function given the key features.

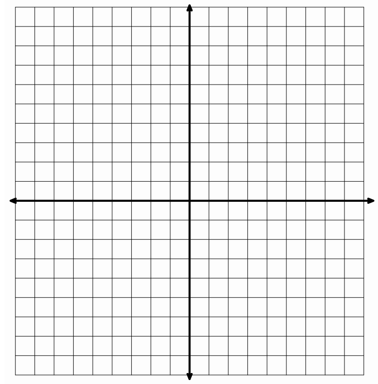
Use the given description of several of the key features of the function to sketch a possible graph of the function.

4. Domain contains all Real numbers between -2 and 3.

   Range contains all Real numbers between 3 and 7.

   The function is increasing from -2 to 0 and decreasing after 0.

   The function is not continuous at every point.

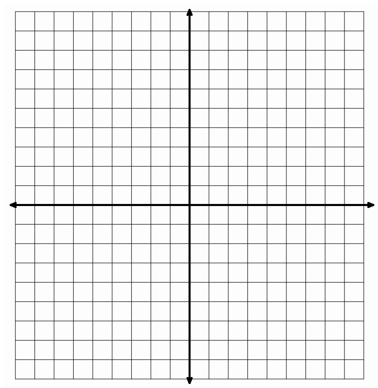


5. The function has a minimum at -5.

   The function has a maximum at 8.

   The function has two intervals on which it is decreasing and one interval on which it is increasing.

   The Domain of the functions contains all Real numbers from 1 to 9.

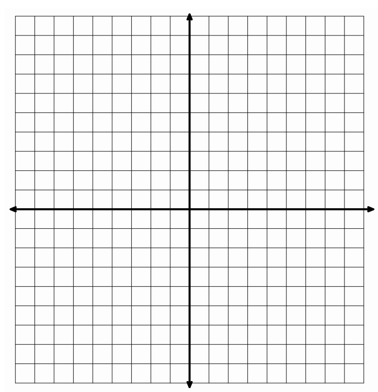


6. This function is not continuous anywhere.

   The function contains only seven elements in its domain.

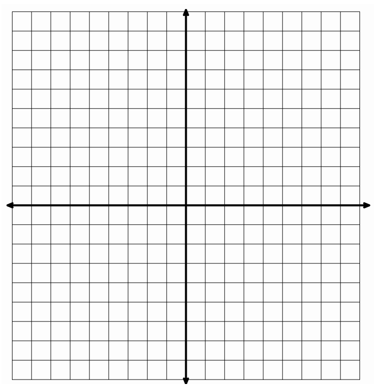
   The values of the domain are between -10 and 2.

   The values of the range are between -1 and 1.



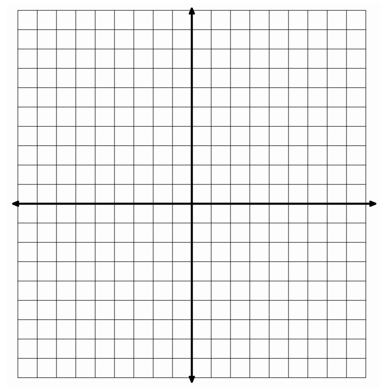
7. The function has three intervals on which its slope is zero.

The function has a maximum and a minimum.



8. The domain of the function is [-5, ¥)

The range of the function is [0, ¥)

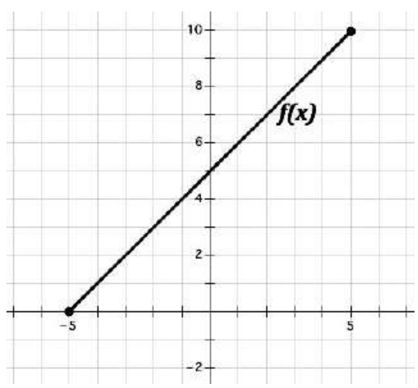


Go

Topic: Determine the following for each function: domain, range, discrete, continuous, increasing, decreasing, etc.

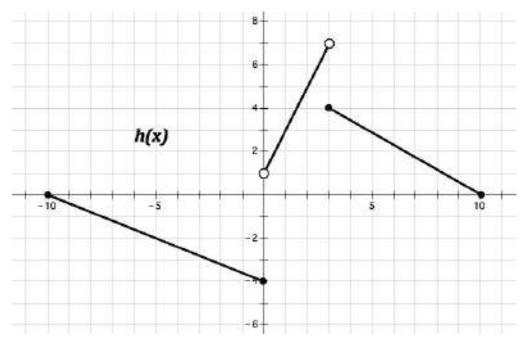
Given the representation of the function(s) provided determine the domain, range, and whether the function is discrete, continuous, increasing, decreasing, etc.

9.



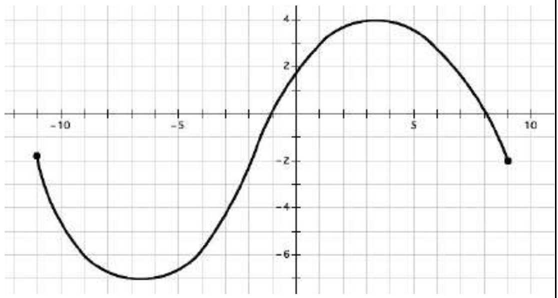
Description of Function:

10.



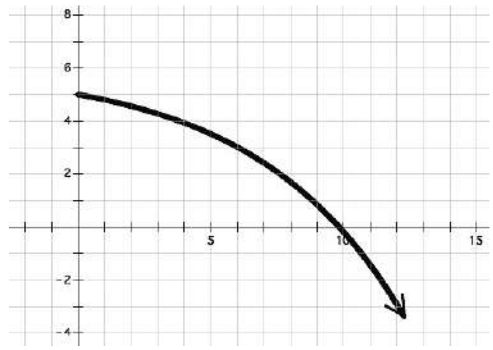
Description of Function:

11.



Description of Function:

12.



Description of Function:

13. $f(0) = 2, f(n+1) = 3(f(n))$ Description of Function:

14. $g(x) = -9 + 4x$ Description of Function:

15. $f(x) = |x|$ Description of Function:

Advanced Mathematics I
Module 6 Advanced Congruence, Constructions and Proof

By

The Mathematics Vision Project:
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Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

**In partnership with the
Utah State Office of Education**

Module 6 – Congruence, Construction and Proof

Classroom Task: Leaping Lizards! - A Develop Understanding Task
Developing the definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.1

Classroom Task: Is It Right? - A Solidify Understanding Task
Examining the slope of perpendicular lines (**G.CO.1, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.2

Classroom Task: Leap Frog– A Solidify Understanding Task
Determining which rigid-motion transformations carry one image onto another congruent image (**G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.3

Classroom Task: Leap Year – A Practice Understanding Task
Writing and applying formal definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.2, G.CO.4, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.4

Classroom Task: Symmetries of Quadrilaterals – A Develop Understanding Task
Finding rotational symmetry and lines of symmetry in special types of quadrilaterals (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.5

Classroom Task: Symmetries of Regular Polygons – A Solidify Understanding Task
Examining characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.6

Classroom Task: Quadrilaterals-Beyond Definition – A Practice Understanding Task
Making and justifying properties of quadrilaterals using symmetry transformations (**G.CO.3, G.CO.4, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.7

Classroom Task: Can You Get There From Here? – A Develop Understanding Task
Describing a sequence of transformations that will carry congruent images onto each other (**G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.8

Classroom Task: Congruent Triangles – A Solidify Understanding Task
Establishing the ASA, SAS and SSS criteria for congruent triangles (**G.CO.6, G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.9

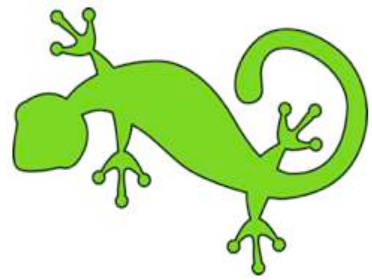
Classroom Task: Congruent Triangles to the Rescue – A Practice Understanding Task
Working with systems of linear equations, including inconsistent and dependent systems
(**G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.10

Classroom Task: Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct rhombuses and squares
(**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.11

Classroom Task: More Things Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct parallelograms, equilateral triangles and inscribed hexagons (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.12

Classroom Task: Justifying Constructions – A Solidify Understanding Task
Examining why compass and straightedge constructions produce the desired objects (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.13

Classroom Task: Construction Blueprints – A Practice Understanding Task
Writing procedures for compass and straightedge constructions (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.14



6.1 Leaping Lizards!

A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns) or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

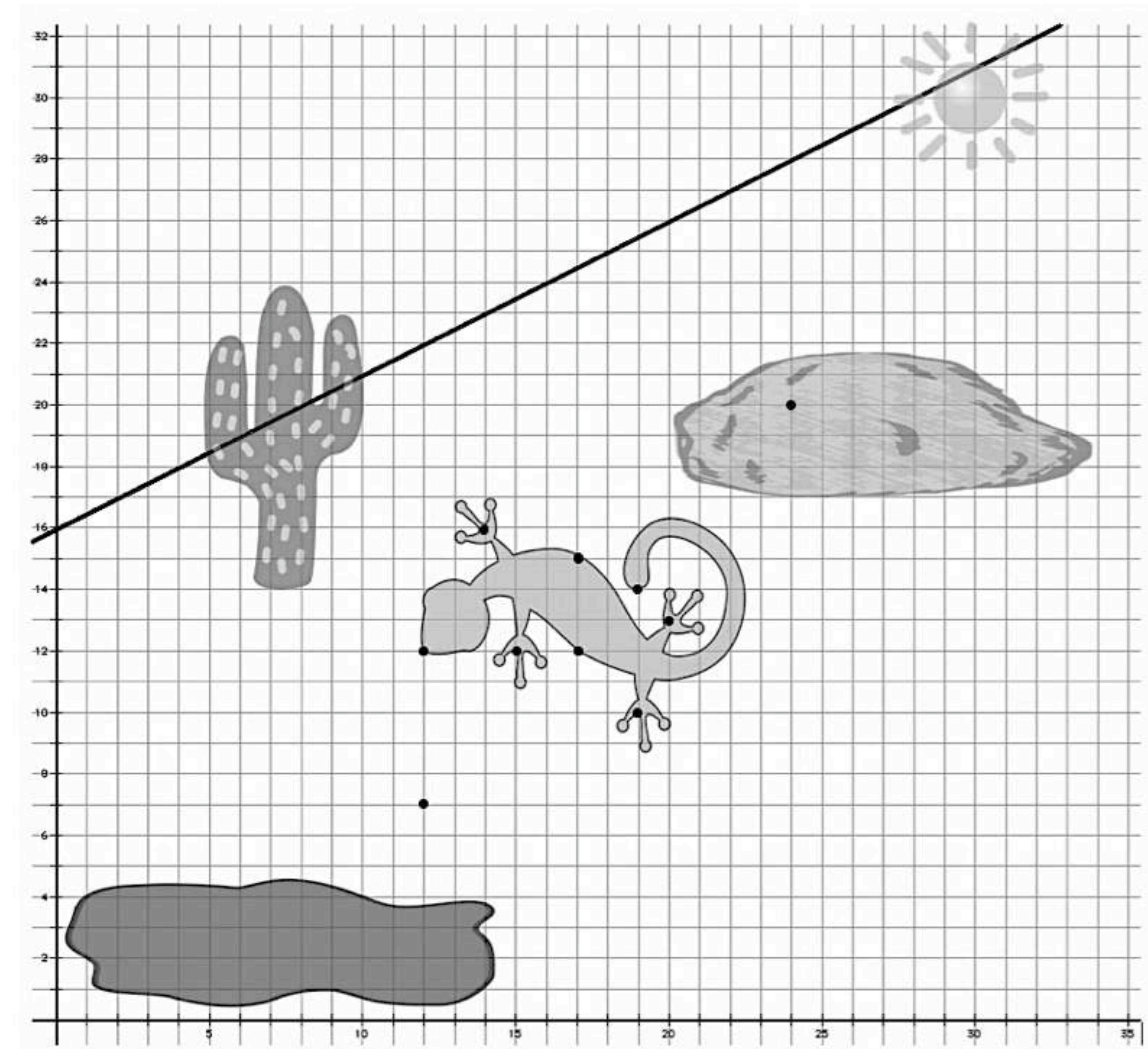
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.



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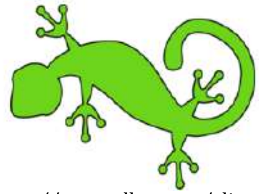
©2012 http://www.clker.com/clipart-brown-mud-puddle

Ready, Set, Go!

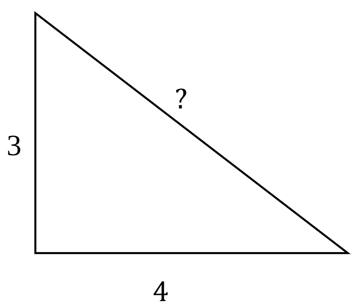


Ready

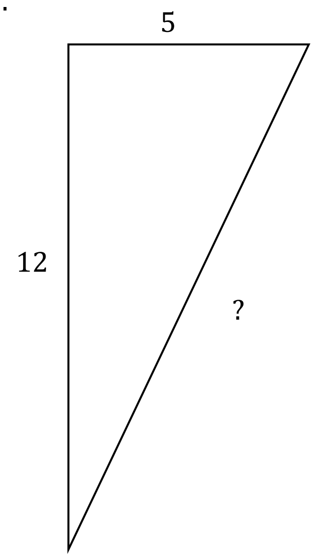
Topic: Pythagorean Theorem

For each of the following right triangles determine the number units measure for the missing side.

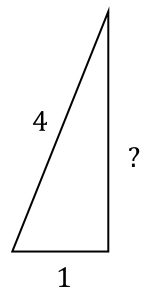
1.



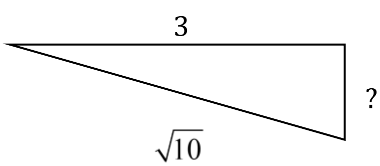
2.



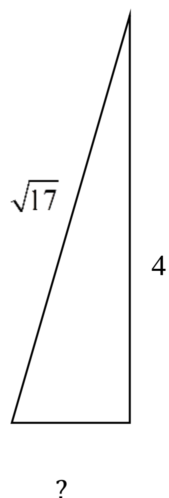
3.



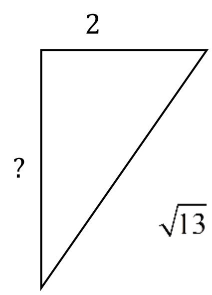
4.



5.



6.



Set

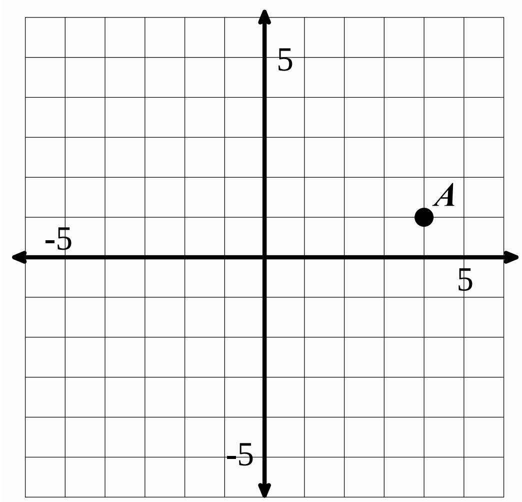
Topic: Transformations

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A'

b. Reflect point A over x-axis, label as A''

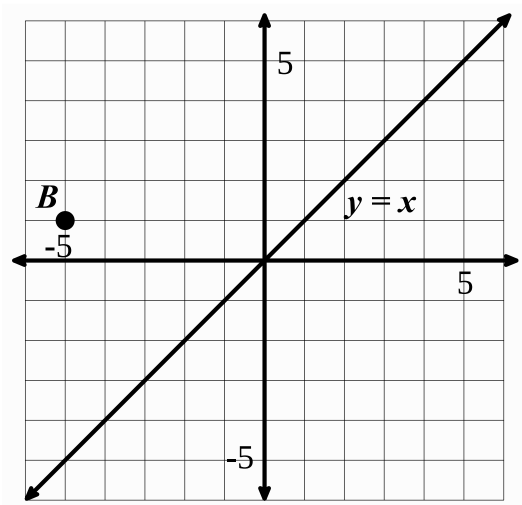
c. Apply the rule $(x-2\,, y-5)$, to point A and label A'''



8a. Reflect point B over the line $y = x$, label as B'

b. Rotate point B 180° about the origin, label as B''

c. Translate point B the point up 3 and right 7 units, label as B'''

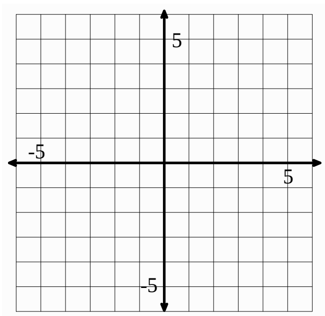


Go

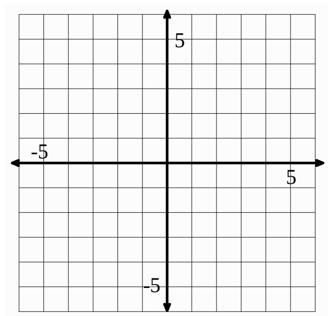
Topic: Graphing linear equations

Graph each equation on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $y = 2x - 3$

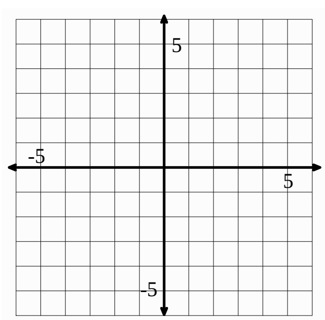


10. $y = -2x - 3$

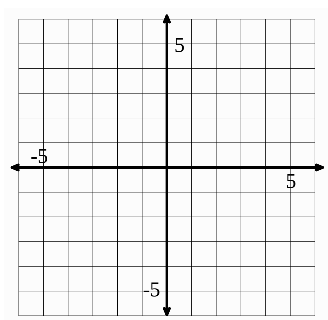


11. What similarities and differences are there between the equations in number 13 and 14?

12. $y = \frac{2}{3}x + 1$

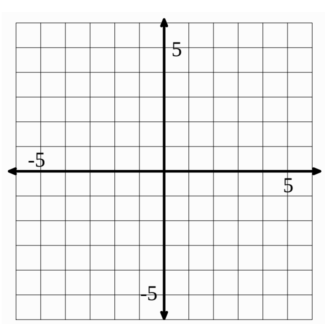


13. $y = -\frac{3}{2}x + 1$

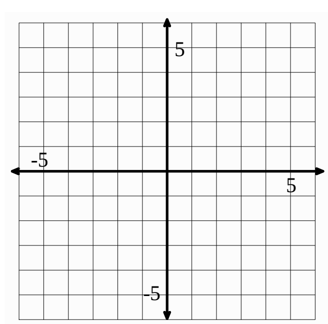


14. What similarities and differences are there between the equations in number 15 and 16?

15. $y = x + 1$



16. $y = x - 3$



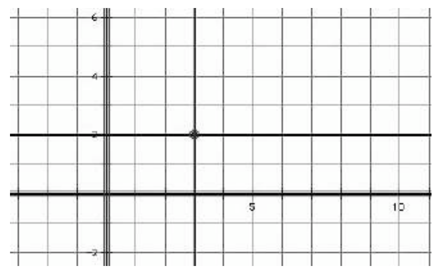
17. What similarities and differences are there between the equations in number 15 and 16?

6.2 Is It Right?
A Solidify Understanding Task

In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a 90° angle or reflecting the lizard across a line.

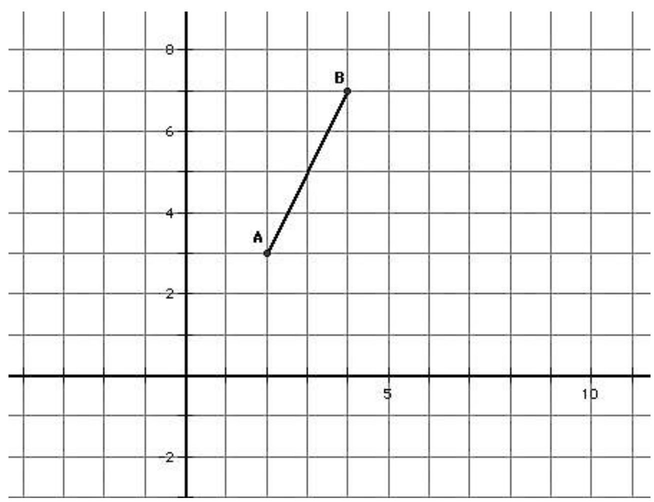
In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical?
How do we determine the slope of a line or line segment that will be perpendicular to it?

<u>Experiment 1</u>



1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

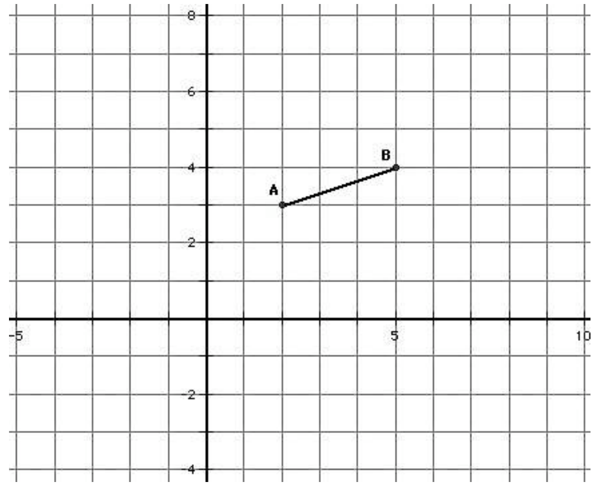
3. Explain how you know that the triangle you formed contains a right angle?

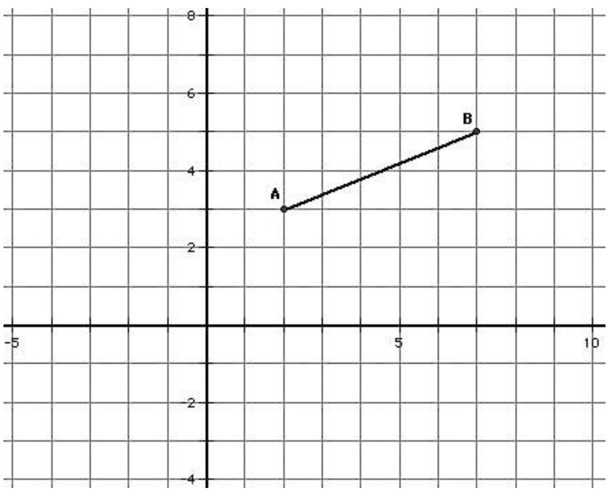
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).



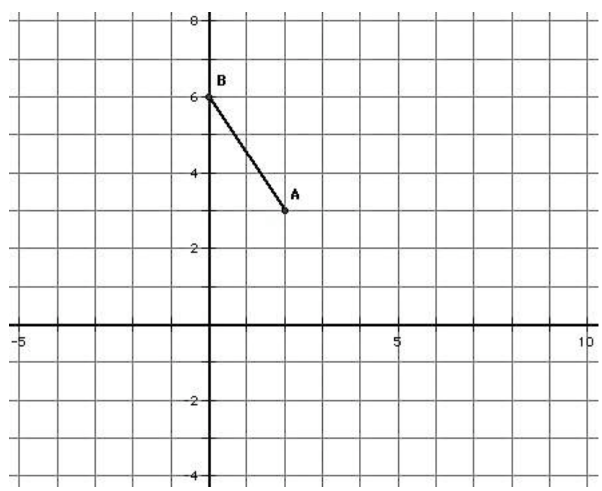


Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).

Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).



Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in the next module.)

Ready, Set, Go!

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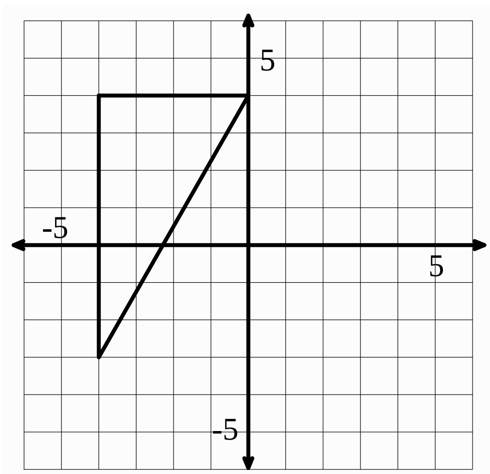
Ready

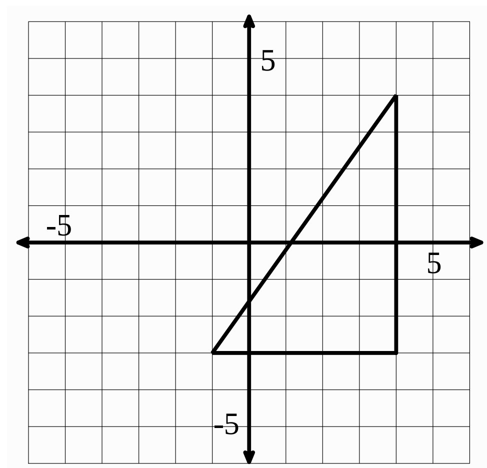
Topic: Finding Distance using Pythagorean Theorem

Use the coordinate grid to find the length of each side of the triangles provided.

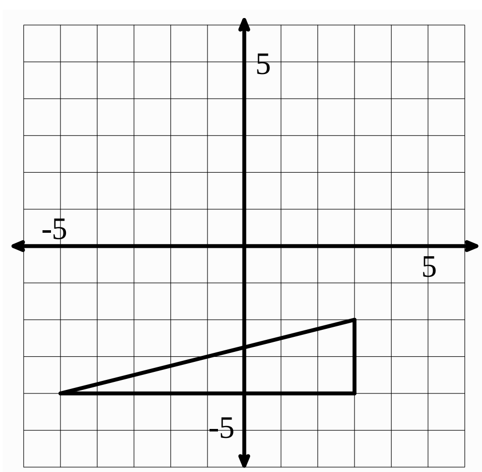
1.



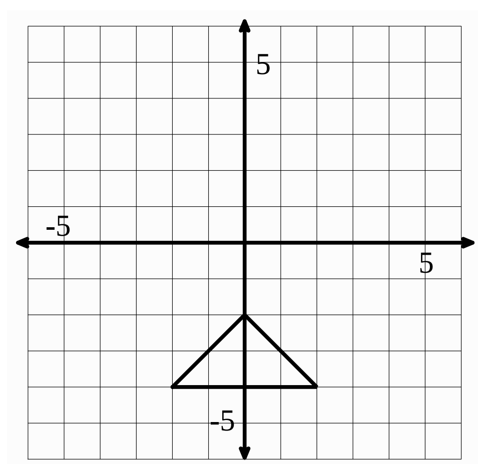
2.



3.



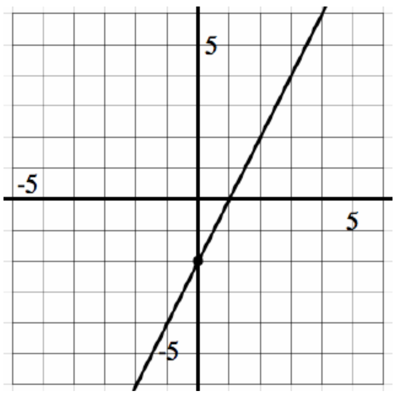
4.



Set

Topic: Slopes of parallel and perpendicular lines.

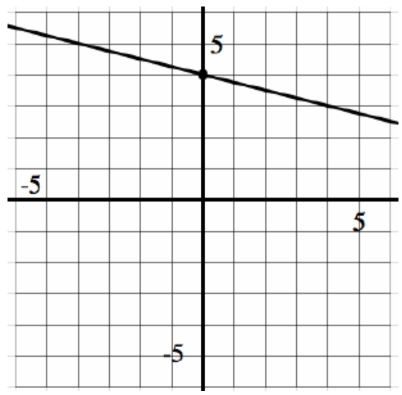
5. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

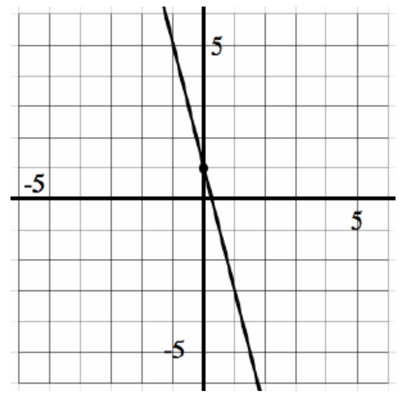
6. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

7. Graph a line *parallel* to the given line.

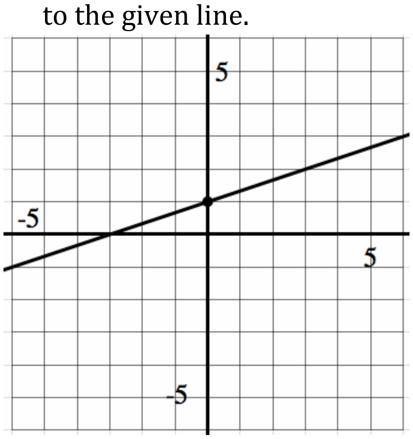


Equation for given line:

Equation for new line:

---

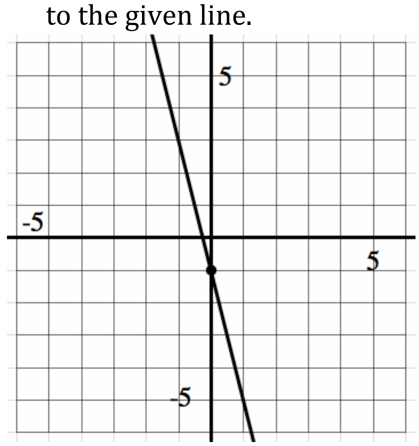
8. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

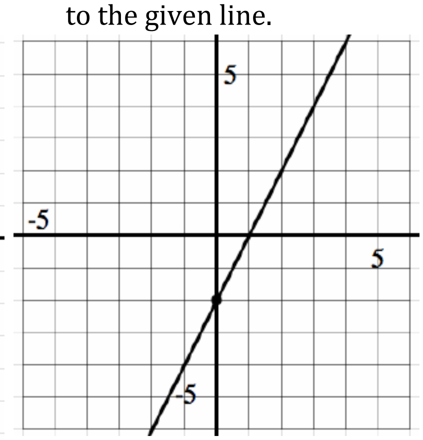
9. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

10. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

Go

Topic: Solve the following equations.

Solve each equation for the indicated variable.

11. $3(x-2)=5x+8$; Solve for *x*.

12. $-3+n=6n+22$; Solve for *n*.

13. $y-5=m(x-2)$; Solve for *x*.

14. $Ax+By=C$; Solve for *y*.

6.3 Leap Frog

A Solidify Understanding Task

Josh is animating a scene where a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.
- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.
- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.
- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	

images this page:
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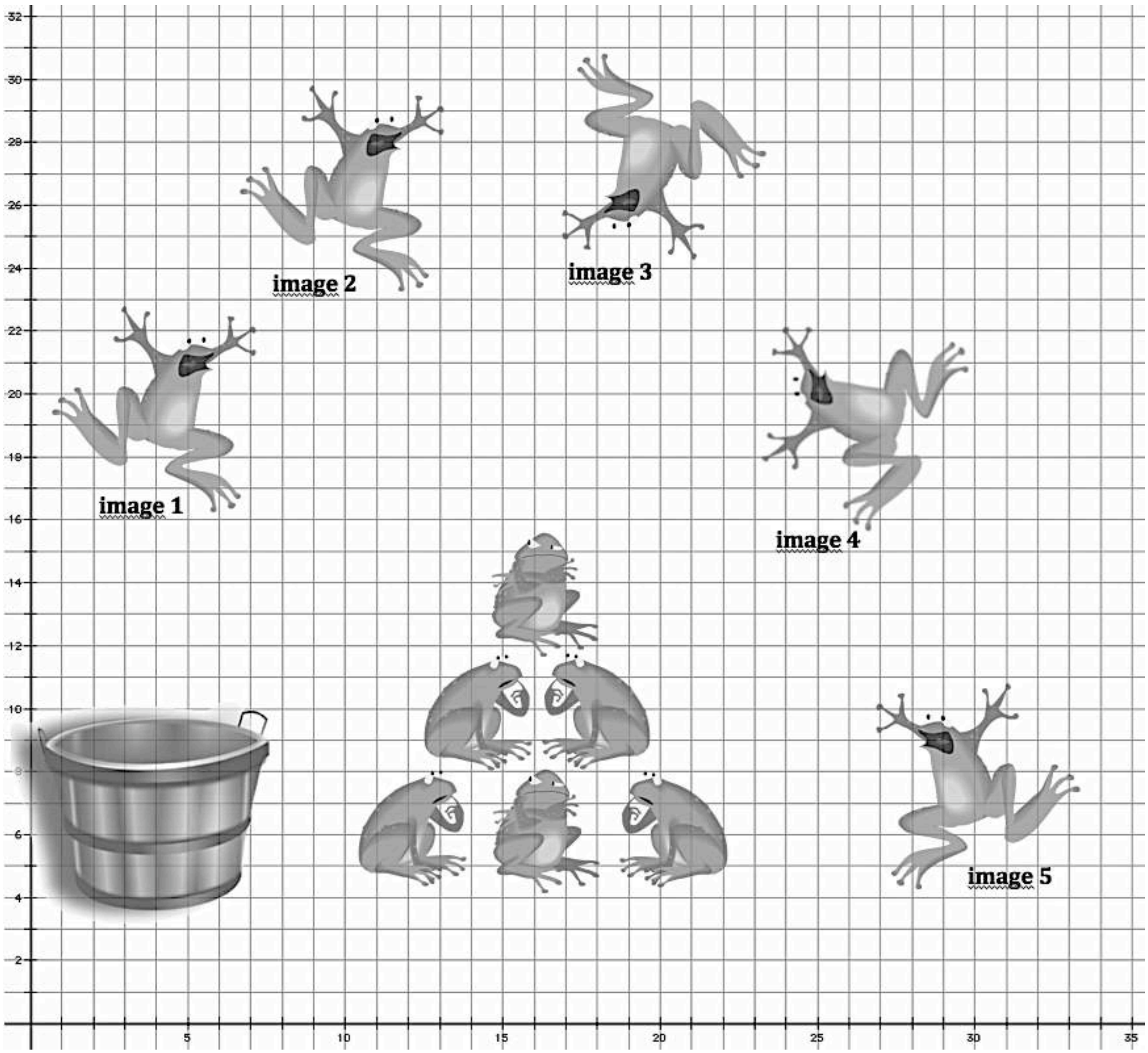
©2012 http://openclipart.org/detail/33979/architetto
©2012 http://openclipart.org/detail/33985/architetto
©2012 http://openclipart.org/detail/170806/hatalar205



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Ready

Topic: Basic Rotations and Reflections of objects

In each problem there will be a preimage and several images based on the give preimage. Determine which of the images are rotations of the given preimage and which of them are reflections of the preimage. If an image appears to be created as the result of a rotation and a reflection then state both.

1.

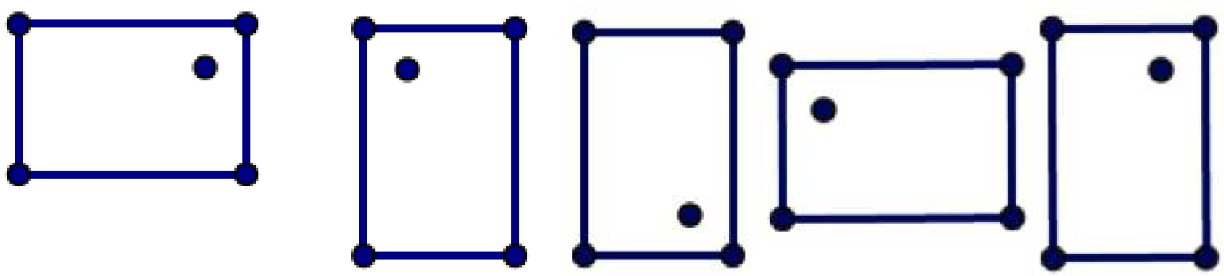
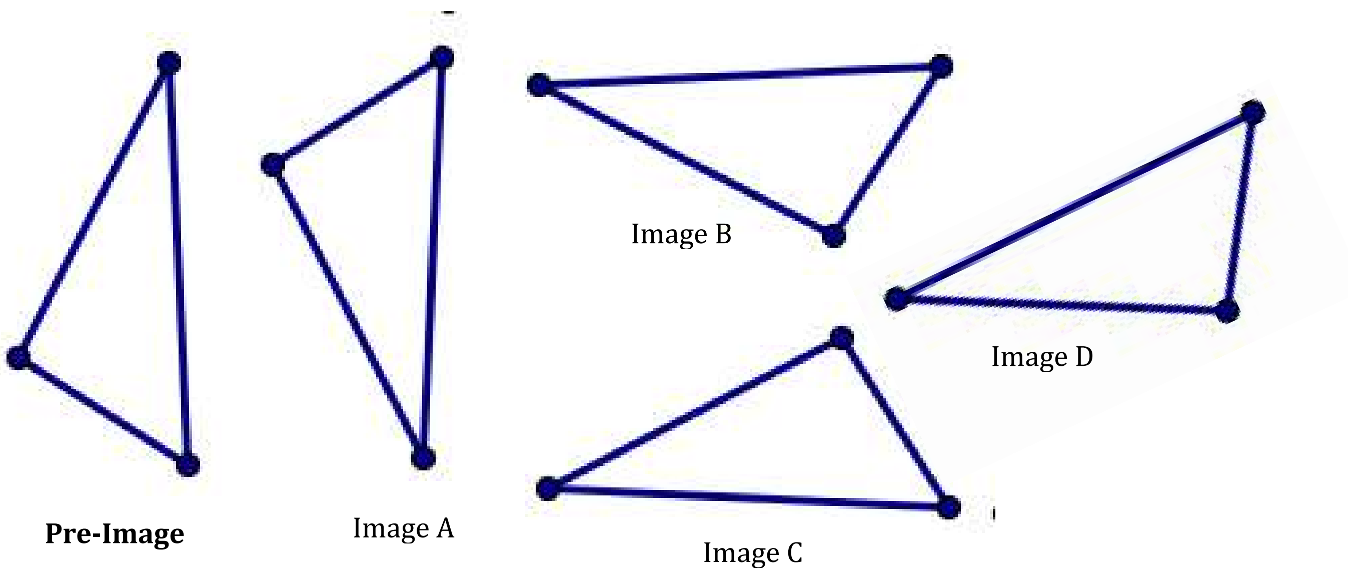


Image A Image B Image C Image D

Pre-Image

2.

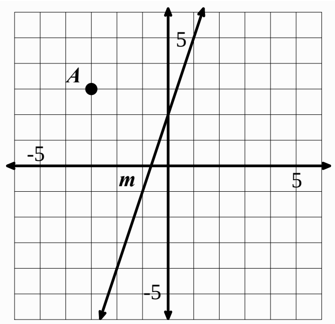


Set

Topic: Reflecting and Rotating points

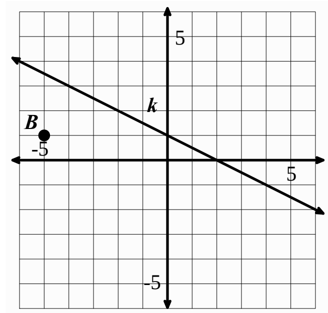
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection.

3.



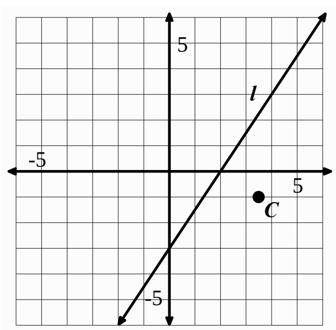
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



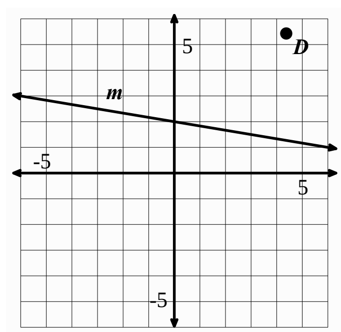
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

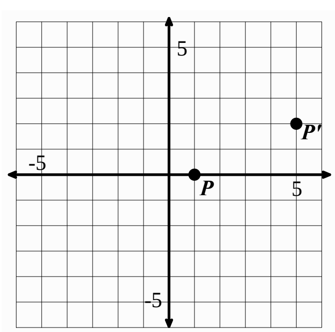
6.



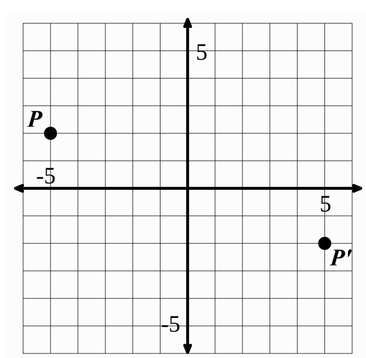
Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.

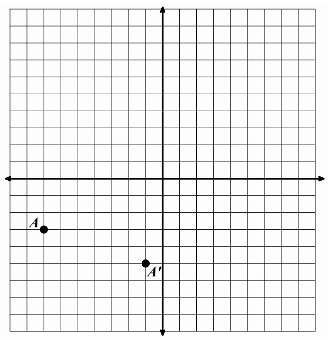


8.

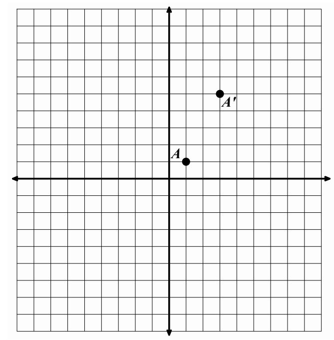


For each pair of point, *A* and *A'* draw in the line of reflection that would need to be used to reflect *A* onto *A'*. Then find the equation of the line of reflection. Also, draw a line connecting *A* to *A'* and find the equation of this line. Compare the slopes of the lines of reflection containing *A* and *A'*.

9.



10.



Go

Topic: Slopes of parallel and perpendicular lines and finding both distance and slope between two points.

For each linear equation write the slope of a line parallel to the given line.

11. $y = -3x + 5$

12. $y = 7x - 3$

13. $3x - 2y = 8$

For each linear equation write the slope of a line perpendicular to the given line.

14. $y = -\frac{2}{7}x + 5$

15. $y = \frac{1}{5}x - 4$

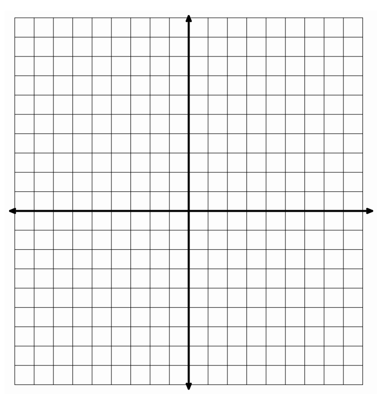
16. $3x + 5y = -15$

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)
a. Slope: b. Distance:

18. (-7, 5) (-2, -7)
a. Slope: b. Distance:

19. (2, -4) (3, 0)
a. Slope: b. Distance:

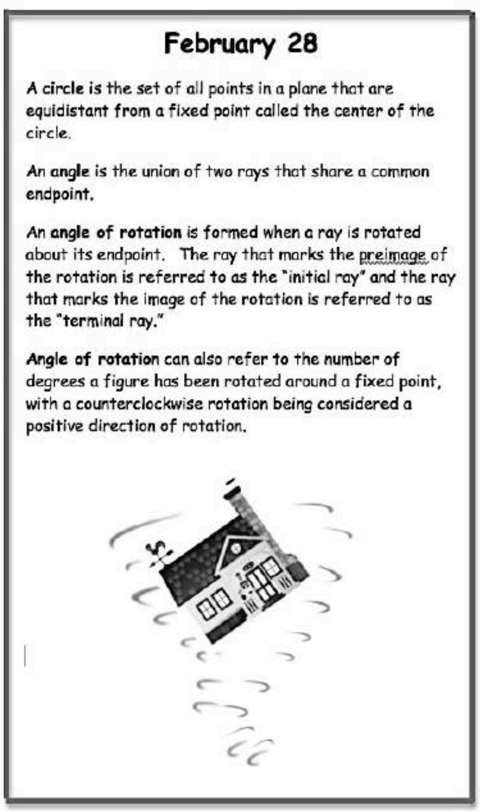


6.4 Leap Year
A Practice Understanding Task

Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages."

So now Carlos and Clarita have the dilemma of having to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.



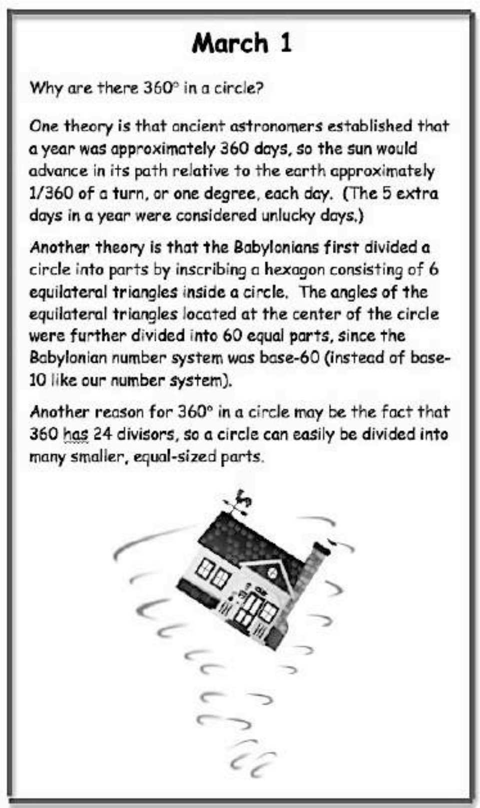
February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures.

Revise your definitions so they also use these words or phrases.

Part 2

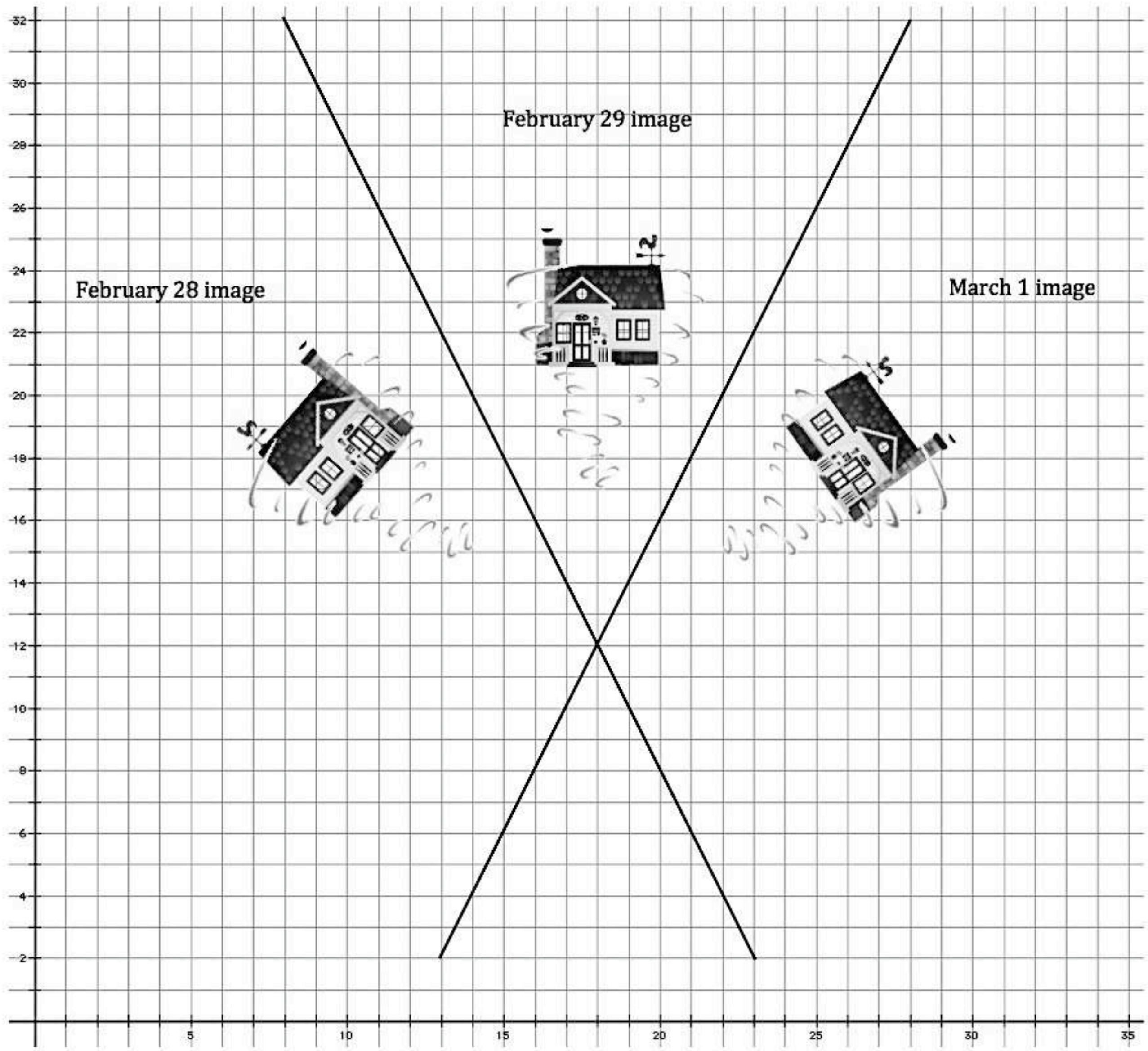
In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



Images this page:
http://openclipart.org/detail/168722/simple-farm-pack-by-viscious-speed
www.clker.com/clipart.tornado-gray

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Ready

Topic: Defining geometric shapes and components

For each of the geometric words below write a definition of the object that addresses the essential elements. Also, list necessary attributes and characteristics.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

5. Rhombus:

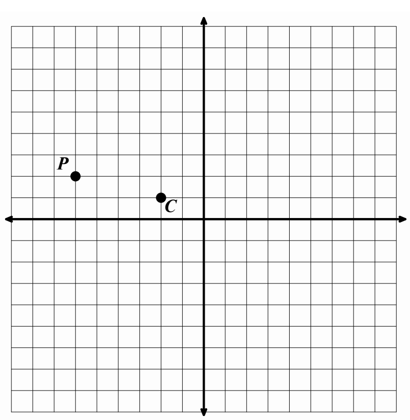
6. Trapezoid:

Set

Topic: Reflections and Rotations, composing reflections to create a rotation

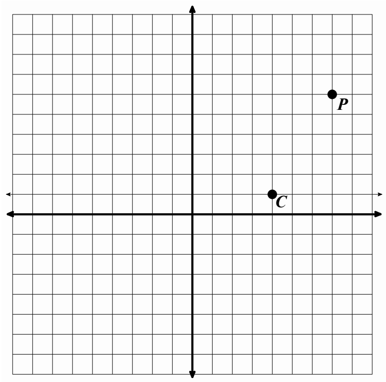
Perform the indicated rotations.

7.



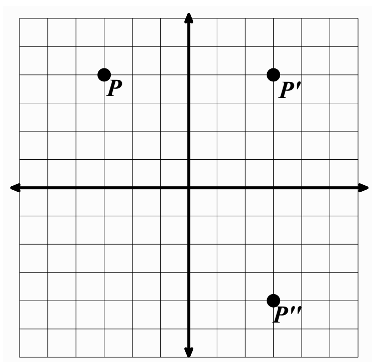
Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P***'.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P***''.

8. 

Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P'***.

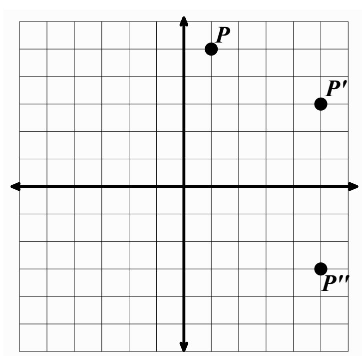
With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P''***.

9. 

a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

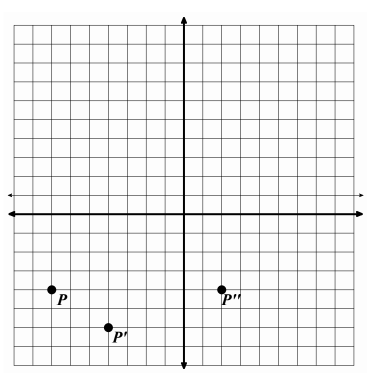
c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10. 

a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11. 

a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

Go

Topic: Rotations about the origin

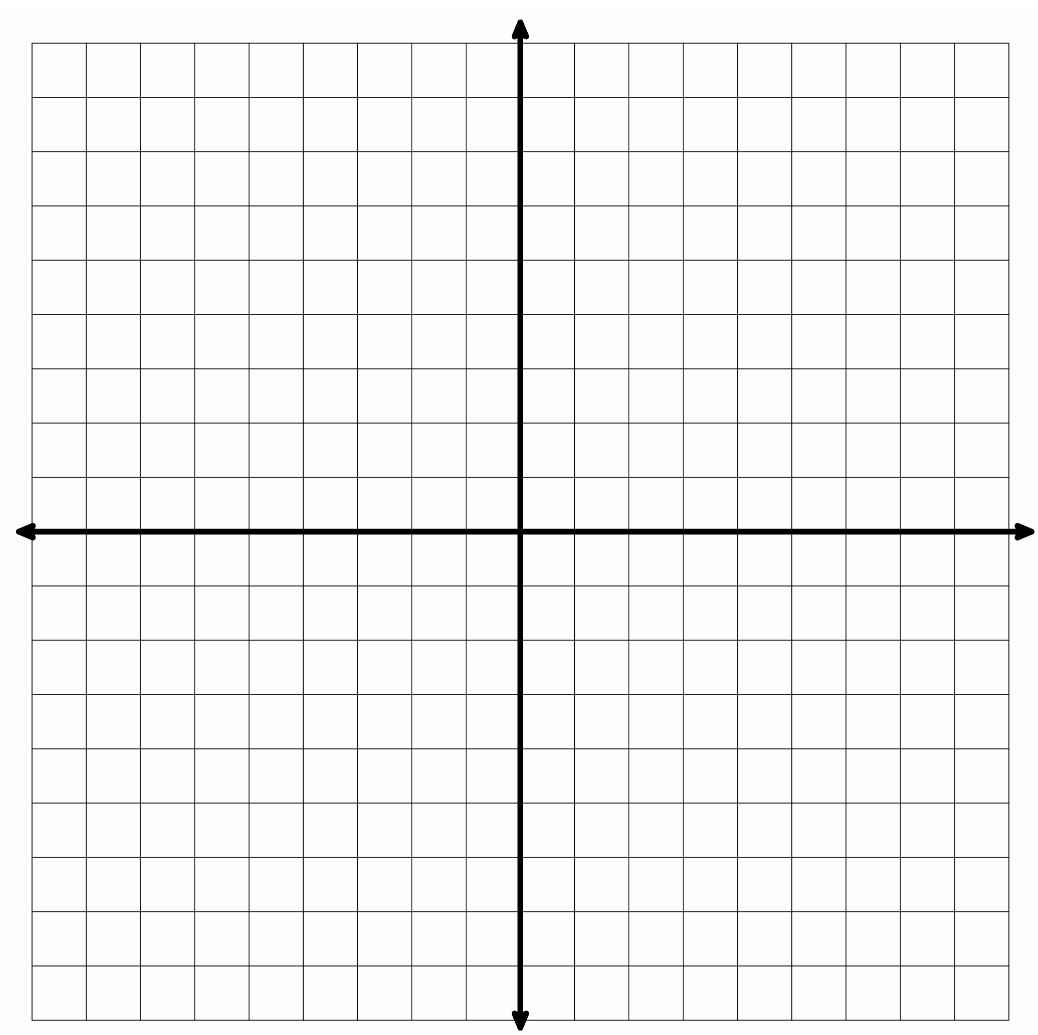
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
    Coordinates for Point ***A'*** (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
    Coordinates for Point ***B'*** (___ , ___)

14. Point ***C*** (-7, 3) rotate 180^0
    Coordinates for Point ***C'*** (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
    Coordinates for Point ***D'*** (___ , ___)



6.5 Symmetries of Quadrilaterals

A Develop Understanding Task

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

1. A **rectangle** is a quadrilateral that contains four right angles. Is it possible to reflect or rotate a rectangle onto itself?

For the rectangle shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rectangle onto itself.

Describe the rotations and/or reflections that carry a rectangle onto itself. (Be as specific as possible in your descriptions.)

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel. Is it possible to reflect or rotate a parallelogram onto itself?

For the parallelogram shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the parallelogram onto itself.

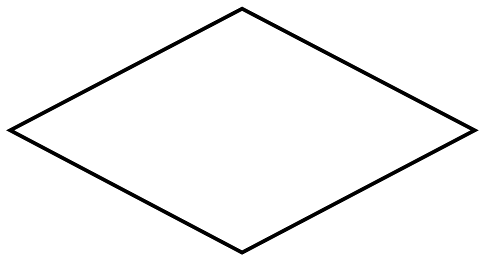
Describe the rotations and/or reflections that carry a parallelogram onto itself. (Be as specific as possible in your descriptions.)

3. A **rhombus** is a quadrilateral in which all sides are congruent. Is it possible to reflect or rotate a rhombus onto itself?

For the rhombus shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rhombus onto itself.



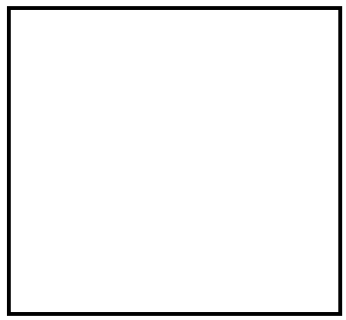
Describe the rotations and/or reflections that carry a rhombus onto itself. (Be as specific as possible in your descriptions.)

4. A **square** is both a rectangle and a rhombus. Is it possible to reflect or rotate a square onto itself?

For the square shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the square onto itself.



Describe the rotations and/or reflections that carry a square onto itself. (Be as specific as possible in your descriptions.)

5. A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find

- any lines of symmetry, or
- any centers of rotational symmetry

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

Ready, Set, Go!

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Ready

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

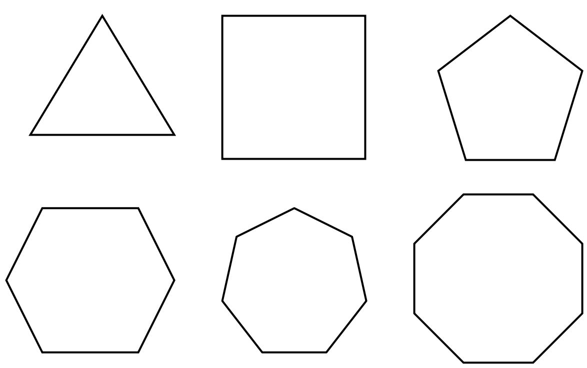
2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

Set

Topic: Lines of symmetry and diagonals

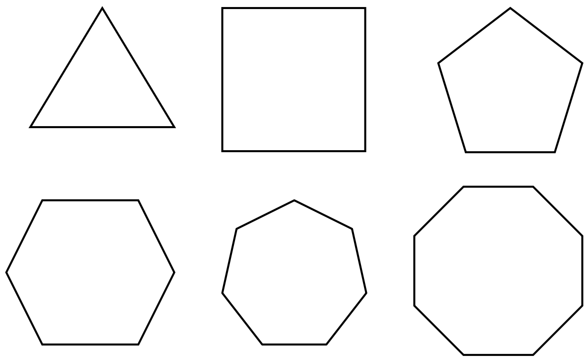
3. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.



Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	

4. Find

all of the diagonals in each regular polygon. Fill in the table including an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

5. Are all lines of symmetry also diagonals? Explain.

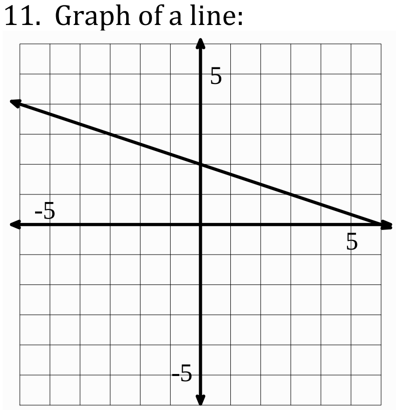
6. Are all diagonals also lines of symmetry? Explain.

7. What shapes will have diagonals that are not lines of symmetry? Name some and draw them.

8. Will all parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

Go

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated point.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated point.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1$.</td><td>a. Parallel line through point (-1 , -7):</td><td>b. Perpendicular to the line line through point (-1 , -7):</td></tr>
<tr><td>10. Table of a line:
<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>-8</td></tr><tr><td>4</td><td>-10</td></tr><tr><td>5</td><td>-12</td></tr><tr><td>6</td><td>-14</td></tr></table></td><td>a. Parallel line through point (3 , 8):</td><td>b. Perpendicular to the line through point (3 , 8):</td></tr>
<tr><td>11. Graph of a line:

</td><td>a. Parallel line through point (2 , -9):</td><td>b. Perpendicular to the line through point (2 , -9):</td></tr>
</table>

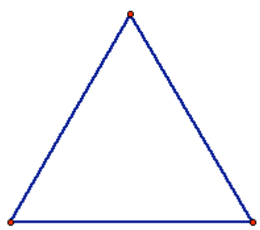
6.6 Symmetries of Regular Polygons

A Solidify Understanding Task

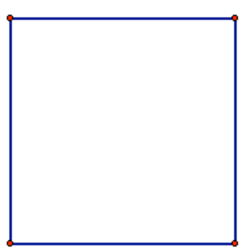
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

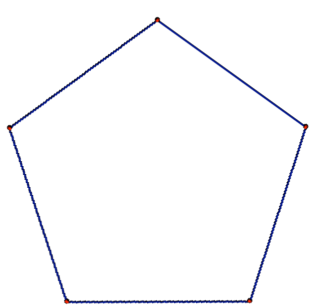
1. An equilateral triangle



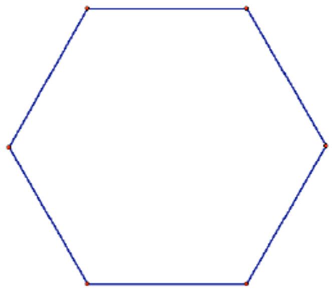
2. A square



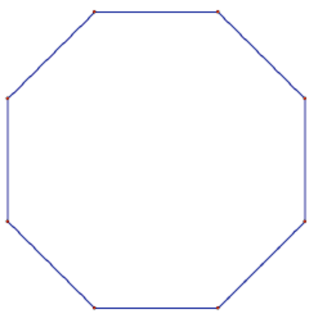
3. A regular pentagon



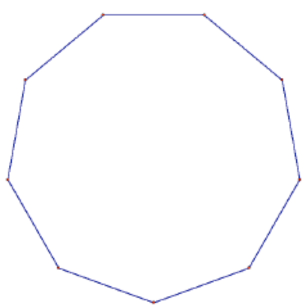
4. A regular hexagon



5. A regular octagon



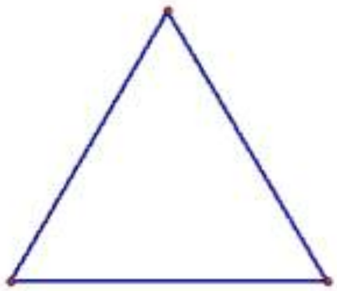
6. A regular nonagon



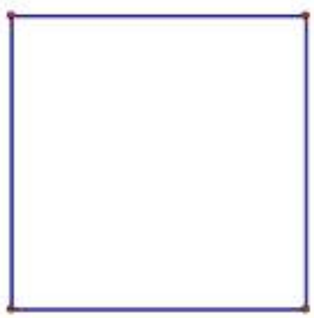
What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

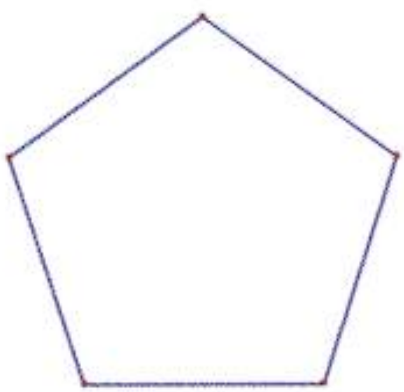
1. An equilateral triangle



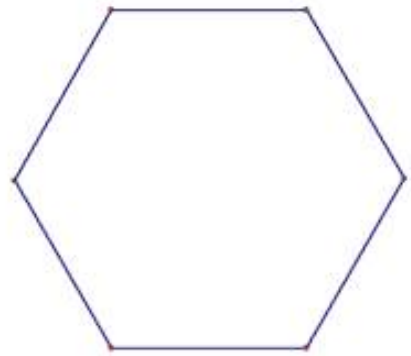
2. A square



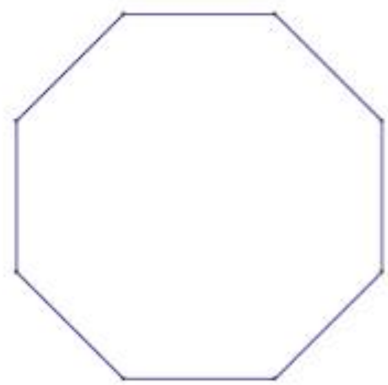
3. A regular pentagon



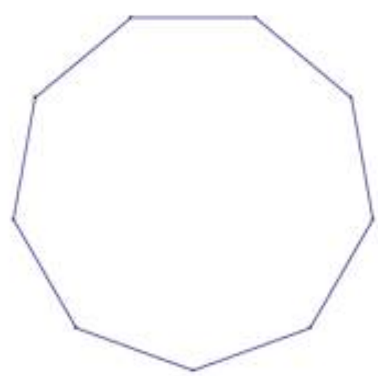
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



Ready, Set, Go!

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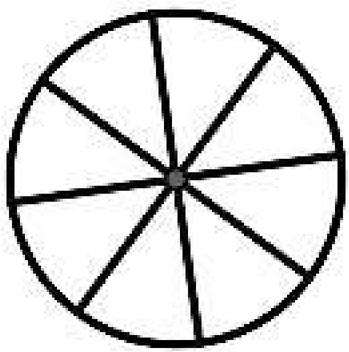
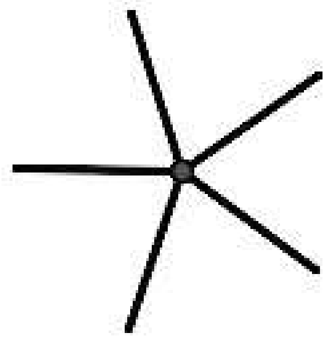
Ready

Topic: Rotation as a transformation, what does it mean?

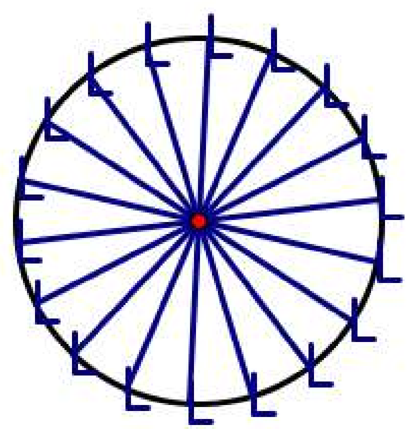
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



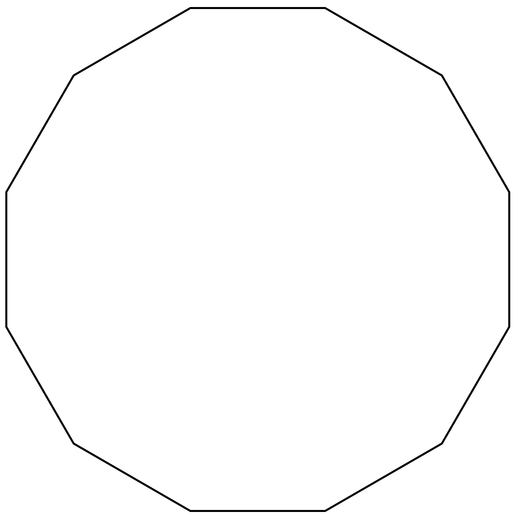
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



Set

Topic: Finding angels of rotation for regular polygons.

4. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



5. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

6. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

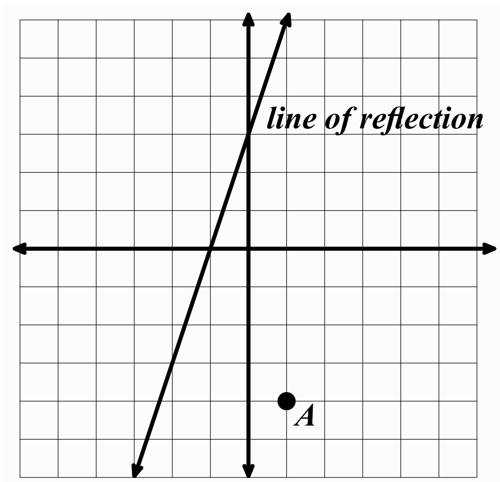
7. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

8. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

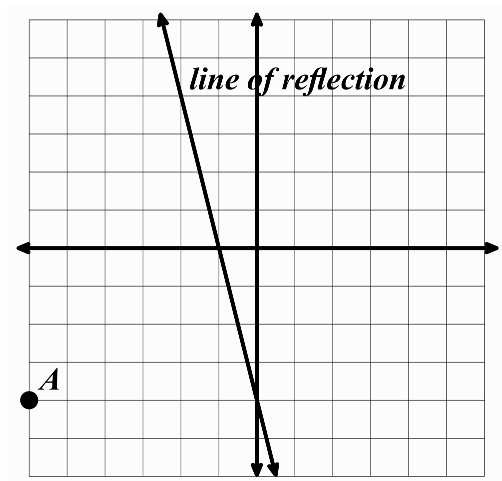
Go

Topic: Reflecting and Rotating points on the coordinate plane.

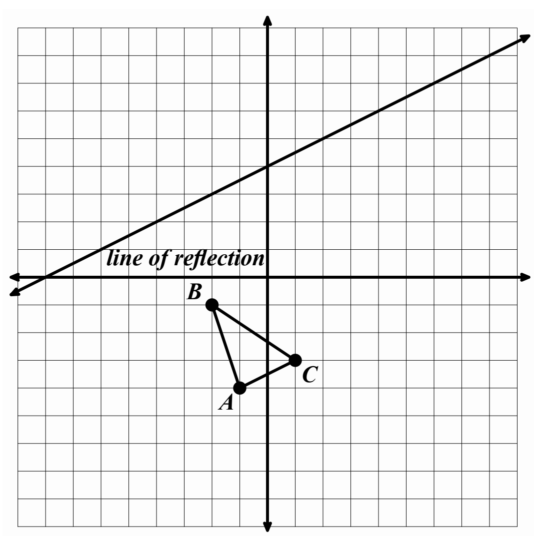
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



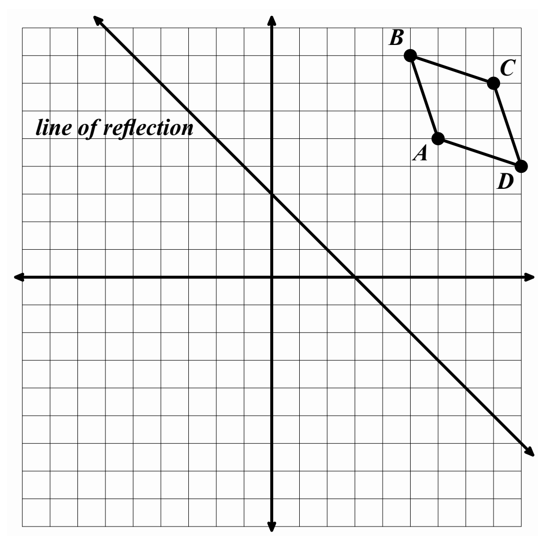
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



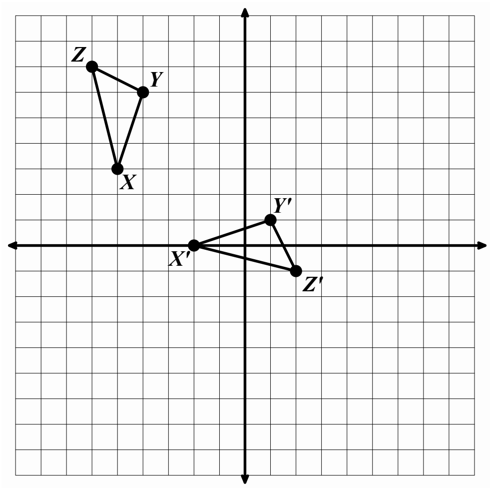
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



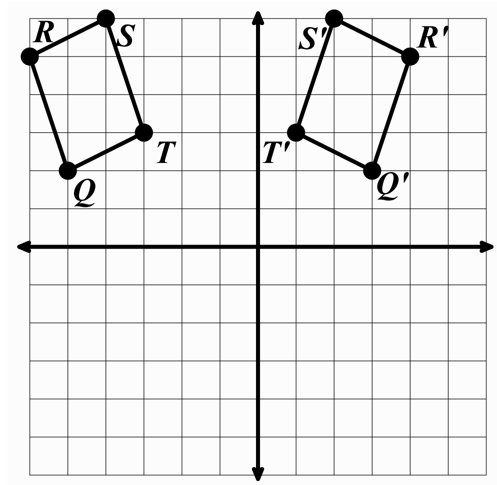
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



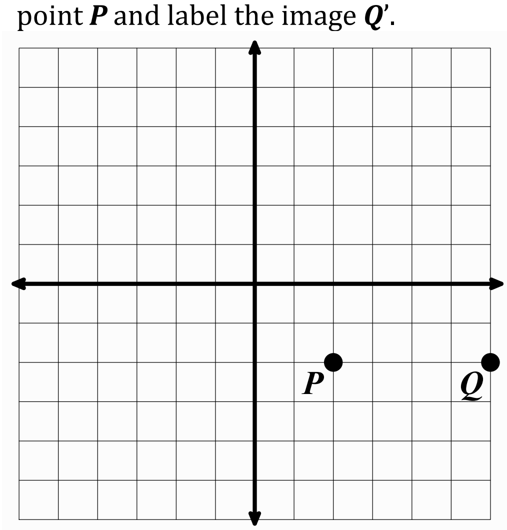
13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



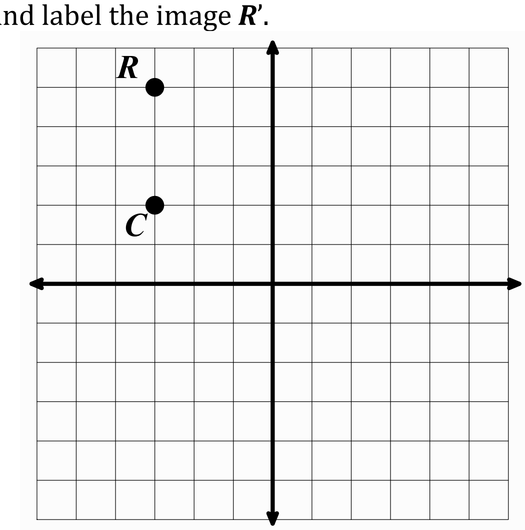
14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



15. Using point ***P*** as a center of rotation. Rotate point ***Q*** 120^0 clockwise about point ***P*** and label the image ***Q'***.



16. Using point ***C*** as the center or rotation. Rotate point ***R*** 270^0 counter-clockwise about point ***C*** and label the image ***R'***.



6.7 Quadrilaterals—Beyond Definition

A Practice Understanding Task

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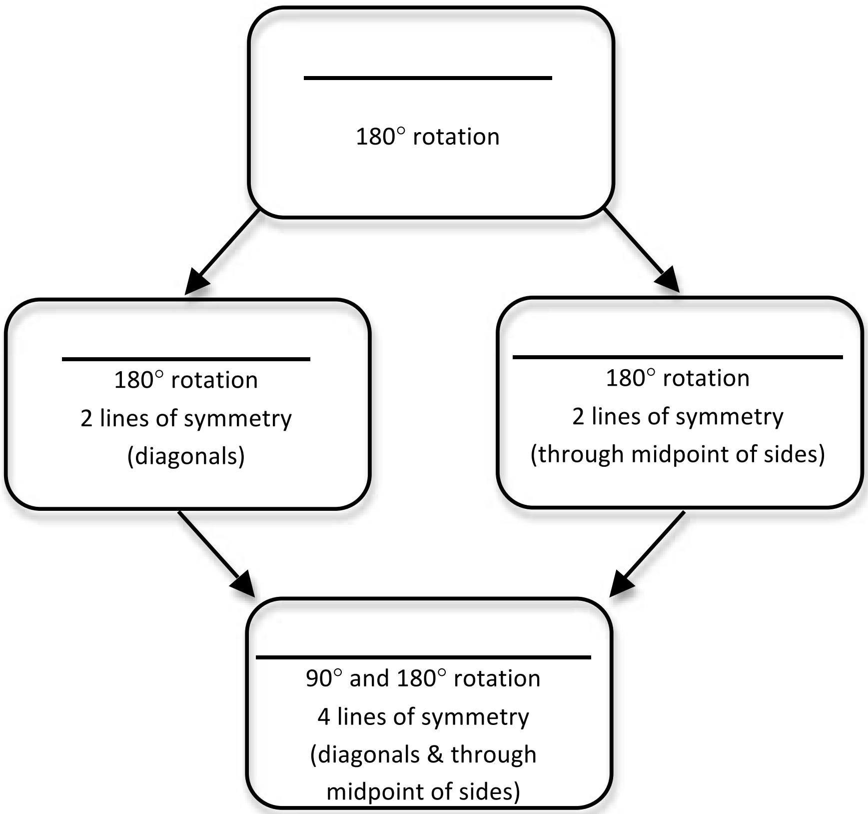
We have found that many different quadrilaterals possess line and/or rotational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

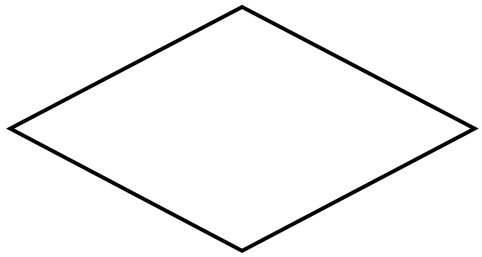
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that all four angles are right angles? Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

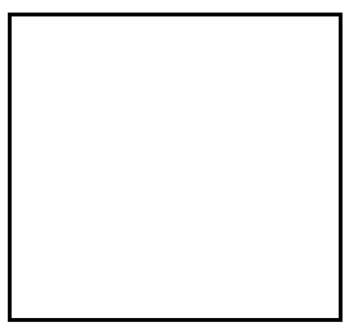
Based on what you know about transformations, what else can we say about parallelograms besides the defining property that opposite sides of a parallelogram are parallel? Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.



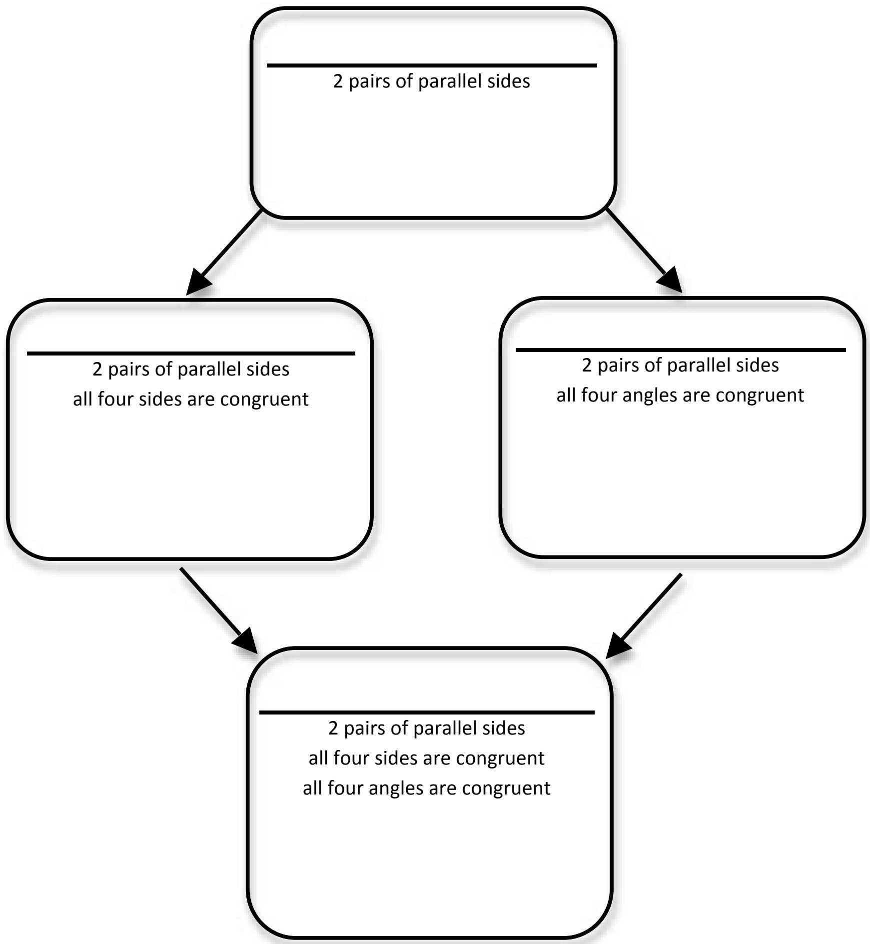
Based on what you know about transformations, what else can we say about a rhombus besides the defining property that all sides are congruent? Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals.

4. A **square** is both a rectangle and a rhombus.



Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and highlighted in the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

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Ready

Topic: Defining Congruence and Similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

Set

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

Go

Topic: Slope and distance

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

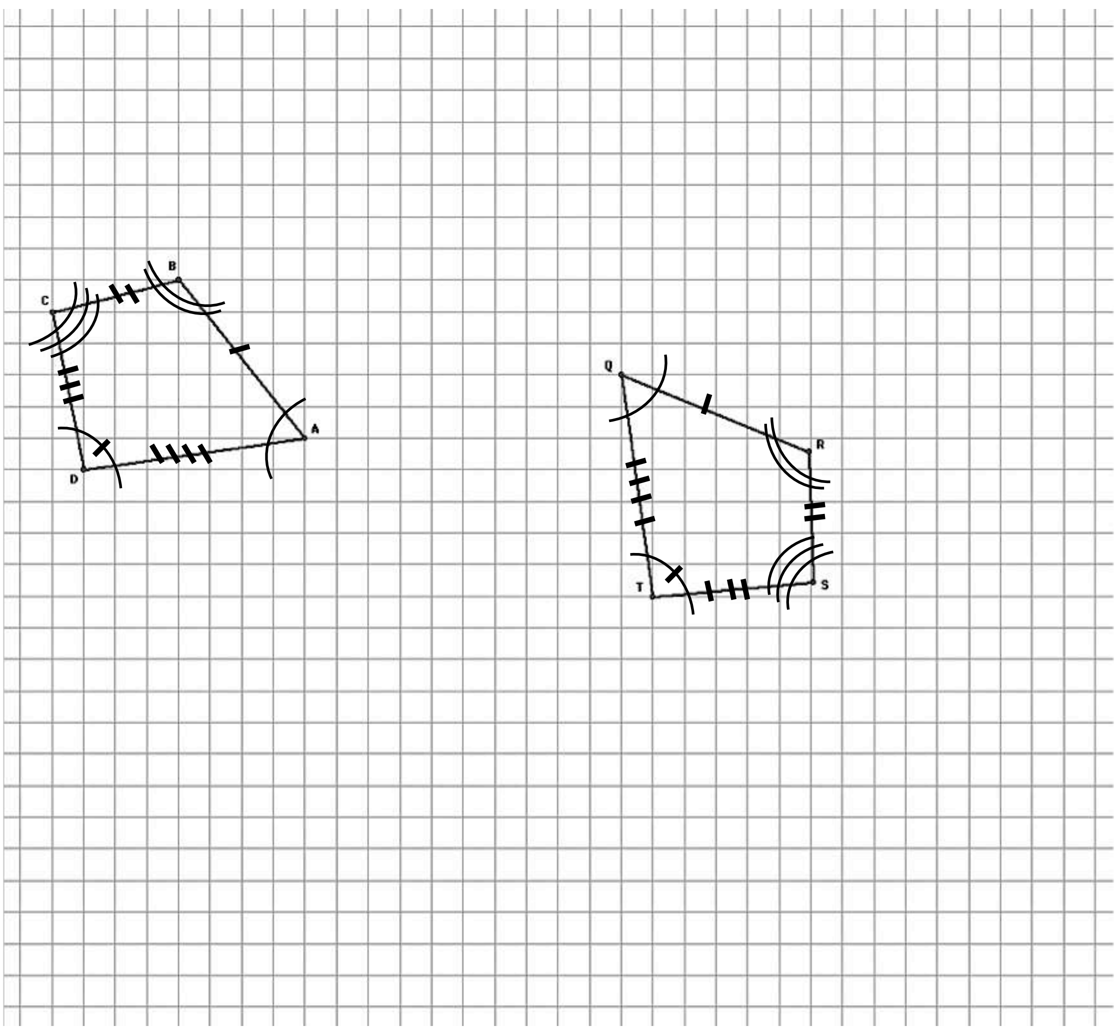
a. Slope: b. Distance:

6. 8 Can You Get There From Here?

A Develop Understanding Task

The two quadrilaterals shown below, quadrilateral *ABCD* and quadrilateral *QRST* are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



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Ready, Set, Go!

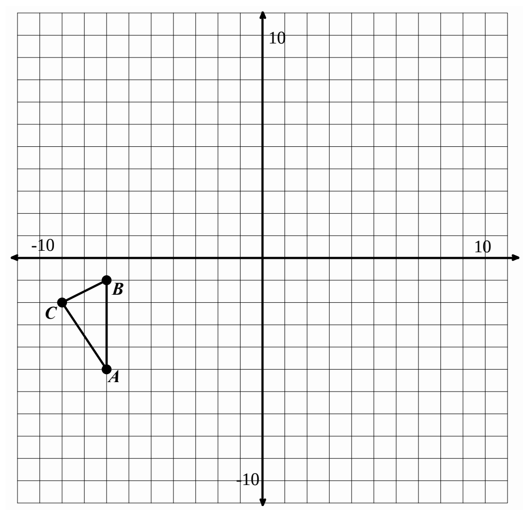
Ready

Topic: Performing a sequence of transformations.

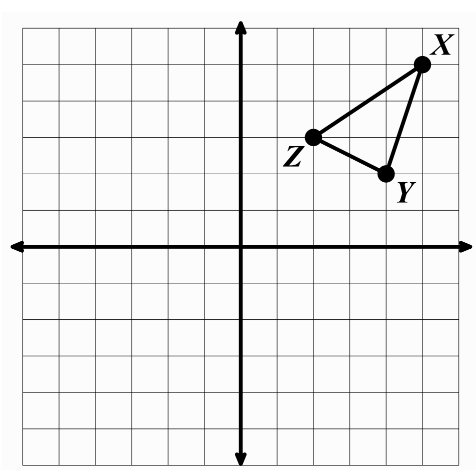
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle ***ABC*** over the line $y = x$ and label the image ***A'B'C'***.

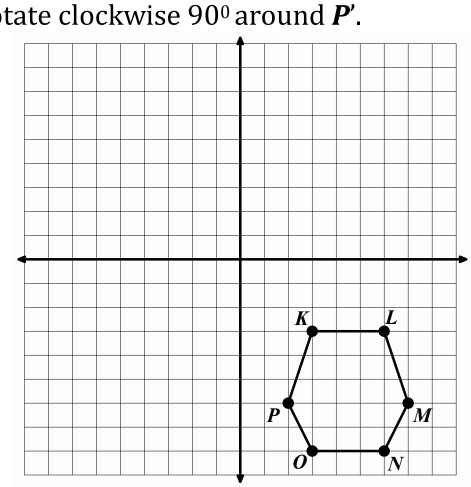
   Rotate triangle ***A'B'C'*** 180^0 counter clockwise around the origin and label the image ***A''B''C''***.



2. Reflect over the line $y = -x$.

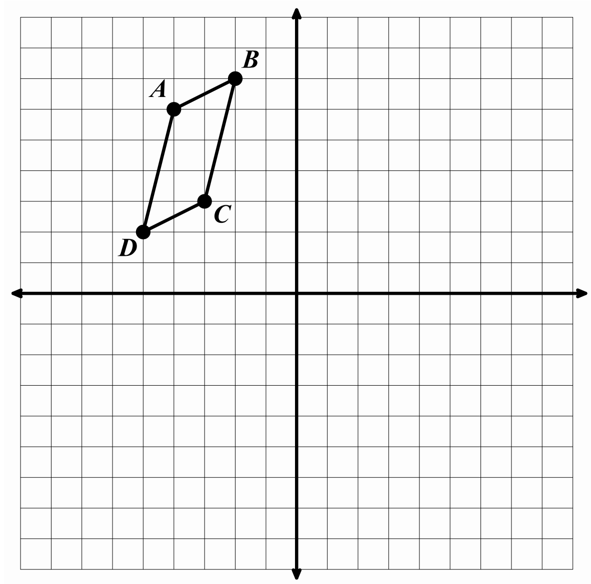


3. Reflect over y-axis and then Rotate clockwise 90^0 around ***P'***.



4. Reflect quadrilateral ABCD over the line $y = 2 + x$ and label the image A'B'C'D'.

Rotate quadrilateral A'B'C'D' counter-clockwise 90^0 around (-2, -3) as the center of rotation label the image A''B''C''C''.

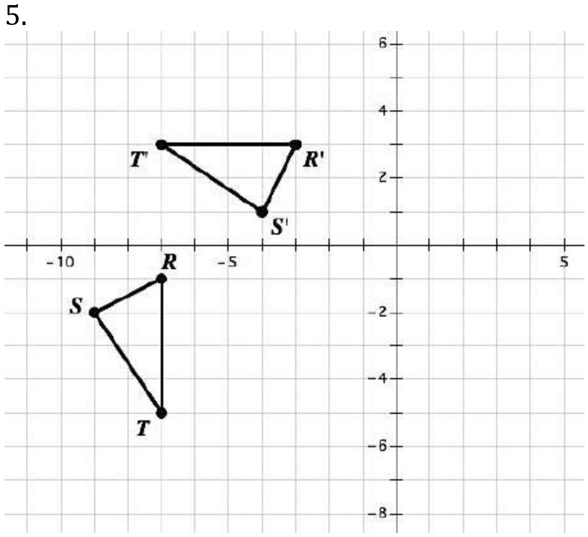


Set

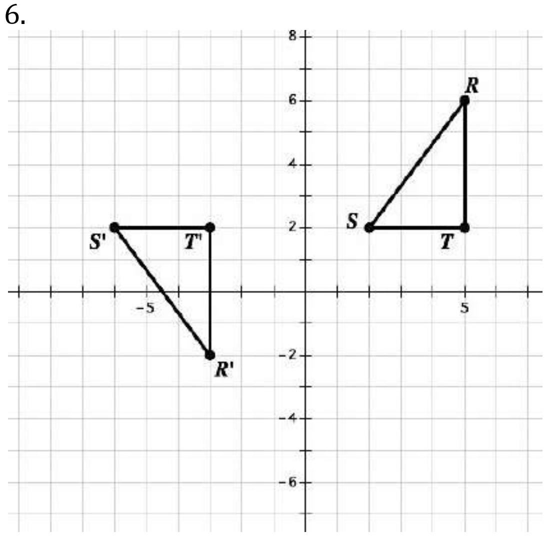
Topic: Find the sequence of transformations.

Find the sequence of transformations that will carry triangle ***RST*** onto triangle ***R'S'T'***. Clearly describe the sequence of transformations below each grid.

5.



6.

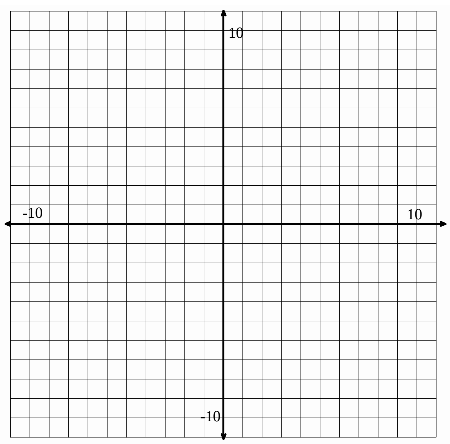


Go

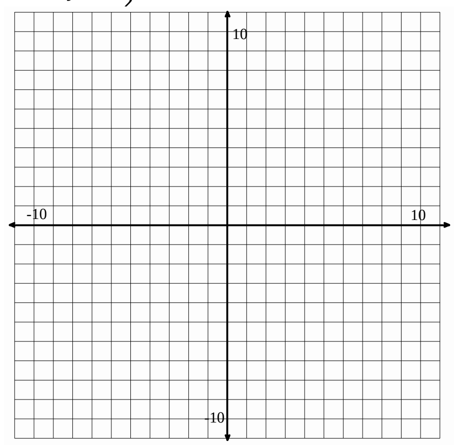
Topic: Graphing functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

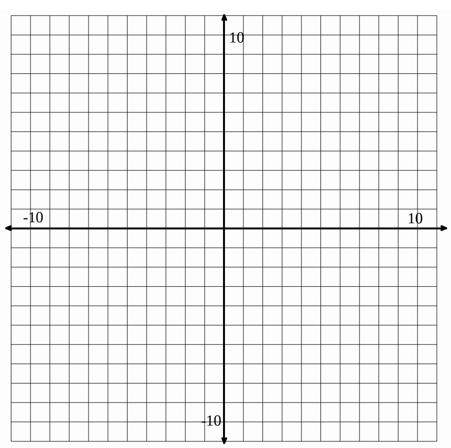
7. $y = \frac{1}{3}x - 1$
$y = -3x - 1$



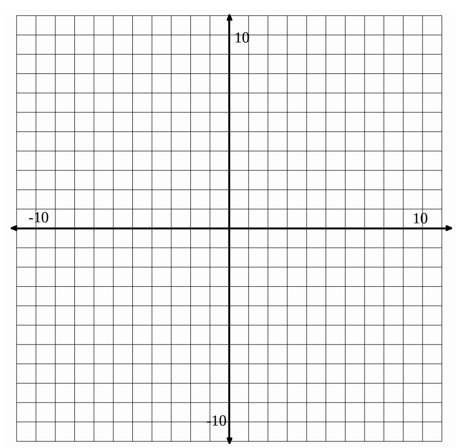
8. $y = -\frac{2}{3}x + 5$
$y = \frac{3}{2}x + 5$



9. $y = \frac{1}{4}x + 2$
$y = -\frac{1}{4} + 2$



10. $y = 2^x$
$y = -2^x$



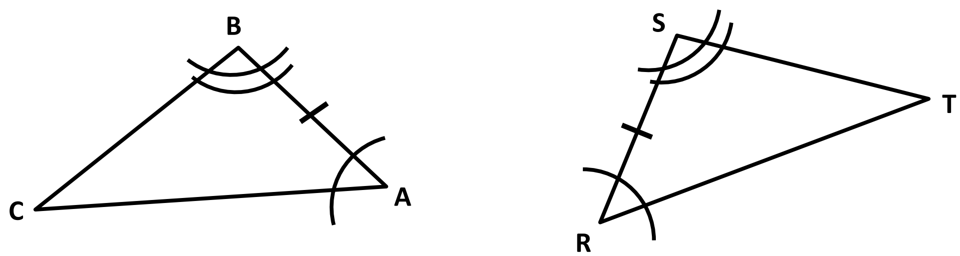
6.9 Congruent Triangles

A Solidify Understanding Task

Zac and Sione are trying to decide how much information they need to know about two triangles before they can convince themselves that the two triangles are congruent.

They are wondering if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria their teacher refers to as ASA—is enough to know that the two triangles are congruent. They are trying to justify that this would be so.

To start reasoning about the congruence of the two triangles, Zac and Sione have created the following diagram in which they have marked an ASA relationship between the triangles.



1. Based on the diagram, which angles have Zac and Sione indicated are congruent? Which sides?

2. To convince themselves that the two triangles are congruent, what else would Zac and Sione need to know?

Zac's Argument

"I know what to do," said Zac. "We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect ΔABC across $\overleftrightarrow{RS}$ and then everything coincides so the triangles are congruent." [Zac and Sione's teacher has suggested they use the word "coincides" when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.]

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included?

3. Write a paragraph explaining your reaction to Zac's argument:

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point *C* coincides with point *T* after you reflect the triangle?"

4. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asked Zac.

Sione replied, "You started using the fact that $\angle A \cong \angle R$ when you translated ΔABC so that vertex *A* coincides with vertex *R*. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac added. "Angles are more than just their vertex points."

5. How might thinking about Zac and Sione's questions help improve Zac's argument?

Sione's Argument

"I would start the same way you did, by translating point *A* until it coincides with point *R*, rotating $\overline{AB}$ about point *R* until it coincides with $\overline{RS}$, and then reflecting ΔABC across $\overleftrightarrow{RS}$," Sione said. "But then I would want to convince myself that points *C* and *T* coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point *C*, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at point *T*, points *C* and *T* must also coincide because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$ because both angles are made up of rays that coincide!"

At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of his statements until it started to slowly make sense.

6. Do the same kind of work that Zac did to make sense of Sione's argument. What parts of his argument are unclear to you? What ideas did sketching out the words of his proof help you to clarify?

Sione's argument suggests that ASA is sufficient criteria for determining if two triangles are congruent. Now Zac and Sione are wondering about other criteria, such as SAS or SSS, or perhaps even AAA (which Zac immediately rejects because he thinks two triangles can have the same angle measures but be different sizes).

7. Draw two triangles that have SAS congruence. Be sure to mark you triangles to show which sides and which angles are congruent.

8. Write out a sequence of transformations to show that the two triangles potentially coincide.

9. If Sione were to examine your work in #8, what questions would he wonder about?

10. How can you use the given congruence criteria (SAS) to resolve Simone's wonderings?

Repeat 7-10 for SSS congruence.

Ready, Set, Go!

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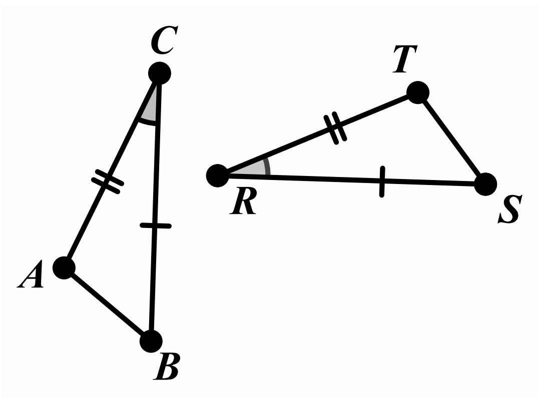
Ready

Topic: Corresponding parts of figures and transformations

Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1, $\angle C \cong \angle R$) Then determine a reflection occurred as part of the sequence of transformations that was used to create the image.

1.

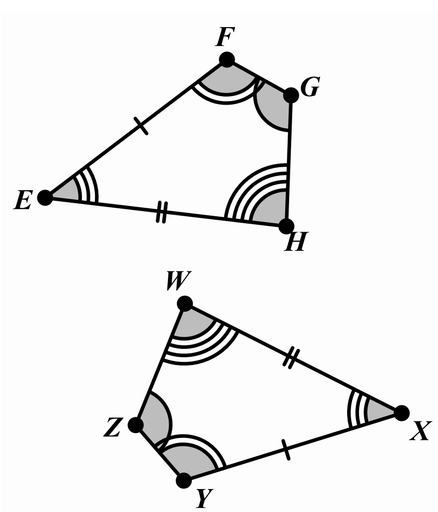


Congruencies

$\angle C \cong \angle R$

Reflected? Yes or No

2.



Congruencies

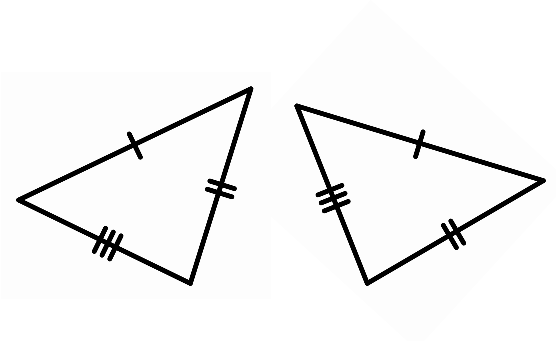
Reflected? Yes or No

Set

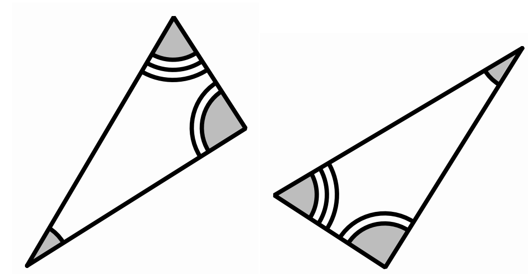
Topic: Triangle Congruencies

Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.

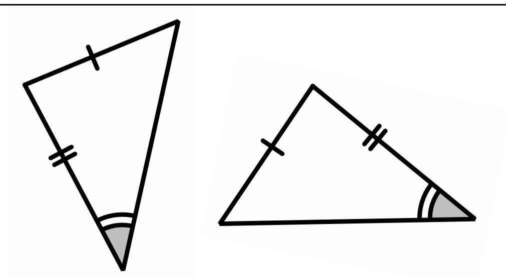
3.



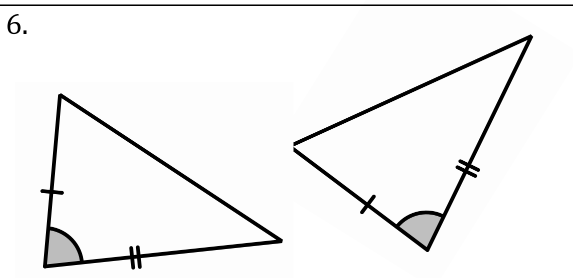
4.



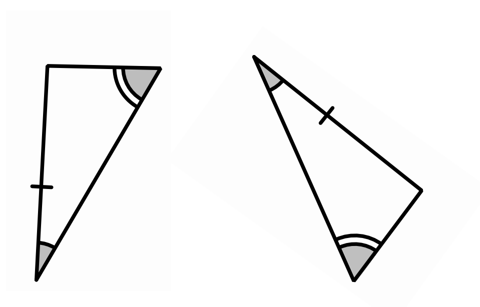
5.



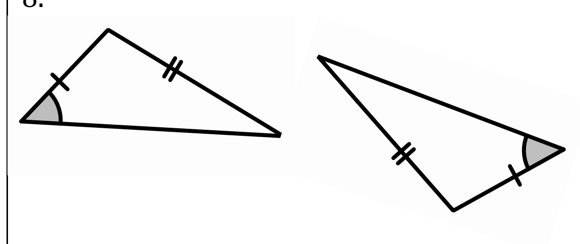
6.



7.



8.



Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

8. $\Delta ABC \cong \Delta PQR$

9. $\Delta XYZ \cong \Delta KLM$

Go

Topic: Review of solving equations and finding recursive rules for sequences.

Solve each equation for *t*.

10. $\frac{3t-4}{5} = 5$

11. $10 - t = 4t + 12 - 3t$

12. $P = 5t - d$

13. $xy - t = 13t + w$

Use the given sequence of number to write a recursive rule for the *n*th value of the sequence.

14. 5, 15, 45, …

15. $\frac{1}{2}$, 0, -$\frac{1}{2}$, -1, …

16. 3, -6, 12, -24, …

17. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, …

6.10 Congruent Triangles to the Rescue

A Practice Understanding Task

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Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent.

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the line through the vertex point of the angle made up of the two congruent sides and the midpoint of the third side in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could she use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N.*

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent.

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

3. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

Ready, Set, Go!

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Ready

Topic: Defining bisectors of angles and perpendicular bisectors.

1. Based on the meaning of "bisect", which means to split into two equal parts, what would it mean to *bisect* an angle? Describe in words and also provide visuals to communicate the meaning of angle bisector.

2. What does it mean if you have a *perpendicular bisector* of a line segment? Provide both written explanation and visual sketches to communicate the meaning of perpendicular bisector.

Set

Topic: Use congruent triangle criteria and transformations to justify conjectures.

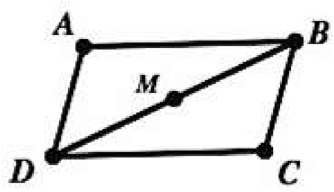
In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

3. True statements:

Point M is the midpoint of $\overline{DB}$

$\angle ABD \cong \angle BDC$

$\overline{AB} \cong \overline{DC}$



Conjecture: $\angle A \cong \angle C$

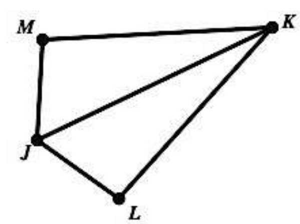
a. Is the conjecture correct?

b. Argument to prove you are right:

4. True statements

$\angle KJL \cong \angle KJM$

$\overline{JL} \cong \overline{JM}$



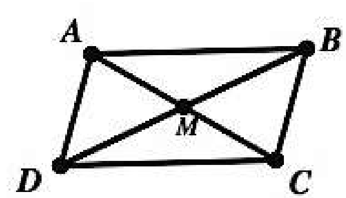
Conjecture: $\overline{JK}$ bisects $\angle MKL$

a. Is the conjecture correct?

b. Argument to prove you are right:

5. True statements

ΔADM is a 180° rotation of ΔCMB



Conjecture: $\Delta ABM \cong \Delta CDM$

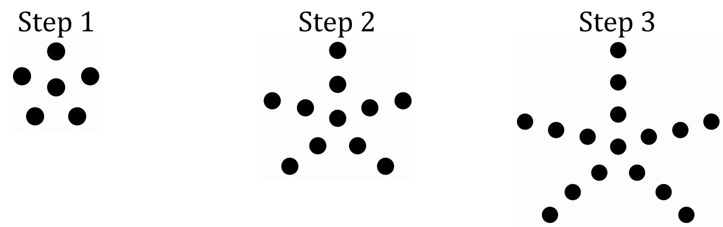
a. Is the conjecture correct?

b. Argument to prove you are right:

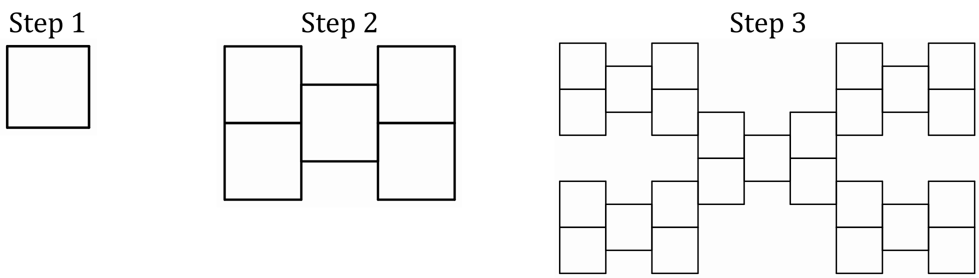
Go

Topic: Create both explicit and recursive rules for the visual patterns.

6. Find an explicit function rule and a recursive rule for dots in step *n*.



7. Find an explicit function rule and a recursive rule for squares in step *n.*



Find an explicit function rule and a recursive rule for the values in each table.

8.

Step	Value
1	1
2	11
3	21
4	31

9.

n	f(n)
2	16
3	8
4	4
5	2

10.

n	f(n)
1	-5
2	25
3	-125
4	625

6.11 Under Construction
A Develop Understanding Task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Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.

Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point *C* on the ray in the diagram above so the distance from *B* to *C* is the same as the distance from *B* to *A*?

1. Describe how you will locate point *C* and how you know $\overline{BC} \cong \overline{BA}$, then construct point *C* on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point *D*, the fourth vertex.

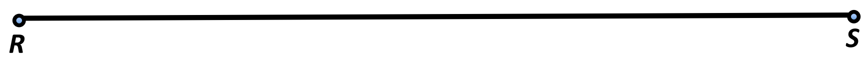
2. Describe how you will locate point *D* and how you know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point *D* on the diagram above.

Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$ below, fold and crease the paper so that point *R* is reflected onto point *S*. Based on the definition of reflection, what do you know about this "crease line"?



You have "constructed" a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the "crease line". When you have located at least two points on the "crease line" use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$ we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram above as point *M.* Using segment $\overline{RM}$ as one side of the square, and the right angle formed by segment $\overline{RM}$ and the perpendicular line drawn through point *M* as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

Ready, Set, Go!

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Ready

Topic: Tools for construction and geometric work.

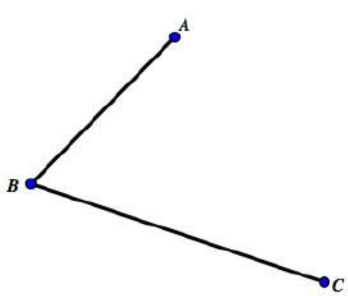
1. Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

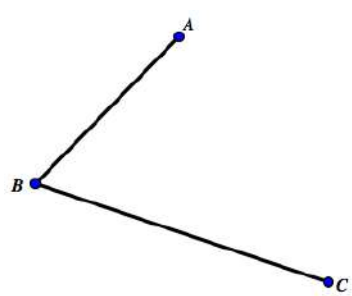
2. In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

Set

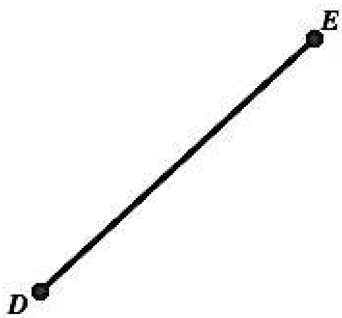
Topic: Constructions with compass and straight edge.

3. Bisect the angle below do it with compass and straight edge as well as with paper folding.

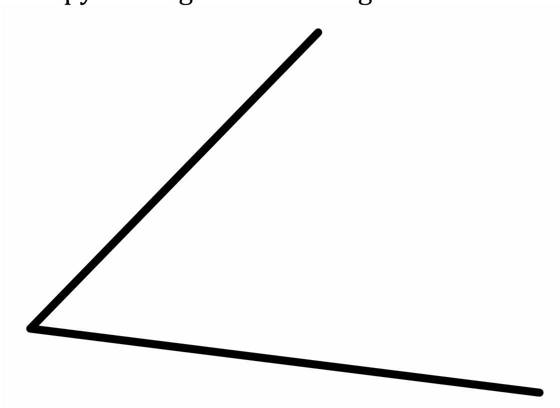




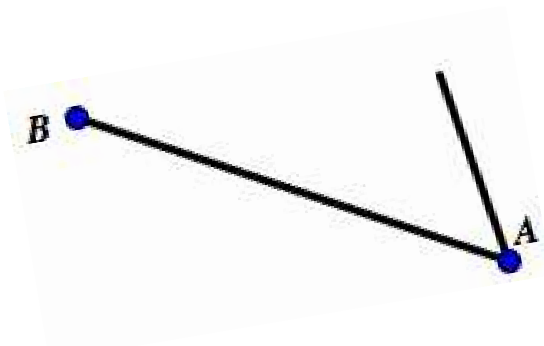
4. Copy the segment below using construction tools of compass and straight edge, label the image ***D'E'***.



5. Copy the angle below using construction tool of compass and straight edge.

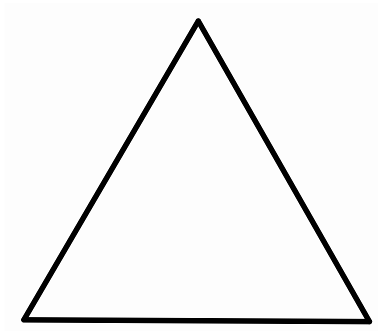


6. Construct a rhombus on the segment AB that is given below and that has point A as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.

8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



Go

Topic: Solving systems of equations review.

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9. $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12. $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

13. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14. $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

6.12 More Things Under Construction
A Develop Understanding Task

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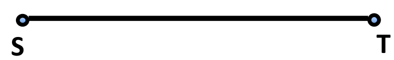
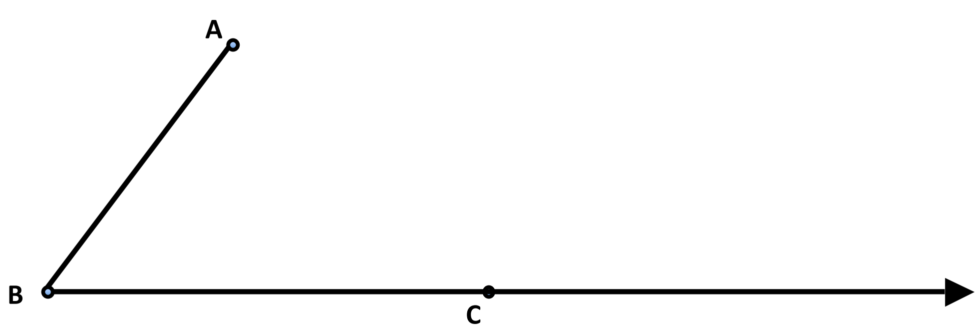
Constructing an Equilateral Triangle

Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ below as one side of the equilateral triangle.



Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment $\overline{AB}$ through point *C* on the diagram below. Since we have observed that parallel lines have the same slope, the line through point *C* will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{CD}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with vertex at point *C* and a side parallel to $\overline{AB}$? (Hint: We know that corresponding parts of congruent triangles are congruent, so perhaps we can begin by constructing some congruent triangles.)



Constructing a Hexagon Inscribed in a Circle

Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

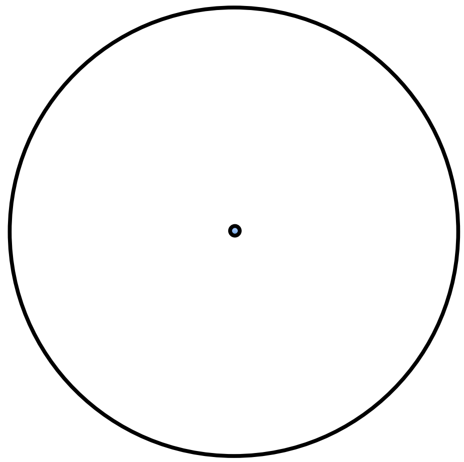
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1. Sketch a diagram of such a decomposition.

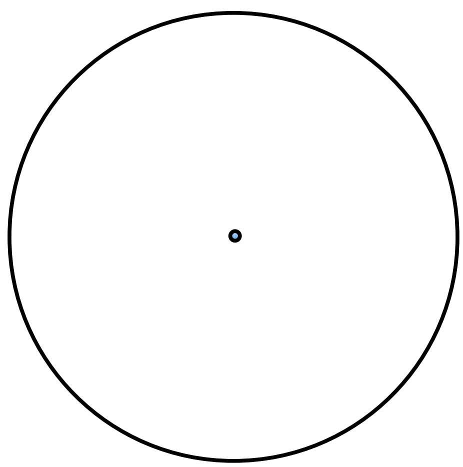
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? Be able to justify how you know this is so.

4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



Describe how you might construct a square inscribed in a circle.

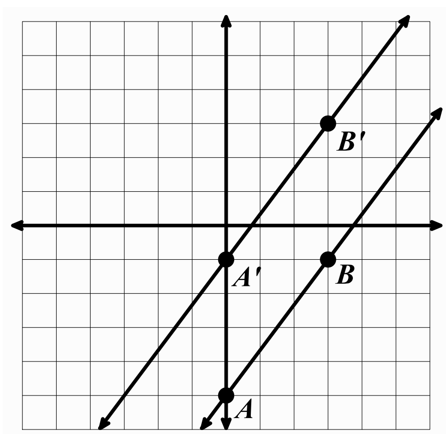
Ready, Set, Go!

Ready

Topic: Transformations of lines, algebraic and geometric thoughts.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, label them, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

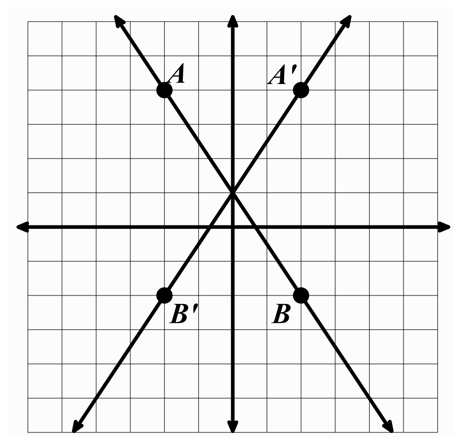


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

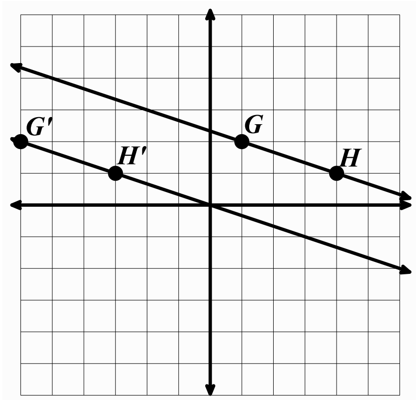


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3.

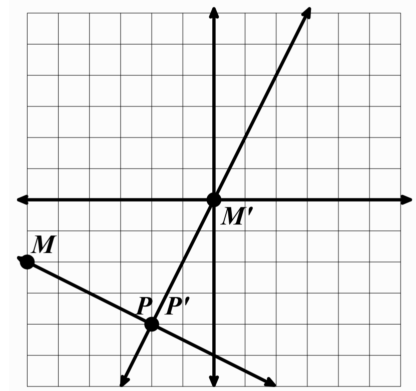


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

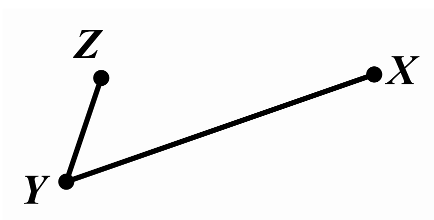
b. Equation for pre-image:

c. Equation for image:

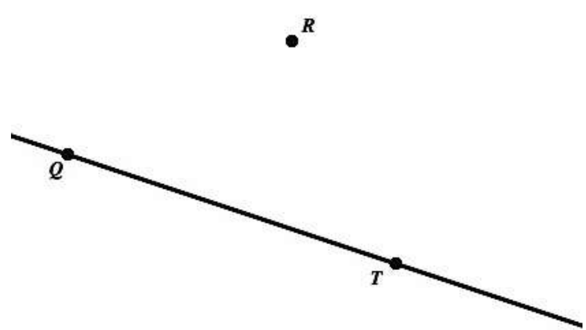
Set

Topic: Geometric Constructions using compass and straight edge.

5. Construct a parallelogram given sides $\overline{XY}$ and $\overline{YZ}$ and $\angle XYZ$.



6. Construct a line parallel to $\overline{QT}$ and through point ***R***.



7. Given segment $\overline{AB}$ show all points ***C*** such that D ABC is an isosceles triangle.

8. Given segment $\overline{AB}$ show all points C such that D ABC is a right triangle.

Go

Topic: Triangle congruence and properties of polygons.

9. What is the minimum amount of information needed to determine that two triangles are congruent? List all possible combinations of needed criteria.

10. What is a line of symmetry and what is a diagonal? Are they the same thing? Could they be the same in a polygon? If so give an example, if not explain why not.

11. How is the number of lines of symmetry for a *regular* polygon connected to the number of sides of the polygon? How is the number of diagonals for a polygon connected to the number of sides?

12. What do right triangles have to do with finding distance between points on a coordinate grid?

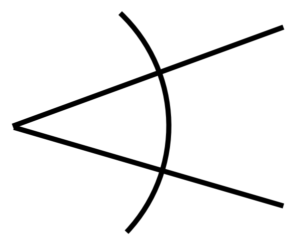
6.13 Justifying Constructions

A Solidify Understanding Task

Compass and straightedge constructions can be justified using such tools as:

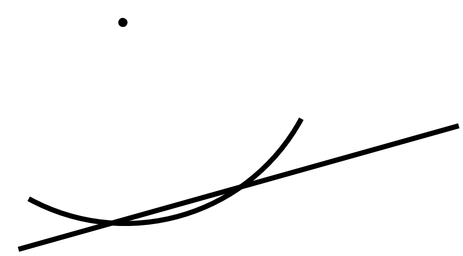
- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

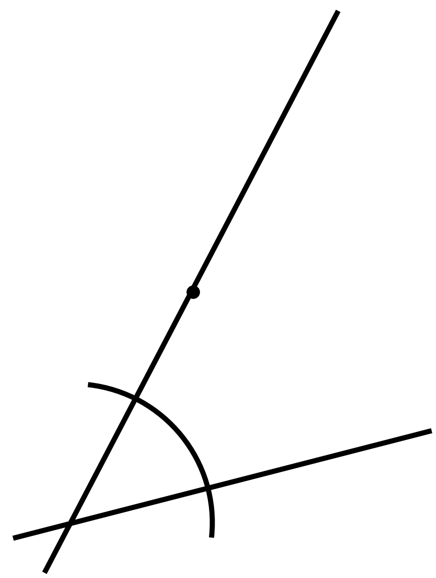
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point not on the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for constructing a line parallel to a given line through a given point.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Ready, Set, Go!

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Ready

Topic: Rotation symmetry for regular polygons and transformations

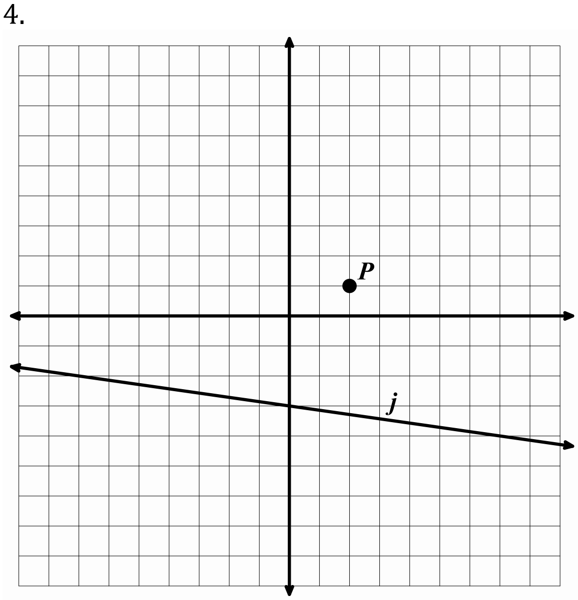
1. What angles of rotational symmetry are there for a pentagon?

2. What angles of rotational symmetry are there for a hexagon?

3. If a regular polygon has an angle of rotational symmetry that is 40^0, how many sides does the polygon have?

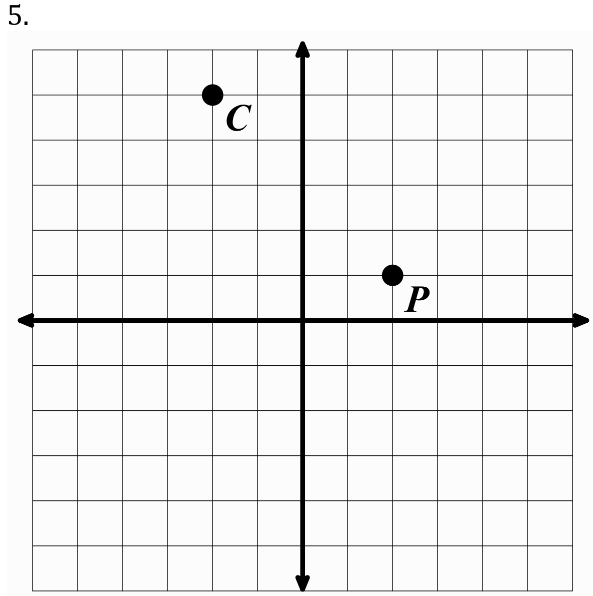
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point ***P*** over line ***j***.

5.

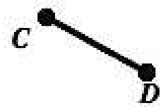


Rotate point ***P*** 90^0 clockwise around point ***C***.

Set

Topic: Constructing regular polygons inscribed in a circle.

6. Construct an isosceles triangle that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the triangle.



7. Construct a hexagon that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the hexagon.

8. Construct a square that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the square.

Go

Topic: Finding distance and slope.

For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.

9. (-2, 8) , (3, -4)
   a. Slope: b. Distance:

10. (-7, -3) , (1, 5))
   a. Slope: b. Distance:

11. (3, 7) , (-5, 9)
   a. Slope: b. Distance:

12. (1, -5) (-7, 1)
   a. Slope: b. Distance:

13. (-10, 31) (20, 11)
   a. Slope: b. Distance:

14. (16, -45) (-34, 75)
   a. Slope: b. Distance:

6.14 Construction Blueprints

A Practice Understanding Task

For each of the following straightedge and compass constructions, illustrate or list the steps for completing the construction and give an explanation for why the construction works. You explanations may be based on rigid-motion transformations, congruent triangles, or properties of quadrilaterals.

Purpose of the construction	Illustration and/or steps for completing the construction	Justification of why this construction works
Copying a segment	1. Set the span of the compass to match the distance between the two endpoints of the segment. 2. Without changing the span of the compass, draw an arc on a ray centered at the endpoint of the ray. The second endpoint of the segment is where the arc intersects the ray.	The given segment and the constructed segment are radii of congruent circles.
Copying an angle		
Bisecting a segment		
Bisecting an angle		
Constructing a perpendicular bisector of a line segment		
Constructing a perpendicular to a line through a given point		

Constructing a line parallel to a given line through a given point		
Constructing an equilateral triangle		
Constructing a regular hexagon inscribed in a circle		

Ready, Set, Go!

Ready

Topic: Connecting tables with transformations.

For each function find the outputs that fill in the table. Then describe the relationship between the outputs in each table.

1. $f(x) = 3x$ $\quad$ $g(x) = 3x - 5$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

2. $t(x) = 2x$ $\quad$ $h(x) = 2x - 5$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

Relationship between $t(x)$ and $h(x)$:

3. $f(x) = 2x$ $\quad$ $g(x) = 2(x - 3)$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

4. $t(x) = 4x$ $\quad$ $h(x) = 4^{(x-3)}$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

Relationship between $t(x)$ and $h(x)$:

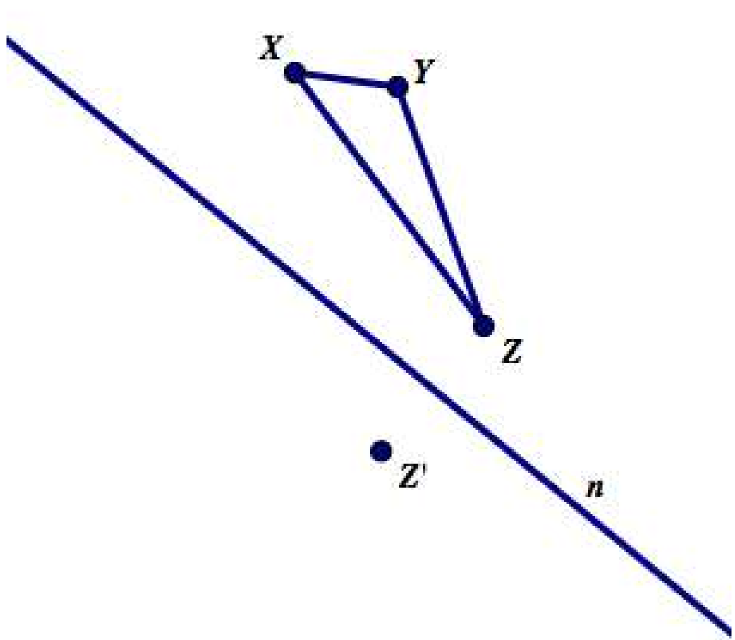
Set

Topic: Constructing transformations

In each problem below use compass and straight edge to construct the transformation that is described.

5. Construct D A'B'C' so that it is a translation of D ABC. (Hint: parallel lines may be useful.)

6. Construct D X'Y'Z' so that it is a reflection of D XYZ over line ***m***.
(Hint: perpendicular lines may be useful.)



Go

Topic: Transformations and triangle congruence.

Determine whether or not the statement is true or false. If true, explain why. If false, explain why not or provide a counterexample.

7. If one triangle can be transformed so that one of its angles and one of its sides coincide with another triangles angle and side then the two triangles are congruent.

8. If one triangle can be transformed so that two of its sides and any one of its angles will coincide with two sides and an angle from another triangle then the two triangles will be congruent.

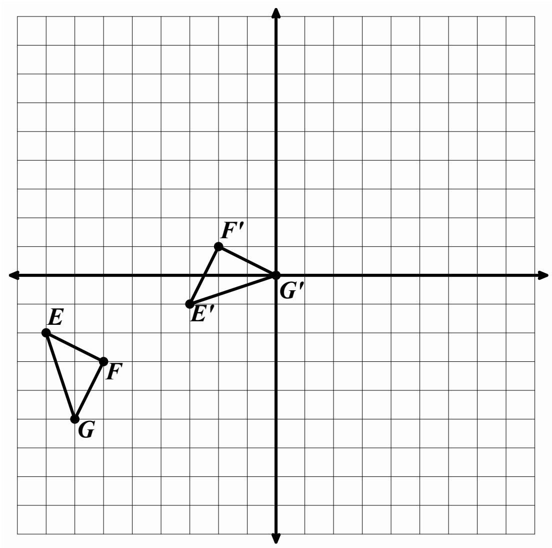
9. If all three angles of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

10. If all three sides of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

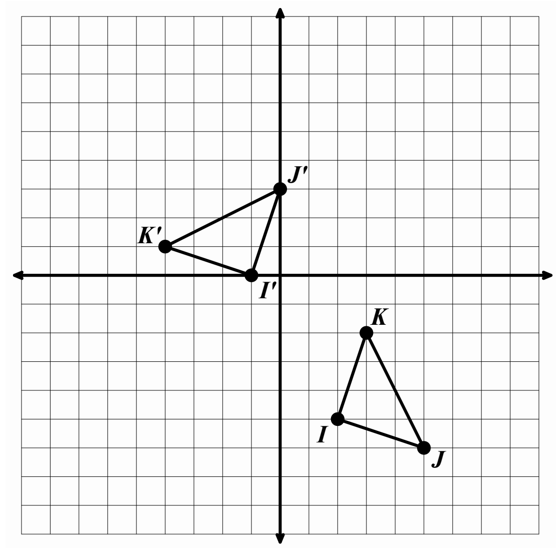
11. For any two congruent polygons there is a sequence of transformations that will transform one of the polygons onto the other.

Find the point of rotation for each of the figures below.

12.

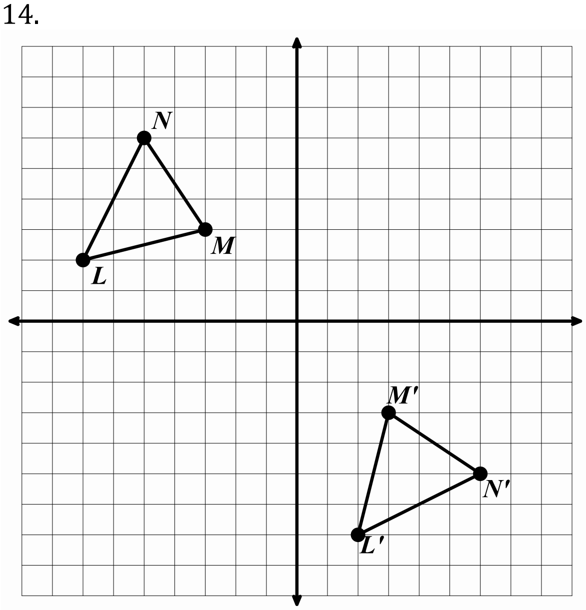


13.

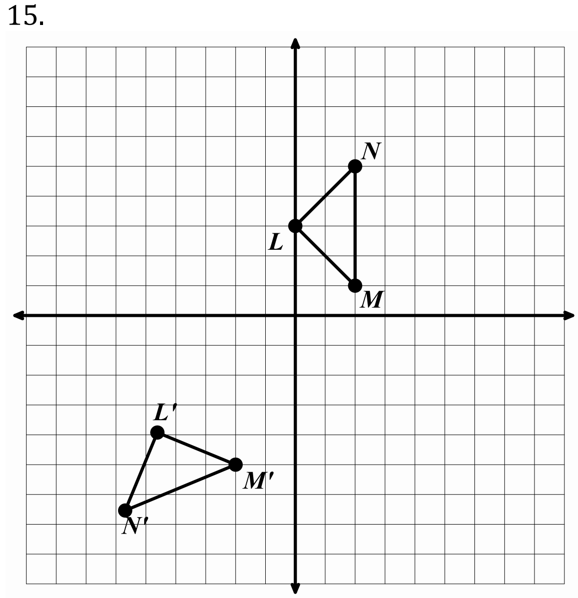


Find the line of reflection for each of the figures drawn below.

14.



15.



Advanced Mathematics I

Module 7 Advanced Connecting Algebra and Geometry

By

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www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

Module 7 – Connecting Algebra and Geometry

Classroom Task: 7.1 Go the Distance- A Develop Understanding Task
Use coordinates to find distances and determine the perimeter of geometric shapes **(G.GPE.7)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.1

Classroom Task: 7.2 Slippery Slopes – A Solidify Understanding Task
Prove slope criteria for parallel and perpendicular lines **(G.GPE.5)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.2

Classroom Task: 7.3 Prove It! – A Solidify Understanding Task
Use coordinates to algebraically prove geometric theorems **(G.GPE.4)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.3

Classroom Task: 7.4 Training Day– A Solidify Understanding Task
Write the equation f(t)= m(t) + k by comparing parallel lines and finding k **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.4

Classroom Task: 7.5 Training Day Part II – A Practice Understanding Task
Determine the transformation from one function to another **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.5

Classroom Task: 7.6 Shifting Functions – A Practice Understanding Task
Translating linear and exponential functions using multiple representations **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.6

Classroom Task: 7.7H The Arithmetic of Vectors – A Solidify Understanding Task
Defining and operating with vectors as quantities with magnitude and direction **(N.VM.1, N.VM.2, N.VM.3, N.VM.4, N.VM.5)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.7H

Classroom Task: 7.8H More Arithmetic of Matrices – A Solidify Understanding Task
Examining properties of matrix addition and multiplication, including identity and inverse properties **(N.VM.8, N.VM.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.8H

Classroom Task: 7.9H The Determinant of a Matrix – A Solidify Understanding Task
Finding the determinant of a matrix and relating it to the area of a parallelogram **(N.VM.10, N.VM.12)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.9H

Classroom Task: 7.10H Solving Systems with Matrices, Revisited – A Solidify Understanding Task
Solving a system of linear equations using the multiplicative inverse matrix **(A.REI.1, UT Honors Standard: Solve systems of linear equations using matrices)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.10H

Classroom Task: 7.11H Transformations with Matrices – A Solidify Understanding Task
Using matrix multiplication to reflect and rotate vectors and images **(N.VM.11, N.VM.12)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.11H

Classroom Task: 7.12H Plane Geometry – A Practice Understanding Task
Solving problems involving quantities that can be represented by vectors **(N.VM.3, N.VM.4a, N.VM.12)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.12H

7.1 Go the Distance
A Develop Understanding Task

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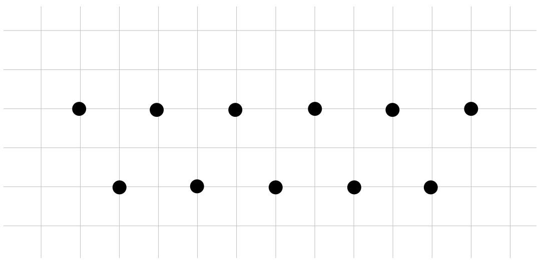
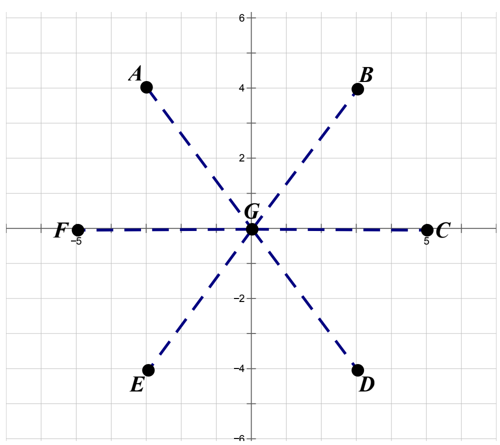
The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one girl in the middle to six other girls. On the grid, their pattern looks like this:

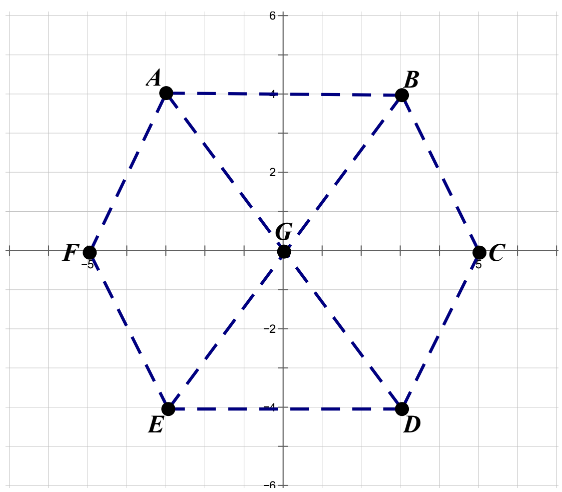


The question the girls have is how long to make the ribbons. Some girls think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

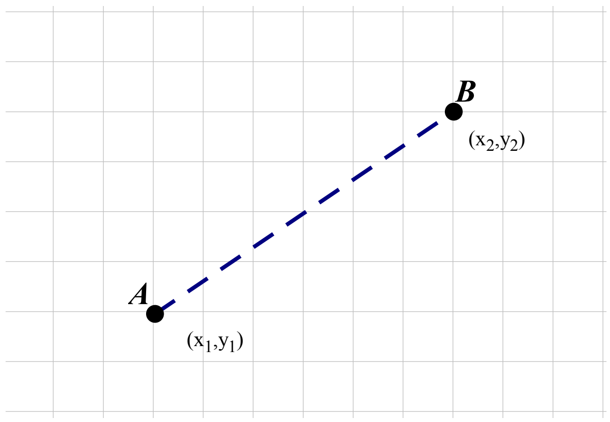
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1,y_1) and B (x_2,y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, using points A and B.

5. Use the process you came up with in #4 to find the distance between two points located at (-1, 5) and (2, -6)

6. Use you process to find the perimeter of the hexagon pattern shown in #3.

Name: Connecting Algebra and Geometry 7.1

http://www.flickr.com/photos/briemckinneyxo/

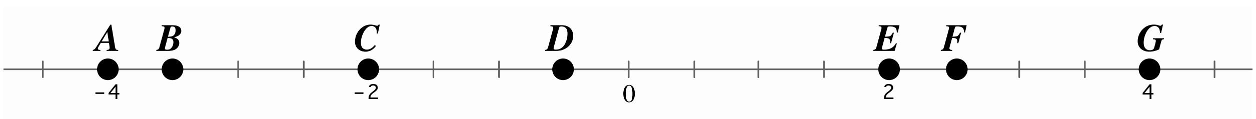
Ready, Set, Go!

Ready

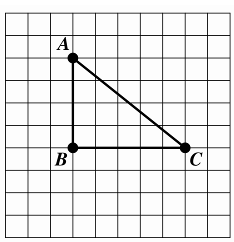
Topic: Finding the distance between two points

Use the number line to find the distance between the given points. (The notation AB means the distance between points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8. 

a. Find AB

b. Find BC

c. Find AC

9. Why is it easier to find the distance between points A and B and points B and C than it is to find the distance between A and C?

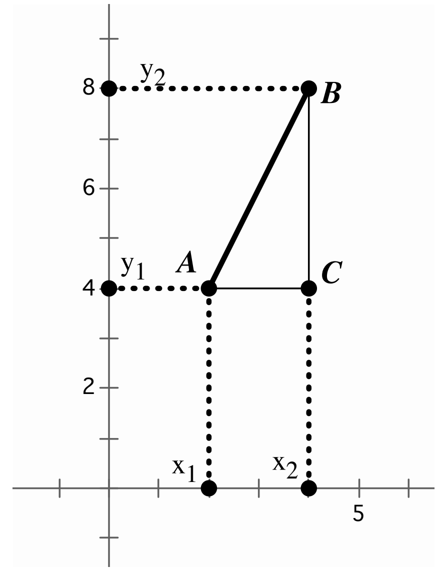
10. Explain how to find the distance between points A and C.

Name: Connecting Algebra and Geometry 7.1

Set

Topic: Slope triangles and the distance formula.

Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_1}$ *where m is the slope.*

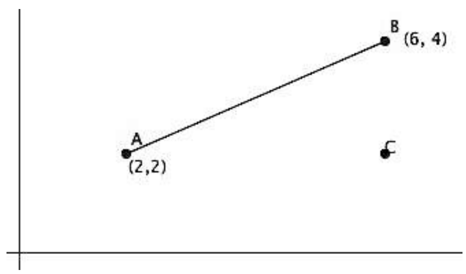


11a. What does the value $(y_2 - y_1)$ tell you?

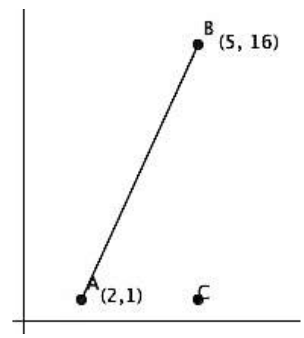
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and performing the Pythagorean Theorem. In this exercise try to develop a more efficient method of finding the length of a line segment by using the meaning of $(y_2 - y_1)$ and $(x_2 - x_1)$ combined with the Pythagorean Theorem.

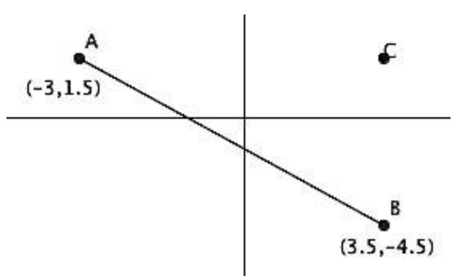
12. Find AB.



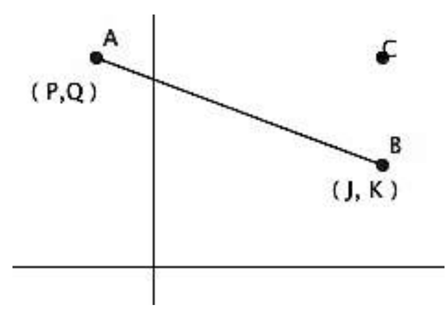
13. Find AB.



14. Find AB.



15. Find AB.



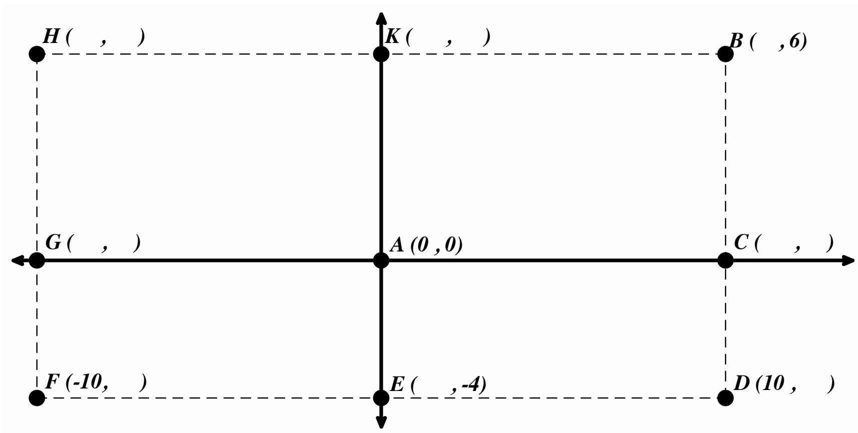
Name: Connecting Algebra and Geometry | 7.1

Go Topic: Rectangular coordinates

Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

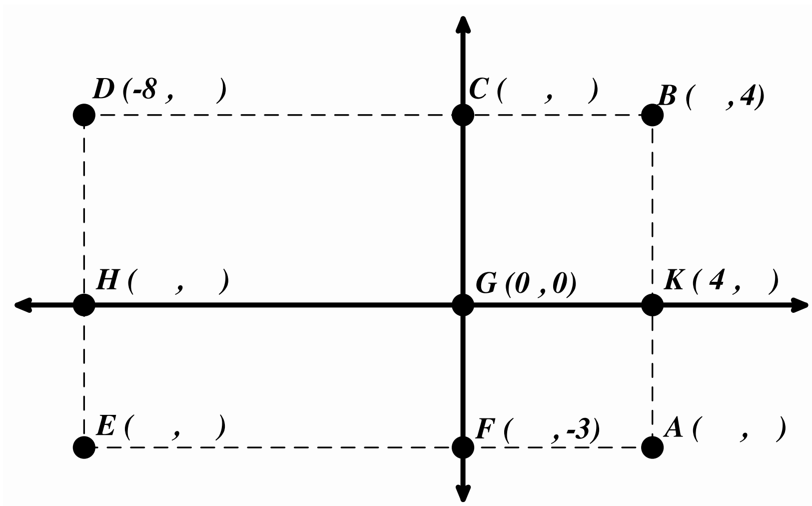
16a. Find HB

b. Find BD



17a. Find DB

b. Find CF



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/the-coordinate-plane

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/distance-formula

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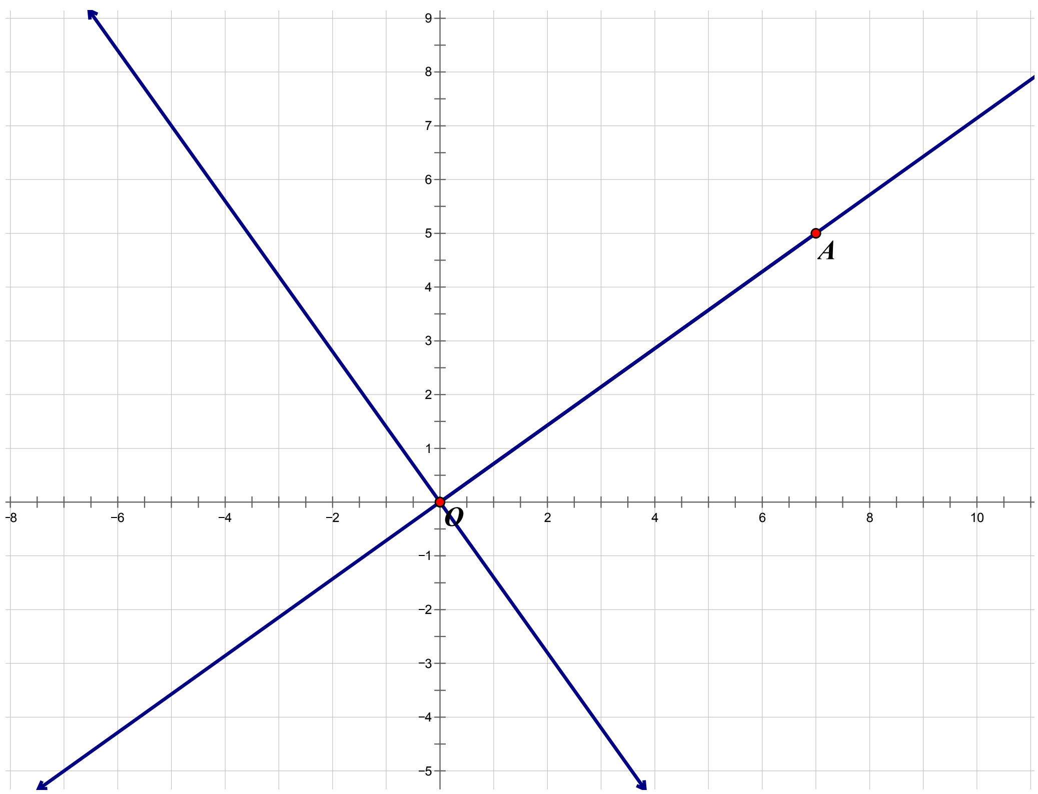
7.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

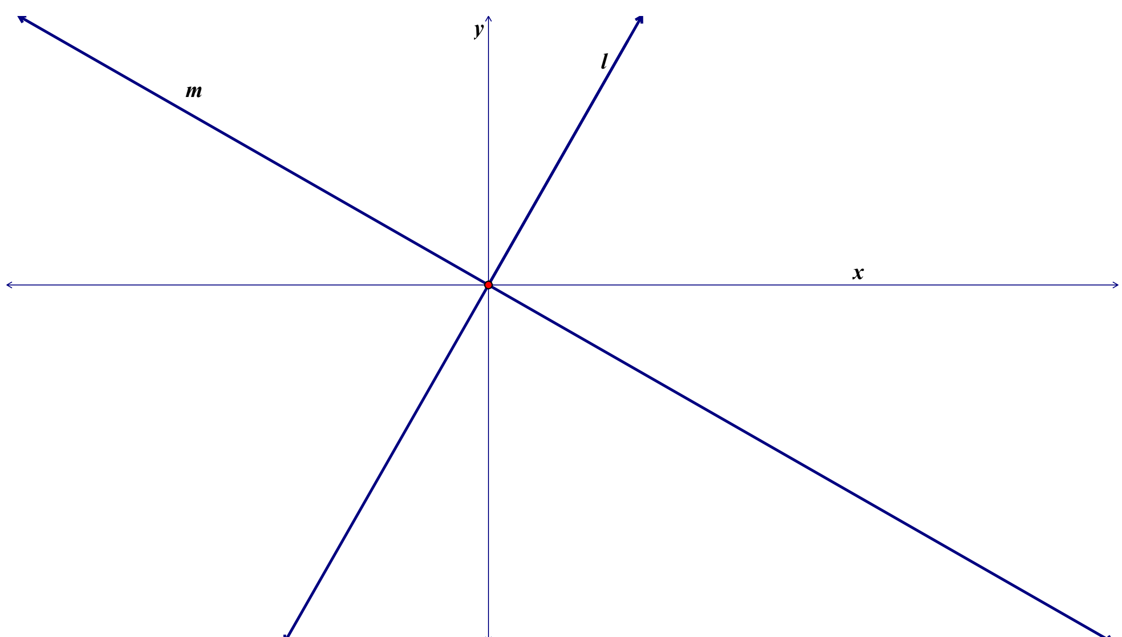
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with this theorem.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle.

6. What is the slope of line *l*?

Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

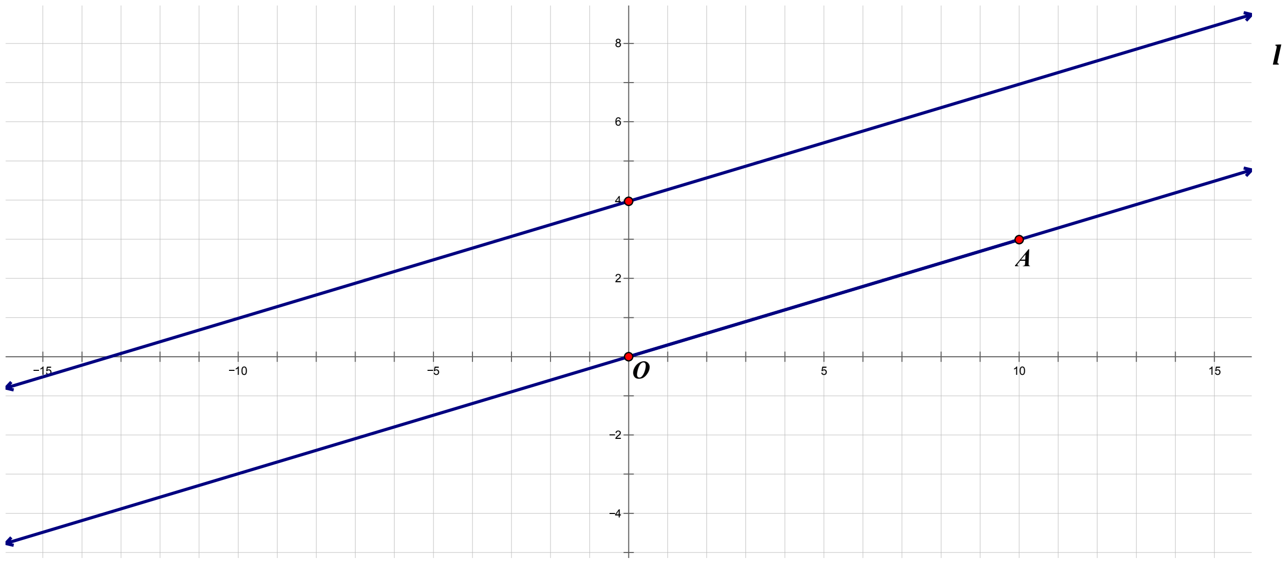
10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

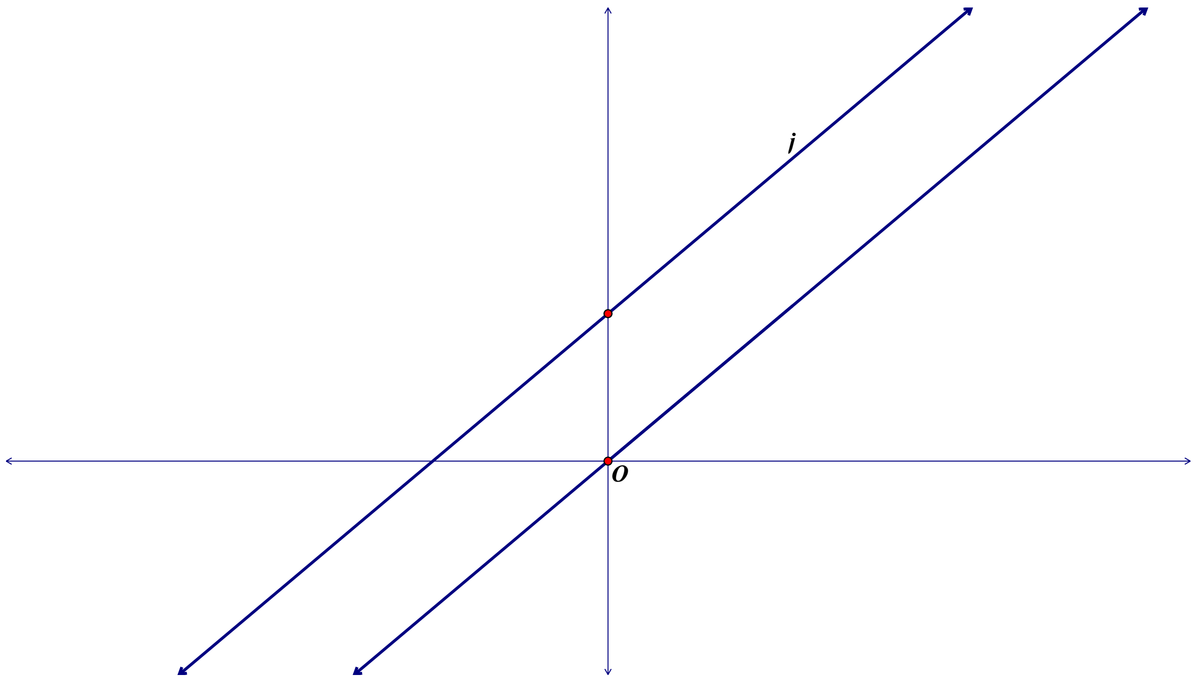
Draw the slope triangle from point A. What is the slope of $\overleftrightarrow{OA}$?



What translation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto line l?

What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

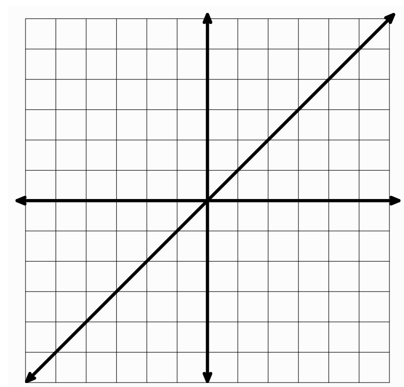
Name: Connecting Algebra and Geometry | 7.2

Ready, Set, Go!

Ready

Topic: Graphing lines.

The graph at the right is of the line $f(x) = x$.



1a. On the same grid, graph a parallel line that is 3 units above it.

b. Write the equation of the new line. ___________________________

c. Write the y-intercept of the new line as an ordered pair.

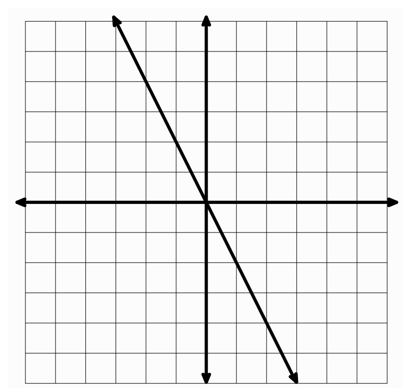
d. Write the x-intercept of the new line as an ordered pair.

e. Write the equation of the new line in point-slope form using the y-intercept.

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

The graph at the right is of the line $f(x) = -2x$.



2a. On the same grid, graph a parallel line that is 4 units below it.

b. Write the equation of the new line. _________________________

c. Write the y-intercept of the new line as an ordered pair.

d. Write the x-intercept as an ordered pair.

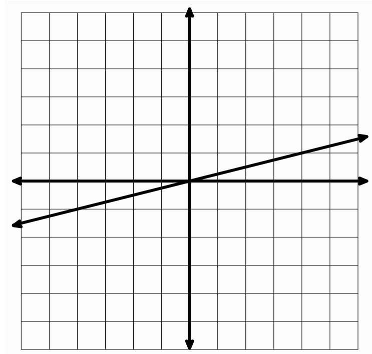
e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

Name: Connecting Algebra and Geometry | 7.2

The graph at the right is of *f(x)* = ¼ x



3a. Graph a parallel line 2 units below.

b. Write the equation of the new line.

c. Write the y-intercept as an ordered pair.

d. Write the x-intercept as an ordered pair.

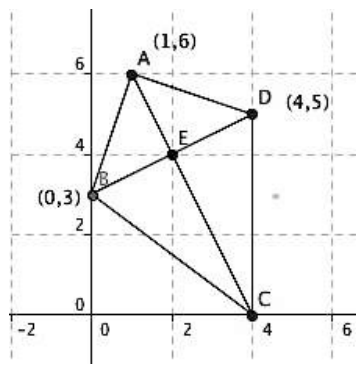
e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept

g. Explain in what way the equations are the same and in what way they are different.

Set

Topic: Verifying and Proving Geometric Relationships



The quadrilateral at the right is called a **kite.**

Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically.
(A symbol may be used more than once.)

$\cong \quad \perp \quad || \quad < \quad > \quad =$

Proof

4. $\overline{BC}$ ________ $\overline{DC}$ ____________________

5. $\overline{BD}$ ________ $\overline{AC}$ ____________________

6. $\overline{AB}$ ________ $\overline{BC}$ ____________________

Name: Connecting Algebra and Geometry | 7.2

7. ΔABC _______ ΔADC ____

____

8. $\overline{BE}$ ________ $\overline{ED}$ ____

____

9. $\overline{AE}$ ________ $\overline{ED}$ ____

____

10. $\overline{AC}$ _________ $\overline{BD}$ ____

____

Go

Topic: Writing equations of lines.

Write the equation of the line in standard form using the given information.

11. Slope: -¼ point (12, 5)

12. A (11, -3) , B (6, 2)

13. x-intercept: -2, y-intercept: -3

14. All x values are -7, y can be anything

15. Slope: ½ x-intercept: 5

16. E (-10, 17) , G (13, 17)

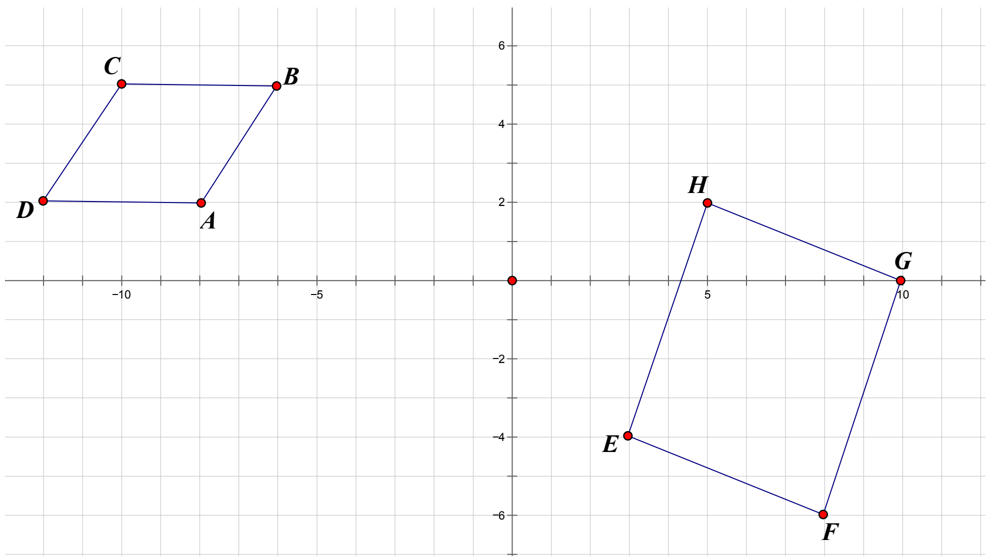
Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-using-x-and-y-intercepts

7.3 Prove It!
A Solidify Understanding Task

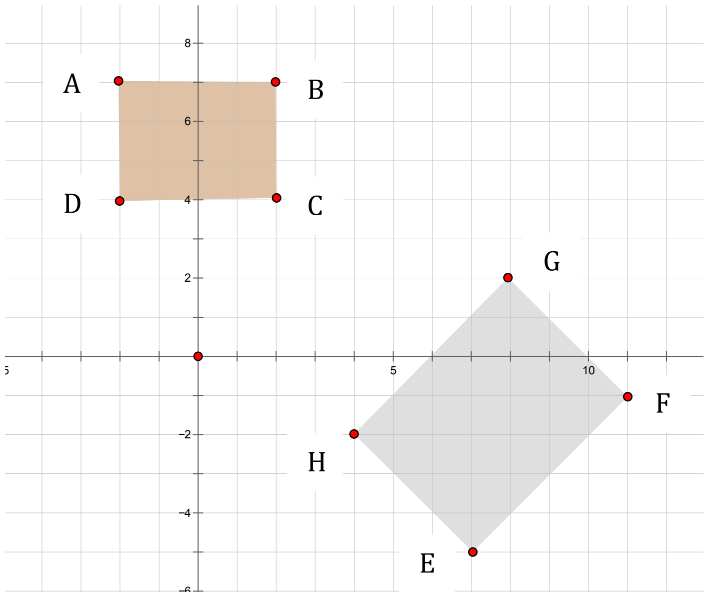

www.flickr.com/photos/safari_vacation

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.



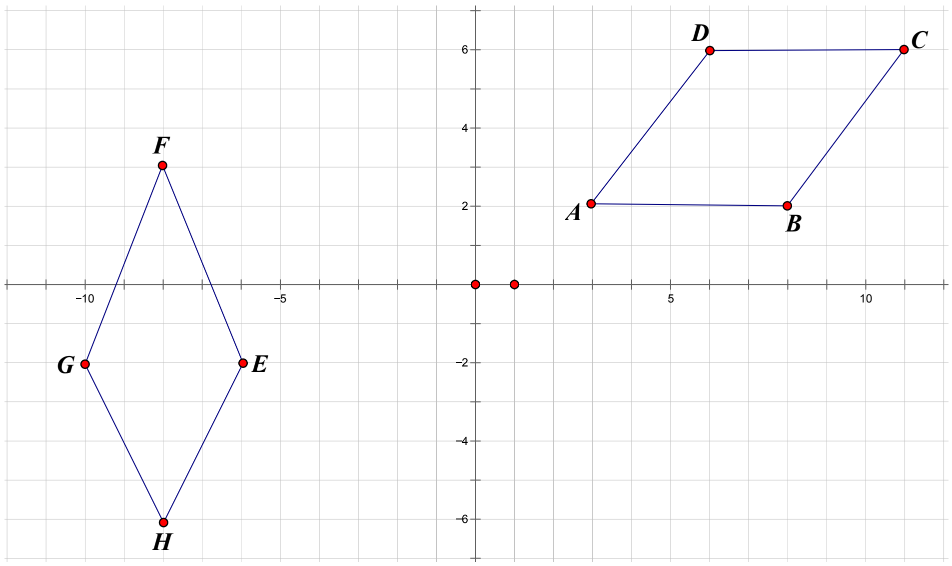
Is ABCD a parallelogram? Explain how you know.

Is EFGH a parallelogram? Explain how you know.



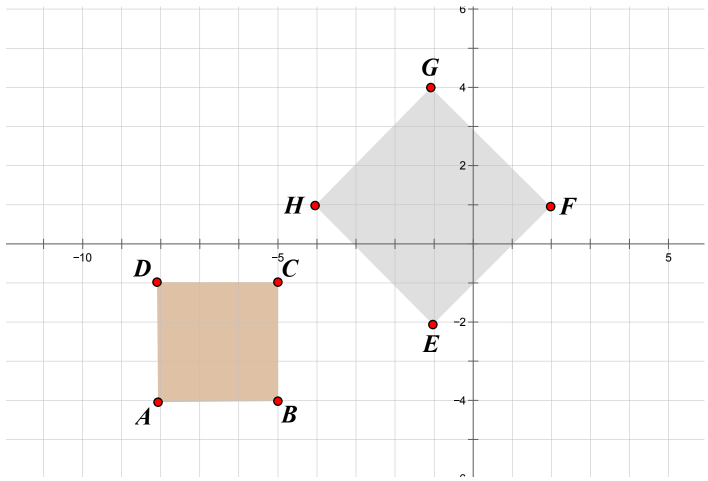
Is ABCD a rectangle? Explain how you know.

Is EFGH a rectangle? Explain how you know.



Is ABCD a rhombus? Explain how you know.

Is EFGH a rhombus? Explain how you know.



Is ABCD a square? Explain how you know.

Is EFGH a square? Explain how you know.

Name: Connecting Algebra and Geometry | 7.3

Ready, Set, Go!

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Ready

Topic: Tables of value

Find the value of *f (x)* for the given domain. Write *x* and *f (x)* as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

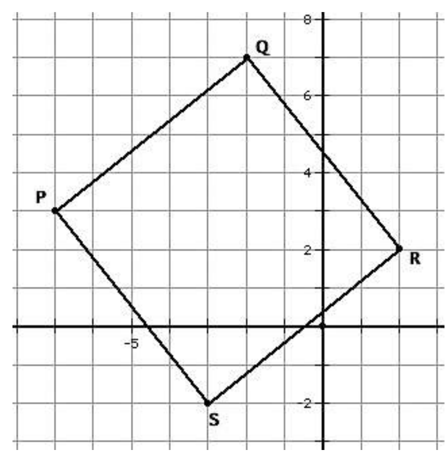
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

Set

Topic: Characteristics of rectangles and squares

4a. Is the figure below a rectangle? (Justify your answer)

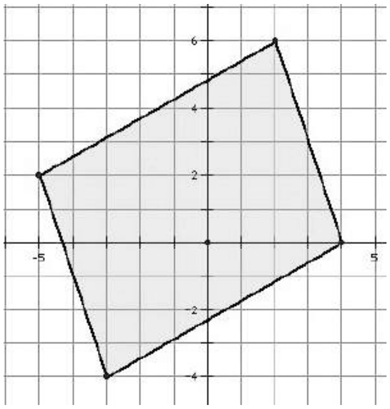
b. Is the figure a square? (Justify your answer)



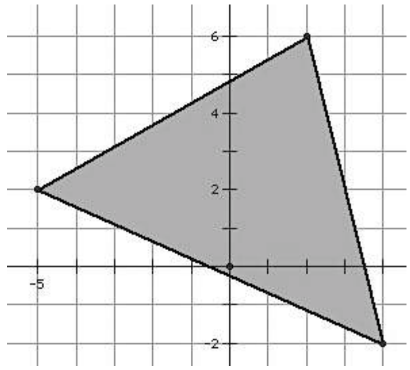
Go

Find the perimeter of each figure below. Round to the nearest hundredth.

5.



6.



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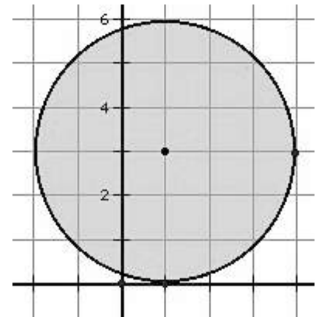

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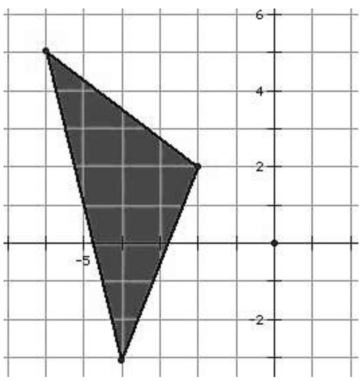

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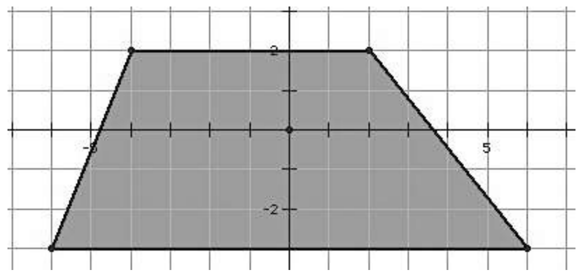
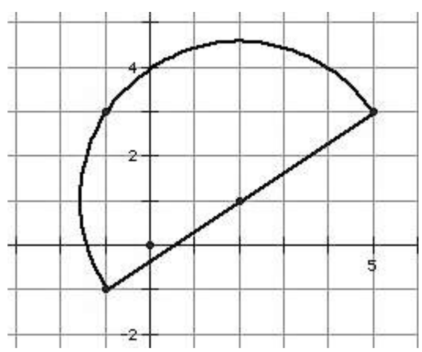


10.



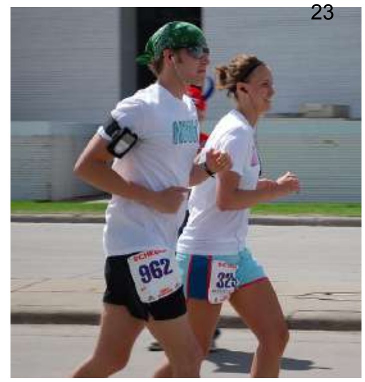
Need Help? Check out these related videos:

http://www.khanacademy.org/math/geometry/basic-geometry/v/perimeter-and-area-of-a-non-standard-polygon

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/distance-formula

7.4 Training Day

A Develop Understanding Task

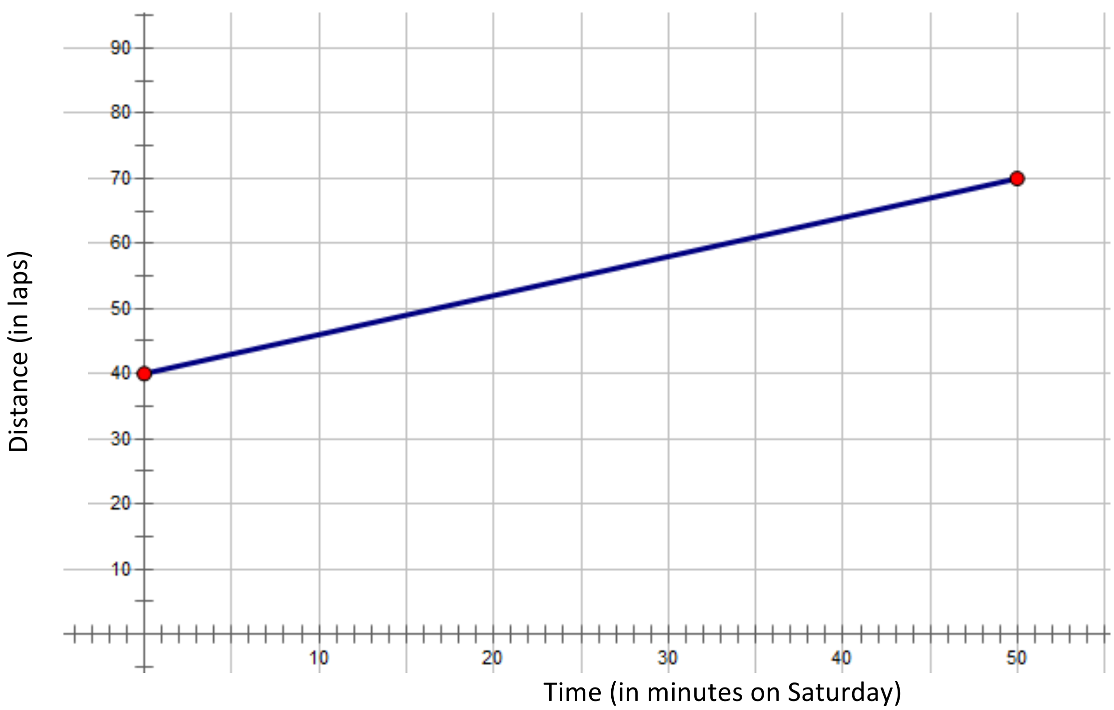


Fernando and Mariah are training for six weeks to run in the Salt Lake half- marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



What observations can be made about the similarities and differences between the two trainers?

1. Write the equation, *m(t),* that models Mariah's distance.

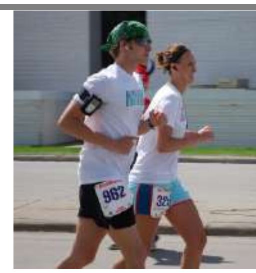
2. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame: The rate of both runners is the same throughout the week, however, Fernando ____.

3. In mathematics, sometimes one function can be used to build another. Write Fernando's equation, *f(t),* by starting with Mariah's equation, *m(t).*

   f(t) =

4. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 3. Write in words how you would explain this new function to your class.

Name: Connecting Algebra and Geometry | 7.4



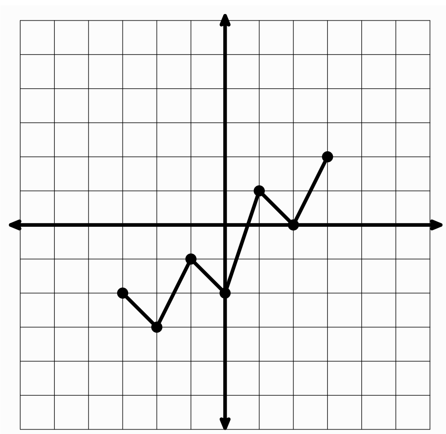
Ready, Set, Go!

http://www.flickr.com/photos/fargomoorheadcvb

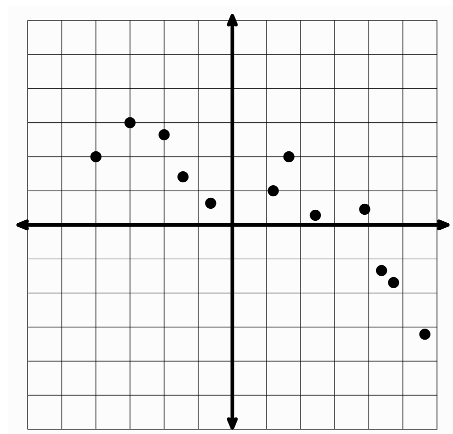
Ready

Topic: Vertical transformations of graphs

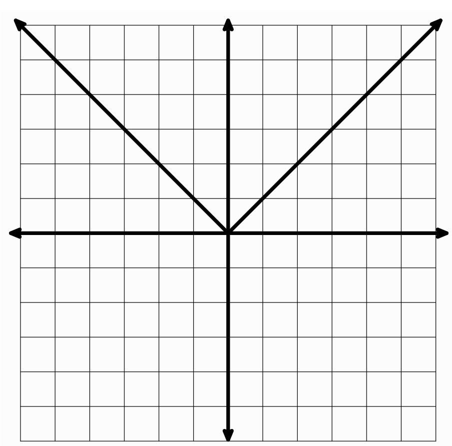
1. Use the graph below to draw a new graph that is translated up 3 units.



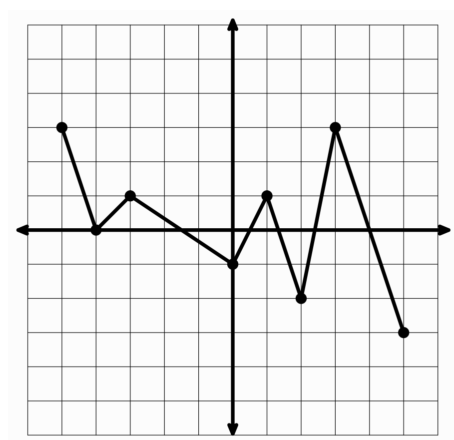
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.



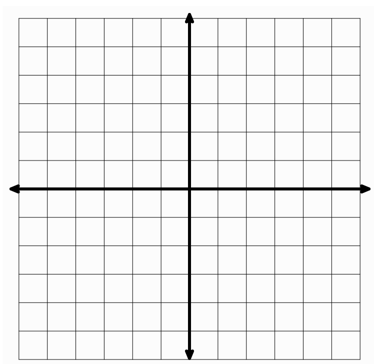
Name: Connecting Algebra and Geometry | 7.4

Set

You are given the equation of *f(x)* and the transformation *g(x) = f(x) + k.* Graph both *f* (*x*) and *g* (*x*) *and* the linear equation for *g(x)* below the graph.

5. $f(x) = 2x - 4$

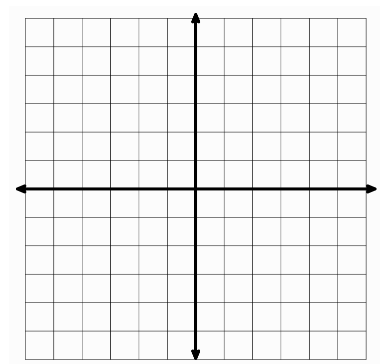
$g(x) = f(x) + 3$



$g(x) =$ ____________________

6. $f(x) = 0.5x$

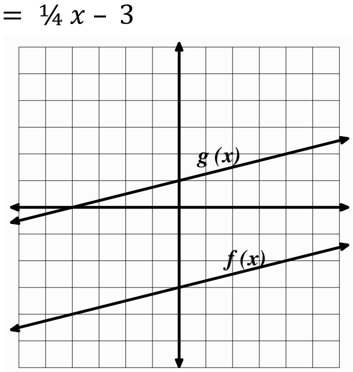
$g(x) = f(x) - 3$



$g(x) =$ ____________________

Based on the given graph, write the equation of *g(x)* in the form of *g(x) = f(x) + k.* Then simplify the equation of *g(x)* into slope-intercept form. The equation of *f(x)* is given.

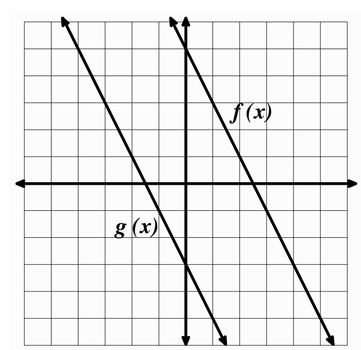
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x) =$ ____________________
Translation form

b. $g(x) =$ ____________________
Slope-Intercept form

8. $f(x) = -2x + 5$



a. $g(x) =$ ____________________
Translation form

b. $g(x) =$ ____________________
Slope-Intercept form

Go

9. Fernando and Mariah are training for a half marathon. The chart below describes their workout for the week just before the half marathon. If four laps are equal to one mile, and if there are 13.1 miles in a half marathon, do you think Mariah and Fernando are prepared for the event? Describe how you think each person will perform in the race. Include who you think will finish first and what each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

7.5 Training Day Part II

A Practice Understanding Task

http://www.flickr.com/photos/pdgoodman

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

Week 2: Mariah had completed 15 more laps than Fernando before they trained on Saturday.

a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: $m(t) = f(t)$ ________

Week 3: On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be $f(t) = m(t) + 30$."

a. What does Fernando's statement mean?

b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

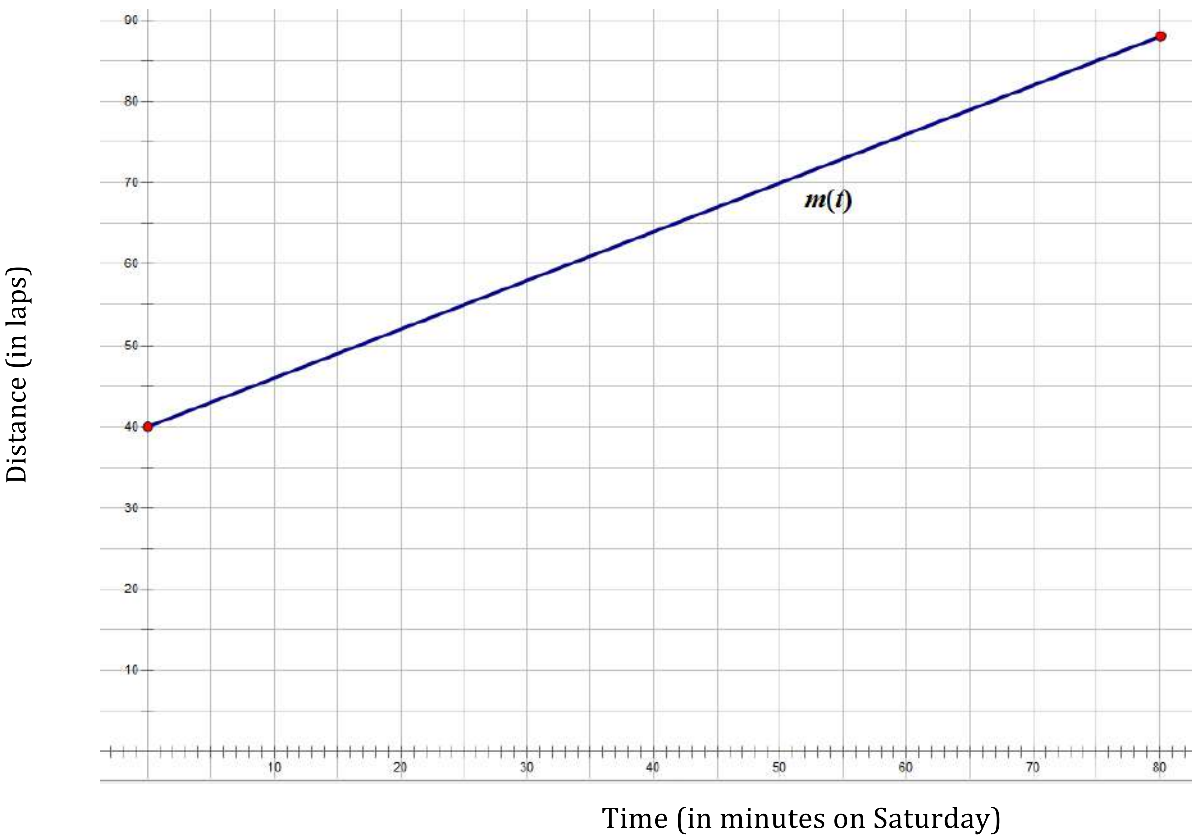
c. Write the equation for both runners:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

Week 4: The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. $f(t) = m(t) - 10$



b. Write the equations for both runners.
c. What do you notice about the two graphs? Would this always be true if one person ran "*k*" laps more or less each week?

Week 5: This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

a. Write the equation for Mariah given that they run at the same speed that they have every week.
b. Write Fernando's equation as a transformation of Mariah's equation.

What conjectures can you make about the general statement: "*g(x) = f(x)+k*" when it comes to linear functions?

Name: Connecting Algebra and Geometry | 7.5

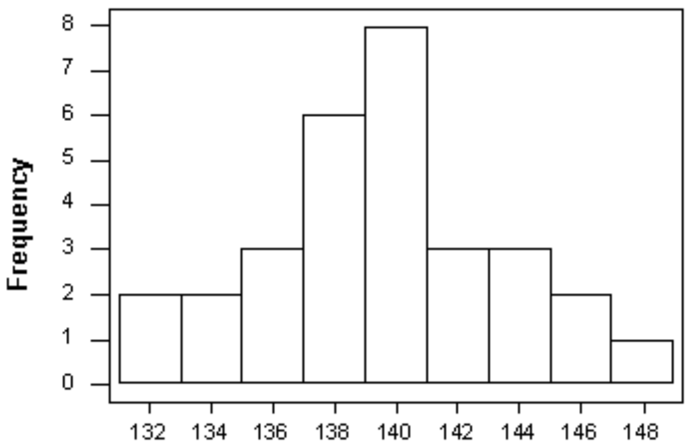
Ready, Set, Go!

http://www.flickr.com/photos/pdgoodman

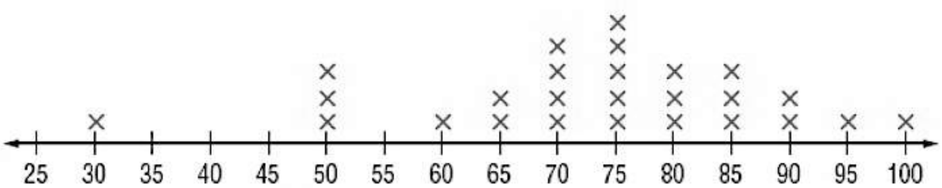
Ready

Topic: Identifying spread.

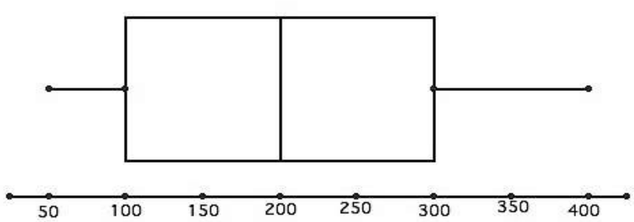
1. Describe the spread in the histogram below.



2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



Name: Connecting Algebra and Geometry | 7.5

Set

You are given information about $f(x)$ and $g(x)$. Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$)

4. $f(x) = 7x + 13$
$g(x) = 7x - 5$

$g(x) =$ ____________________
Translation form

5. $f(x) = 22x - 12$
$g(x) = 22x + 213$

$g(x) =$ ____________________
Translation form

6. $f(x) = -15x + 305$
$g(x) = -15x - 11$

$g(x) =$ ____________________
Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x) =$ ____________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x) =$ ____________________
Translation form

9.

x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x) =$ ____________________
Translation form

Go

Topic: Vertical and horizontal translations

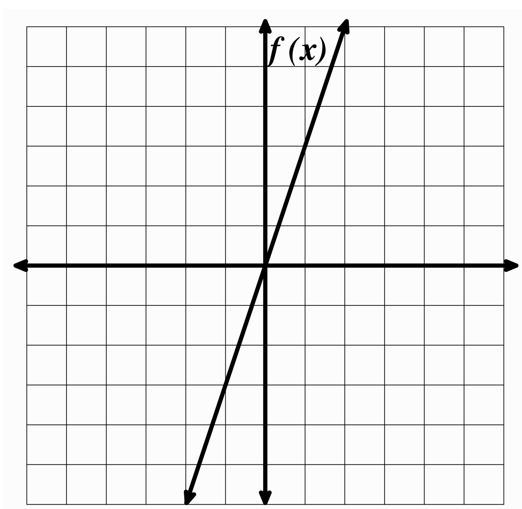
10. Use the graph of $f(x) = 3x$ to answer the following questions.

a. Sketch the graph of $g(x) = 3x - 2$ on the same grid.

b. Sketch the graph of $h(x) = 3(x - 2)$.

c. Describe how $f(x)$, $g(x)$, and $h(x)$ are different and how they are the same.

d. Explain in what way the parentheses affect the graph. Why do you think this is so?



7.6 Shifting Functions

A Practice Understanding Task

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth $20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

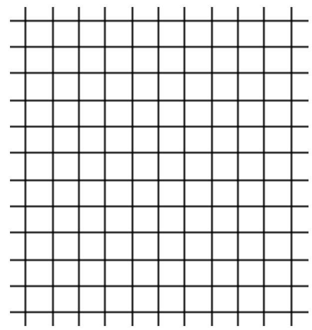
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

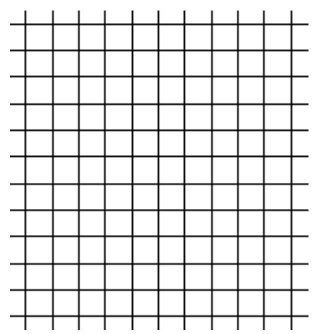
3. $If\ \ g(x) = 3(2)^x\ and\ \ f(x) =\ g(x) - 5, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



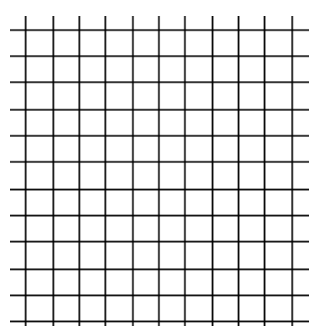
4. $If\ \ g(x) = 4(.5)^x\ and\ \ f(x) =\ g(x) + 3,\ then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



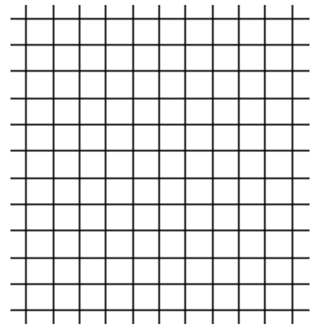
5. $If\ \ g(x) = 4x + 3\ and\ \ f(x) =\ g(x) + 7, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



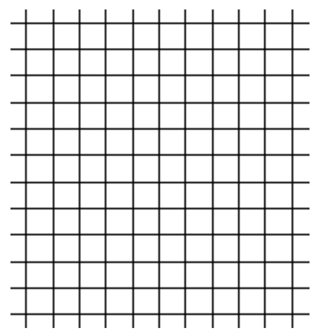
6. $If\ g(x) = 2x + 1\ and\ f(x) = g(x) - 4, then\ f(x) =$ ______________________________

x				
f(x)				
g(x)				



7. $If\ g(x) = -x\ and\ f(x) = g(x) + 3, then\ f(x) =$ ______________________________

x				
f(x)				
g(x)				



Part III: Communicate your understanding.

8. If *f(x) = g(x) + k,* describe the relationship between *f(x)* and *g(x).* Support your answers with tables and graphs.

Name: Connecting Algebra and Geometry | 7.6

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Ready, Set, Go!

Ready

Topic: Finding percentages.

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
 a.How many girls sang soprano?

 b. What percent of the entire chorus sang soprano?

4. Only 30% of the boys could sing bass.
 a. How many boys were in the bass section?

 b. What percent of the entire chorus sang bass?

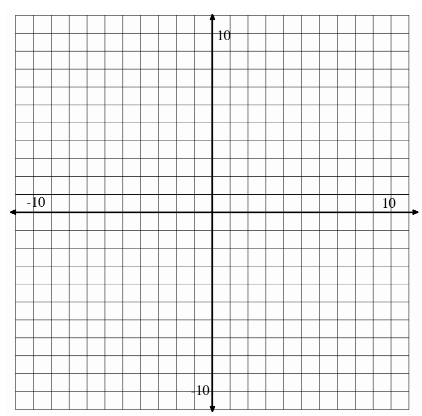
5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

Set

Topic: Graphing exponential equations

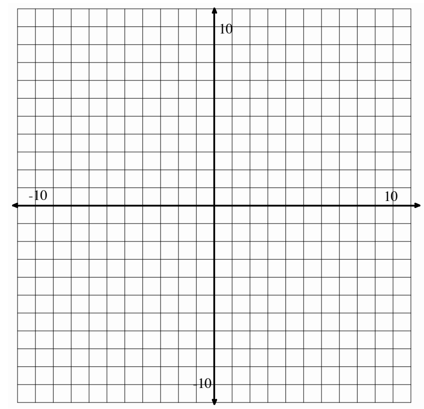
6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.
 a. Predict what you think is the same and what is different.

 b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed.



7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$

a. Predict what you think is the same and what is different.



b. Use your calculator to graph both equations on the same grid.
Explain what stayed the same and what changed.
Identify in what way it changed.

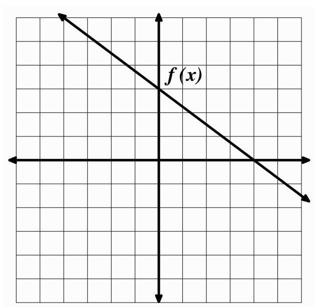
Go

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

a.

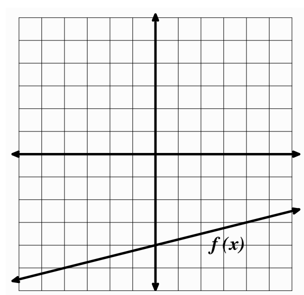


b. $f(x) =$ ________________

c. $g(x) =$ ________________
Slope-Intercept form

9. $g(x) = f(x) + 4$

a.

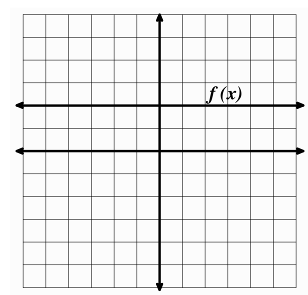


b. $f(x) =$ ________________

c. $g(x) =$ ________________
Slope-Intercept form

10. $g(x) = f(x) - 6$

a.



b. $f(x) =$ ________________

c. $g(x) =$ ________________
Slope-Intercept form

Need Help? Check out these related videos:

http://www.khanacademy.org/math/arithmetic/percents/v/identifying-percent-amount-and-base

7.7H The Arithmetic of Vectors

A Solidify Understanding Task

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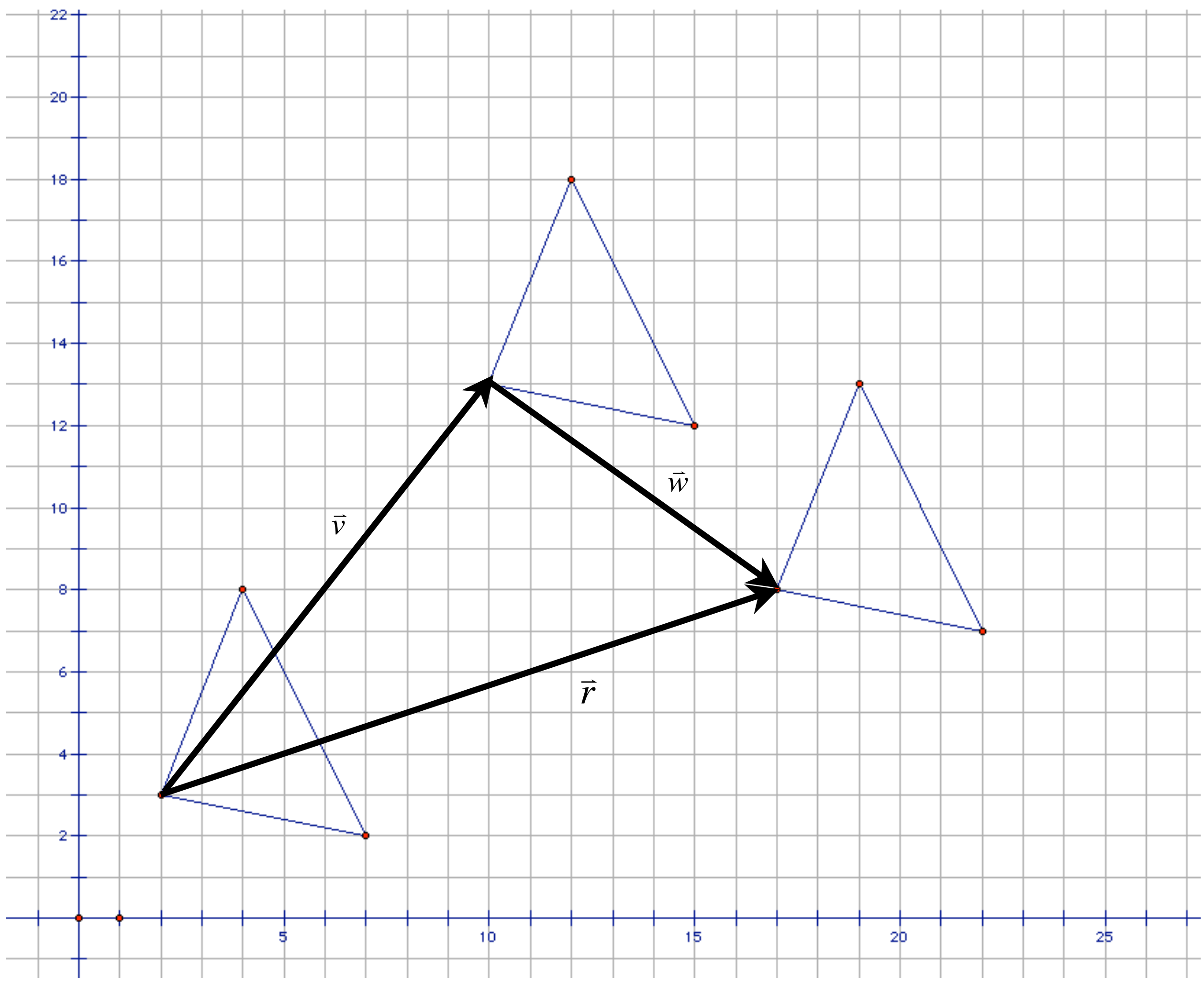
The following diagram shows a triangle that has been translated to a new location, and then translated again. Arrows have been used to indicate the movement of one of the vertex points through each translation. The result of the two translations can also be thought of as a single translation, as shown by the third arrow in the diagram.



Draw arrows to show the movement of the other two vertices through the sequence of translations, and then draw an arrow to represent the resultant single translation. What do you notice about each set of arrows?

A **vector** is a quantity that has both **magnitude** and **direction**. The arrows we drew on the diagram represent both translations as vectors—each translation has *magnitude* (the distance moved) and *direction* (the direction in which the object is moved). Arrows, or *directed line segments*, are one way of representing a vector.

Addition of Vectors

1. In the example above, two vectors $\vec{v}$ and $\vec{w}$ were combined to form vector $\vec{r}$. This is what is meant by "adding vectors". Study each of the following methods for adding vectors, then try each method to add vectors $\vec{s}$ and $\vec{t}$ given in the diagram below to find $\vec{q}$, such that $\vec{s} + \vec{t} = \vec{q}$.

2. Explain why each of these methods gives the same result.

Method 1: *End-to-end*

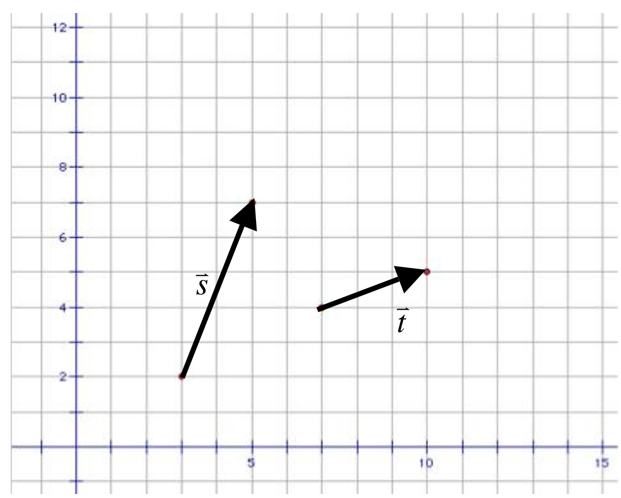
The diagram given above illustrates the end-to-end strategy of adding two vectors to get a resultant vector that represents the sum of the two vectors. In this case, the resulting vector shows that a single translation could accomplish the same movement as the combined sum of the two individual translations, that is $\vec{v} + \vec{w} = \vec{r}$.

Method 2: *The parallelogram rule*

Since we can relocate the arrow representing a vector, draw both vectors starting at a common point. Often both vectors are relocated so they have their *tail* ends at the origin. These arrows form two sides of a parallelogram. Draw the other two sides. The resulting sum is the vector represented by the arrow drawn from the common starting point (for example, the origin) to the opposite vertex of the parallelogram. Question to think about: How can you determine where to put the missing vertex point of the parallelogram?

Method 3: *Using horizontal and vertical components*

Each vector consists of a horizontal component and a vertical component. For example, vector $\vec{v}$ can be thought of as a movement of 8 units horizontally and 13 units vertically. This is represented with the notation $\langle 8, 13 \rangle$. Vector $\vec{w}$ consists of a movement of 7 units horizontally and -5 units vertically, represented by the notation $\langle 7, -5 \rangle$. Question to think about: How can the components of the individual vectors be combined to determine the horizontal and vertical components of the resulting vector $\vec{r}$?



3. Examine vector $\vec{s}$ given above. While we can relocate the vector, in the diagram the *tail* of the vector is located at (3, 2) and the *head* of the vector is located at (5, 7). Explain how you can determine the horizontal and vertical components of a vector from just the coordinates of the point at the tail and the point at the head of the vector? That is, how can we find the horizontal and vertical components of movement without counting across and up the grid?

Magnitude of Vectors

The symbol $\|\vec{v}\|$ is used to denote the magnitude of the vector, in this case the length of the vector.

Devise a method for finding the magnitude of a vector and use your method to find the following. Be prepared to describe your method for finding the magnitude of a vector.

4. $\|\vec{v}\|$

5. $\|\vec{w}\|$

6. $\|\vec{v} + \vec{w}\|$

Scalar Multiples of Vectors

We can stretch a vector by multiplying the vector by a scale factor. For example, $2\vec{v}$ represents the vector that has the same direction as $\vec{v}$, but whose magnitude is twice that of $\vec{v}$.

Draw each of the following vectors on a coordinate graph:

7. $3\vec{s}$

8. $-2\vec{t}$

9. $3\vec{s} + (-2\vec{t})$

10. $3\vec{s} - 2\vec{t}$

Other Applications of Vectors

We have illustrated the concept of a vector using translation vectors in which the magnitude of the vector represents the distance a point gets translated. There are other quantities that have magnitude and direction, but the magnitude of the vector does not always represent length.

For example, a car traveling 55 miles per hour along a straight stretch of highway can be represented by a vector since the speed of the car has magnitude, 55 miles per hour, and the car is traveling in a specific direction. Pushing on an object with 25 pounds of force is another example. A vector can be used to represent this push since the force of the push has magnitude, 25 pounds of force, and the push would be in a specific direction.

10. A swimmer is swimming across a river with a speed of 20 ft/sec and at a 45° angle from the bank of the river. The river is flowing at a speed of 5 ft/sec. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the two vectors representing the motion of the swimmer and the flow of the river.

11. Two teams are participating in a tug-of-war. One team exerts a combined force of 200 pounds in one direction while the other team exerts a combined force of 150 pounds in the other direction. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the vectors representing the efforts of the two teams.

Ready, Set, Go!

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Ready

Topic: Solving Equations Using Properties of Arithmetic

1. Here are the steps Zac used to solve the following equation. State or describe the properties of arithmetic or the properties of equality he is using in each step.

$2(x + 5) + 7x = 4x + 15$	*the distributive property*	$9x - 4x = 4x + 5 - 4x$	i.
$(2x + 10) + 7x = 4x + 15$	a.	$(9 - 4)x = 4x + 5 - 4x$	j.
$2x + (10 + 7x) = 4x + 15$	b.	$5x = 4x + 5 - 4x$	k.
$2x + (7x + 10) = 4x + 15$	c.	$5x = 4x - 4x + 5$	l.
$(2x + 7x) + 10 = 4x + 15$	d.	$5x = 0 + 5$	m.
$(2 + 7)x + 10 = 4x + 15$	e.	$5x = 5$	n.
$9x + 10 = 4x + 15$	f.	$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 5$	o.
$9x + 10 - 10 = 4x + 15 - 10$	g.	$1x = 1$	p.
$9x + 0 = 4x + 5$	h.	$x = 1$	q.
$9x = 4x + 5$			

Solve each of the following equations for *x*, carefully recording each step. Then state or describe the properties of arithmetic (for example, the distributive property, or the associative property of multiplication, etc.) or properties of equality (for example, the addition property of equality) that justify each step.

2. $2(3x + 5) = 4(2x - 1)$

3. $\frac{4}{5}x + 3 = 2x - 1$

Set

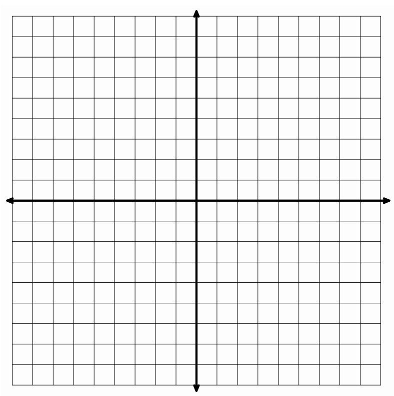
Topic: Adding vectors

Two vectors are described in component form in the following way:

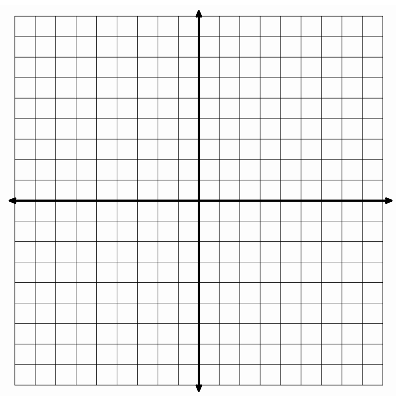
$$\vec{v} : \langle -2 , 3 \rangle \text{ and } \vec{w} : \langle 3 , 4 \rangle$$

On the grids below, create vector diagrams to show:

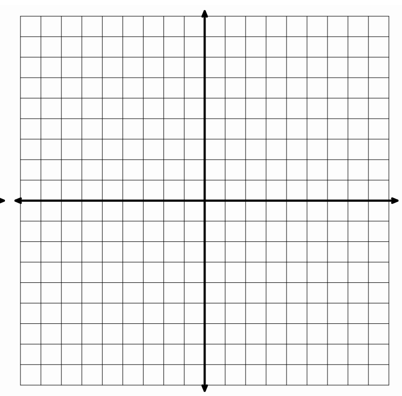
4. $\vec{v} + \vec{w} =$



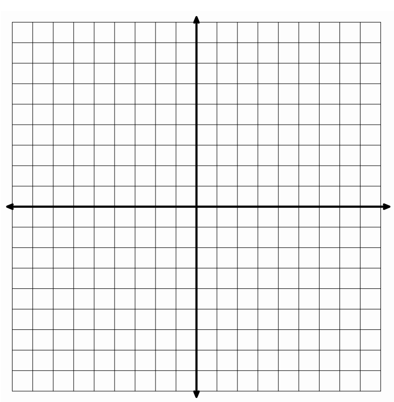
5. $\vec{v} - \vec{w} =$



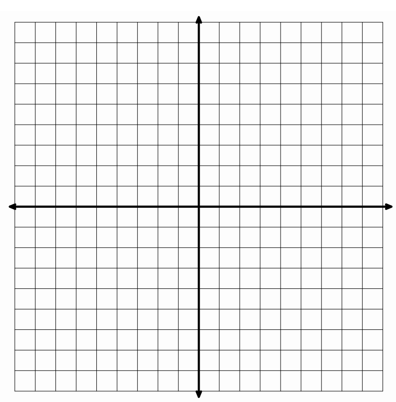
6. $3\vec{v} =$



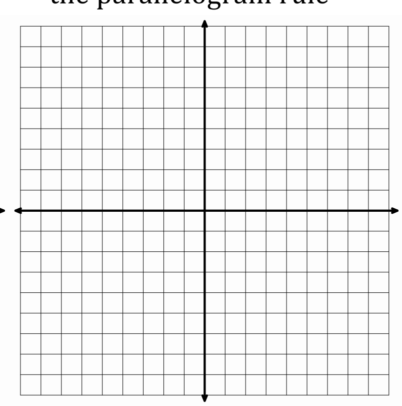
7. $-2\vec{w} =$



8. $3\vec{v} - 2\vec{w} =$



9. Show how to find $\vec{v} + \vec{w}$ using the parallelogram rule



Go

Topic: The arithmetic of matrices

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix},\ B = \begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix},\ \textbf{and}\ C = \begin{bmatrix} 4 & 2 & -1 \\ 5 & 2 & 3 \end{bmatrix}$$

Find the following sums, differences or products, as indicated. If the sum, difference or product is undefined, explain why.

10. $A + B$

11. $A + C$

12. $2A - B$

13. $A \cdot B$

14. $B \cdot A$

15. $A \cdot C$

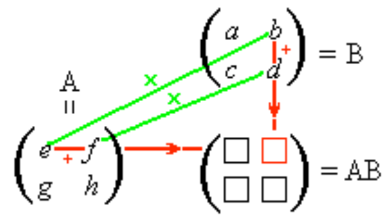
16. $C \cdot A$

Need Help? Check out these related videos:

http://www.khanacademy.org/science/physics/mechanics/v/visualizing-vectors-in-2-dimensions

7.8 H More Arithmetic of Matrices

A Solidify Understanding Task



http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

In this task you will have an opportunity to examine some of the properties of matrix addition and matrix multiplication. We will restrict this work to square 2×2 matrices.

The table below defines and illustrates several properties of addition and multiplication for real numbers and asks you to determine if these same properties hold for matrix addition and matrix multiplication. While the chart asks for a single example for each property, you should experiment with matrices until you are convinced that the property holds or you have found a counter-example to show that the property does not hold. Can you base your justification on more that just trying out several examples?

Property	Example with Real Numbers	Example with Matrices
Associative Property of Addition $(a + b) + c = a + (b + c)$		
Associative Property of Multiplication $(ab)c = a(bc)$		
Commutative Property of Addition $a + b = b + a$		
Commutative Property of Multiplication $ab = ba$		
Distributive Property of Multiplication Over Addition $a(b + c) = ab + ac$		

In addition to the properties listed in the table above, addition and multiplication of real numbers include properties related to the numbers 0 and 1. For example, the number 0 is referred to as the *additive identity* because $a + 0 = 0 + a = a$, and the number 1 is referred to as the *multiplicative identity* since $a \cdot 1 = 1 \cdot a = a$. Once the additive and multiplicative identities have been identified, we can then define additive inverses a and $-a$ since $a + -a = 0$, and multiplicative inverses a and $1/a$ since $a \cdot \frac{1}{a} = 1$. To decide if these properties hold for matrix operations, we will need to determine if there is a matrix that plays the role of 0 for matrix addition, and if there is a matrix that plays the role of 1 for matrix multiplication.

<u>The Additive Identity Matrix</u>

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of 0, or the additive identity matrix, for the following matrix addition. Will this same matrix work as the additive identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

<u>The Multiplicative Identity Matrix</u>

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of 1, or the multiplicative identity matrix, for the following matrix multiplication. Will this same matrix work as the multiplicative identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Now that we have identified the additive identity and multiplicative identity for 2×2 matrices, we can search for additive inverses and multiplicative inverses of given matrices.

Finding an Additive Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of the additive inverse of the first matrix. Will this same process work for finding the additive inverse of all 2×2 matrices?

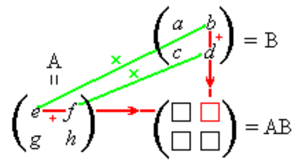
$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding a Multiplicative Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of the multiplicative inverse of the first matrix. Will this same process work for finding the multiplicative inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Ready, Set, Go!



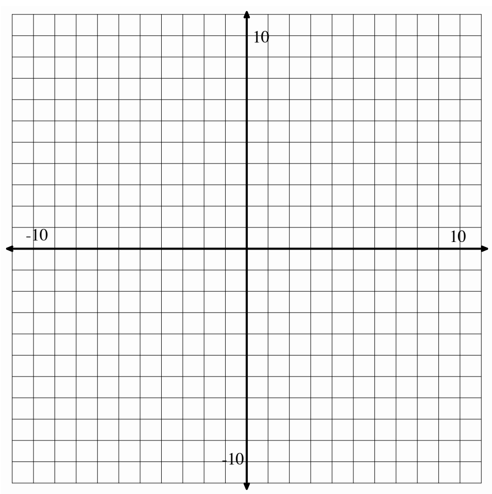
http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

Ready

Topic: Solving systems of linear equations

1. Solve the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

a. By Graphing:



b. By substitution:

c. By elimination:

Set

Topic: Inverse matrices

2. Given: Matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

a. Find the additive inverse of matrix A

b. Find the multiplicative inverse of matrix A

3. Given: Matrix $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

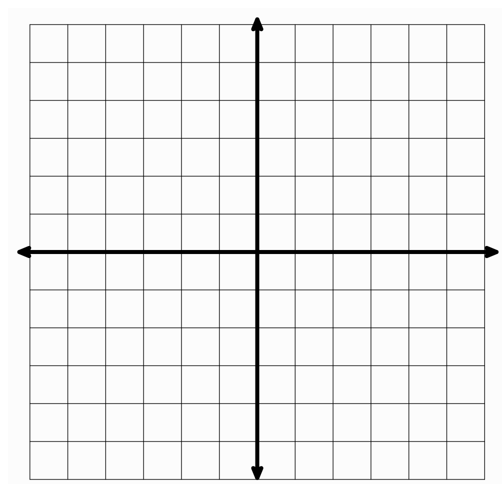
a. Find the additive inverse of matrix *B*.

b. Find the multiplicative inverse of matrix *B*

Go

Topic: Parallel lines, perpendicular lines and length from a coordinate geometry perspective

Given the four points: *A* (2, 1), *B* (5, 2), *C* (4, 5), and *D* (1, 4)



4. Is *ABCD* a parallelogram? Provide convincing evidence for your answer.

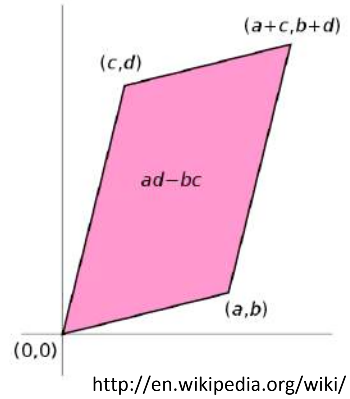
5. Is *ABCD* a rectangle? Provide convincing evidence for your answer.

6. Is *ABCD* a rhombus? Provide convincing evidence for your answer.

7. Is *ABCD* a square? Provide convincing evidence for your answer.

7.9H The Determinant of a Matrix

A Solidify Understanding Task



http://en.wikipedia.org/wiki/File:Area_parallellogram_as_determinant.svg

In the previous task we learned how to find the multiplicative inverse of a matrix. Use that process to find the multiplicative inverse of the following two matrices.

1. $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. Were you able to find the multiplicative inverse for both matrices?

There is a number associated with every square matrix called the **determinant**. If the determinant is not equal to zero, then the matrix has a multiplicative inverse.

For a 2×2 matrix the determinant can be found using the following rule: (note: the vertical lines, rather than the square brackets, are used to indicate that we are finding the determinant of the matrix)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

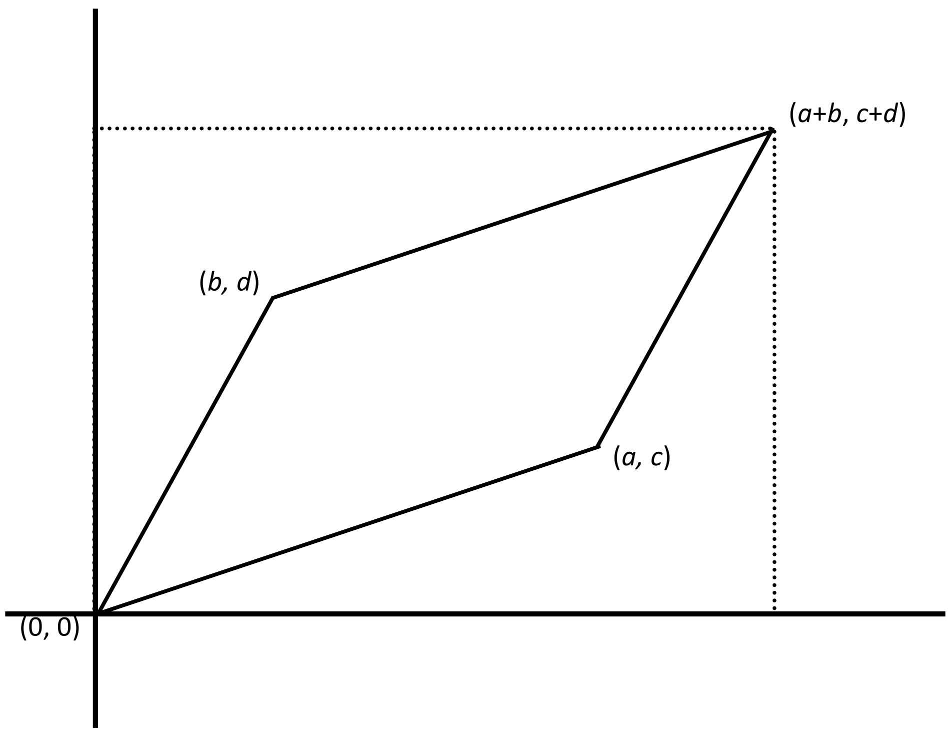
4. Using this rule, find the determinant of the two matrices given in problems 1 and 2 above.

The absolute value of the determinant of a 2×2 matrix can be visualized as the area of a parallelogram, constructed as follows.

- Draw one side of the parallelogram with endpoints at (0, 0) and (a, c).
- Draw a second side of the parallelogram with endpoints at (0, 0) and (b, d).
- Locate the fourth vertex that completes the parallelogram.

(Note that the elements in the columns of the matrix are used to define the endpoints of the vectors that form two sides of the parallelogram.)

5. Use the following diagram to show that the area of the parallelogram is given by $ad - bc$.



6. Draw the parallelograms whose areas represent the determinants of the two matrices listed in questions 1 and 2 above. How does a zero determinant show up in these diagrams?

7. Create a matrix for which the determinant will be negative. Draw the parallelogram associated with the determinant of your matrix and find the area of the parallelogram.

The determinant can be used to provide an alternative method for finding the inverse of 2×2 matrix.

8. Use the process you used previously to find the inverse of a generic 2×2 matrix whose elements are given by the variables *a, b, c* and *d*. For now, we will refer to the elements of the inverse matrix as M_1, M_2, M_3 and M_4 as illustrated in the following matrix equation. Find expressions for M_1, M_2, M_3 and M_4 in terms of the elements of the first matrix, *a, b, c* and *d*.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$M_1 =$

$M_2 =$

$M_3 =$

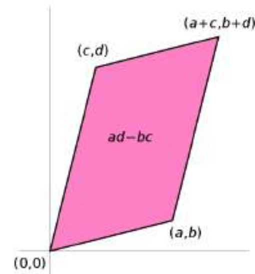
$M_4 =$

Use your work above to explain this strategy for finding the inverse of a 2×2 matrix: (note: the $^{-1}$ superscript is used to indicate that we are finding the multiplicative inverse of the matrix)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ where $ad - bc$ is the determinant of the matrix

Name: Connecting Algebra and Geometry | 7.9H

Ready, Set, Go!



http://en.wikipedia.org/wiki/File:Area_parallellogram_as_determinant.svg

Ready

Topic: Solving systems of linear equations using row reduction

Given the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

1. Zac started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -5 & -17 \\ 2 & 1 & 10 \end{bmatrix}$

Describe what Zac did to get from the matrix on the left to the matrix on the right.

2. Lea started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 3 \\ 1 & \frac{1}{2} & 5 \end{bmatrix}$

Describe what Lea did to get from the matrix on the left to the matrix on the right.

3. Using either Zac's or Lea's first step, continue solving the system using row reduction. Show each matrix along with notation indicating how you got from one matrix to another. Be sure to check your solution.

Set

Topic: The determinant of a 2 × 2 matrix

4. Use the determinant of each 2 × 2 matrix to decide which matrices have multiplicative inverses, and which do not.

a. $\begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

5. Find the multiplicative inverse of each of the matrices in 4, provided the inverse matrix exists.

a.

b.

c.

6. Generally matrix multiplication is not commutative. That is, if A and B are matrices, typically $A \cdot B \neq B \cdot A$. However, multiplication of inverse matrices is commutative. Test this out by showing that the pairs of inverse matrices you found in question 7 give the same result when multiplied in either order.

Go

Topic: Parallel and perpendicular lines

Determine if the following pairs of lines are parallel, perpendicular or neither. Explain how you arrived at your answer.

7. $3x + 2y = 7$ and $6x + 4y = 9$

8. $y = \frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 7$

9. $y = \frac{3}{4}x - 2$ and $4x + 3y = 3$

10. Write the equation of a line that is parallel to $y = \frac{4}{5}x - 2$ and has a y-intercept at (0, 4).

11. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 3$ and passes through the point (2, 5).

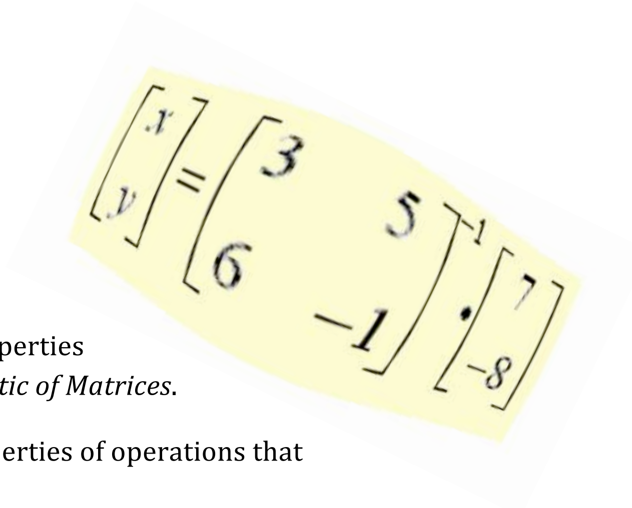
12. Write the equation of a line that is parallel to $y = -\frac{2}{3}x + 3$ and passes through the point (2, 5).

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/equations-of-parallel-and-perpendicular-lines

7.10H Solving Systems with Matrices, Revisited

A Solidify Understanding Task



When you solve linear equations, you use many of the properties of operations that were revisited in the task *More Arithmetic of Matrices*.

1. Solve the following equation for *x* and list the properties of operations that you use during the equation solving process.

$$\frac{2}{3}x = 8$$

2. The list of properties you used to solve this equation probably included the use of a multiplicative inverse and the multiplicative identity property. If you didn't specifically list those properties, go back and identify where they might show up in the equation solving process for this particular equation.

Systems of linear equations can be represented with matrix equations that can be solved using the same properties that are used to solve the above equation. First, we need to recognize how a matrix equation can represent a system of linear equations.

3. Write the linear system of equations that is represented by the following matrix equation. (Think about the procedure for multiplying matrices you developed in previous tasks.)

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

4. Using the relationships you noticed in question 3, write the matrix equation that represents the following system of equations.

$$\begin{cases} 2x + 3y = 14 \\ 3x + 4y = 20 \end{cases}$$

5. The rational numbers $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses. What is the multiplicative inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$? Note: The inverse matrix is usually denoted by $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$.

6. The following table lists the steps you may have used to solve $\frac{2}{3}x = 8$ and asks you to apply those same steps to the matrix equation you wrote in question 4. Complete the table using these same steps.

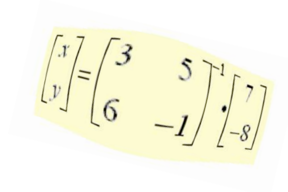
Original equation	$\frac{2}{3}x = 8$	$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$
Multiply both sides of the equation by the multiplicative inverse	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$	
The product of multiplicative inverses is the multiplicative identity on the left side of the equation	$1 \cdot x = \frac{3}{2} \cdot 8$	
Perform the indicated multiplication on the right side of the equation	$1 \cdot x = 12$	
Apply the property of the multiplicative identity on the left side of the equation	$x = 12$	

7. What does the last line in the table in question 6 tell you about the system of equations in question 4?

8. Use the process you have just examined to solve the following system of linear equations.

$$\begin{cases} 3x + 5y = -1 \\ 2x + 4y = 4 \end{cases}$$

Ready, Set, Go!



Ready

Topic: Reflections and rotations

1. The following three points form the vertices of a triangle: (3, 2), (6, 1), (4, 3)

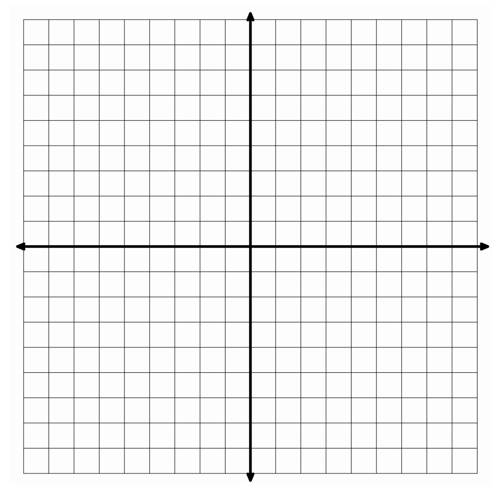
a. Plot these three points on the coordinate grid and then connect them to form a triangle.

b. Reflect the original triangle over the *y*-axis and record the coordinates of the vertices here:

c. Reflect the original triangle over the *x*-axis and record the coordinates of the vertices here:

d. Rotate the original triangle 90° counter-clockwise about the origin and record the coordinates of the vertices here:

e. Rotate the original triangle 180° about the origin and record the coordinates of the vertices here.



Set

Topic: Solving systems using inverse matrices

Two of the following systems have unique solutions (that is, the lines intersect at a single point).

2. Use the determinant of a 2 × 2 matrix to decide which systems have unique solutions, and which one does not.

a. $\begin{cases} 8x - 2y = -2 \\ 4x + y = 5 \end{cases}$

b. $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = -5 \end{cases}$

c. $\begin{cases} 4x + 2y = 0 \\ 3x + y = 2 \end{cases}$

3. For each of the systems in #2 which have a unique solution, find the solution to the system by solving a matrix equation using an *inverse matrix*.

a. b. c.

Go

Topic: Properties of arithmetic

Match each example on the left with the name of a property of arithmetic on the right. Not all answers will be used.

____ 4. $2(x + 3y) = 2x + 6y$

____ 5. $(2x + 3y) + 4y = 2x + (3y + 4y)$

____ 6. $2x + 3y = 3y + 2x$

____ 7. $2(3y) = (2 \cdot 3)y = 6y$

____ 8. $\frac{2}{3} \cdot \frac{3}{2} x = 1x$

____ 9. $x + -x = 0$

____ 10. $xy = yx$

a. multiplicative inverses

b. additive inverses

c. multiplicative identity

d. additive identity

e. commutative property of addition

f. commutative property of multiplication

g. associative property of addition

h. associative property of multiplication

i. distributive property of addition over multiplication

7.11H Transformations with Matrices

A Solidify Understanding Task

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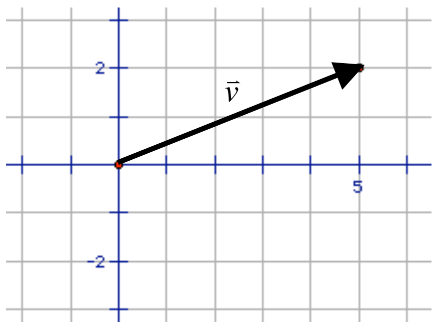
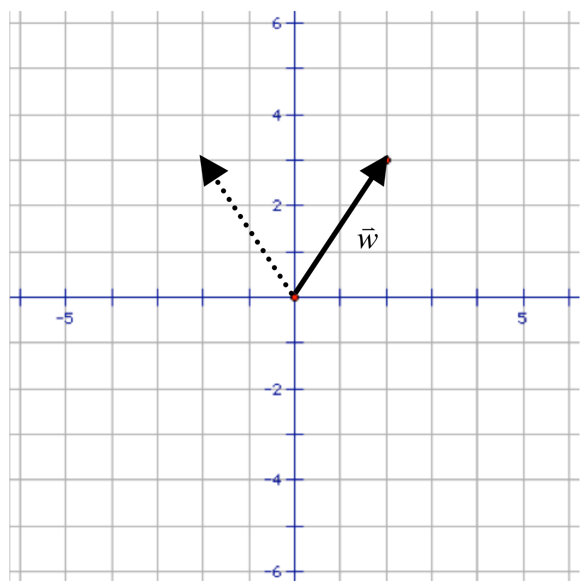
Various notations are used to denote vectors: bold-faced type, ***v***; a variable written with a harpoon over it, $\vec{v}$; or listing the horizontal and vertical components of the vector, $\langle v_x, v_y \rangle$. In this task we will represent vectors by listing their horizontal and vertical components in a matrix with a single column, $\begin{bmatrix} v_x \\ v_y \end{bmatrix}$.

1. Represent the vector labeled $\vec{v}$ in the diagram below as a matrix with one column.



Matrix multiplication can be used to transform vectors and images in a plane.

Suppose we want to reflect $\vec{w}$ over the y-axis. We can represent $\vec{w}$ with the matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and the reflected vector with the matrix $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$.



2. Find the 2×2 matrix that we can multiply the matrix representing the original vector by in order to obtain the matrix that represents the reflected vector. That is, find *a, b, c* and *d* such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

3. Find the matrix that will reflect $\vec{w}$ over the *x*-axis.

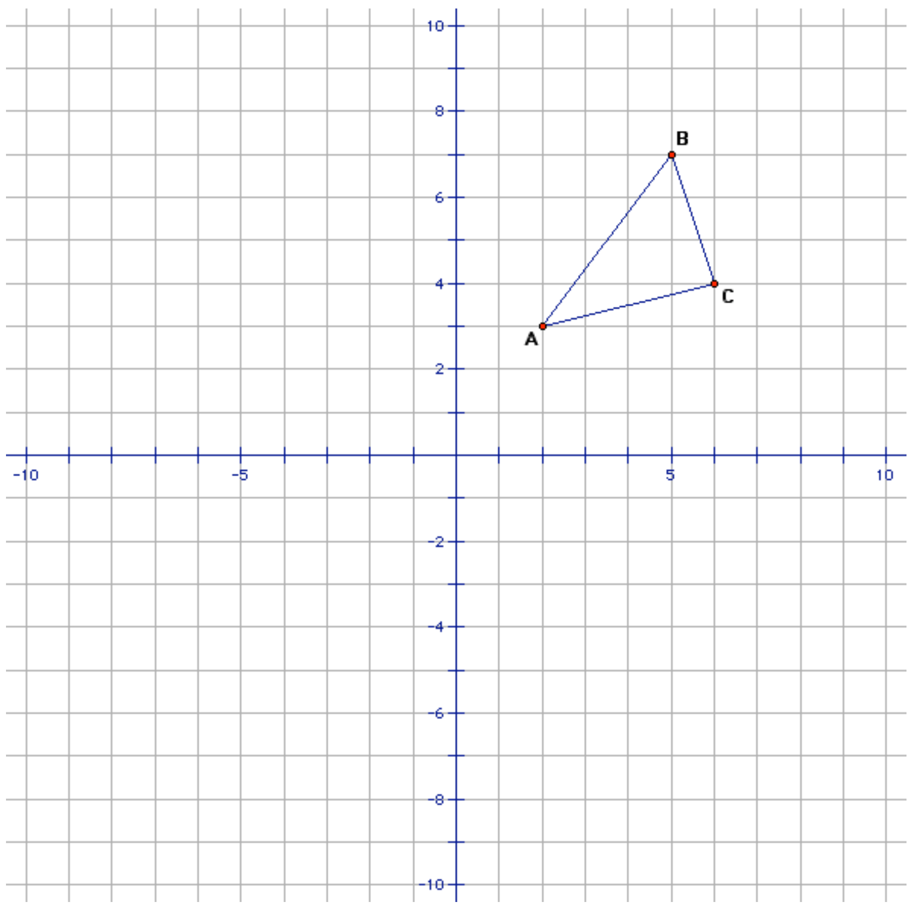
4. Find the matrix that will rotate $\vec{w}$ 90° counterclockwise about the origin.

5. Find the matrix that will rotate $\vec{w}$ 180° counterclockwise about the origin.

6. Find the matrix that will rotate $\vec{w}$ 270° counterclockwise about the origin.

7. Is there another way to obtain a rotation of 270° counterclockwise about the origin other than using the matrix found in question 6? If so, how?

We can represent polygons in the plane by listing the coordinates of its vertices as columns of a matrix. For example, the triangle below can be represented by the matrix $\begin{bmatrix} 2 & 5 & 6 \\ 3 & 7 & 4 \end{bmatrix}$.



8. Multiply this matrix, which represents the vertices of ΔABC, by the matrix found in question 2. Interpret the product matrix as representing the coordinates of the vertices of another triangle in the plane. Plot these points and sketch the triangle. How is this new triangle related to the original triangle?

9. How might you find the coordinates of the triangle that is formed after ΔABC is rotated 90° counterclockwise about the origin using matrix multiplication? Find the coordinates of the rotated triangle.

10. How might you find the coordinates of the triangle that is formed after ΔABC is reflected over the x-axis using matrix multiplication? Find the coordinates of the reflected triangle.

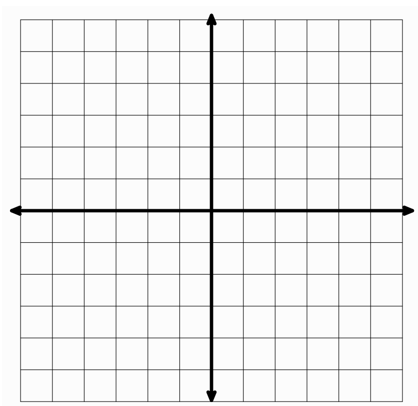
Ready, Set, Go!

Ready

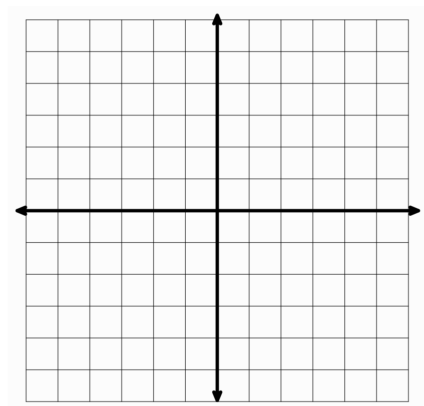
Topic: Adding vectors

Given vectors $\vec{v}:\langle -2,\ 4\rangle$ **and** $\vec{w}:\langle 5,-2\rangle$**, find the following using the parallelogram rule:**

1. $\vec{v}+\vec{w}=$



2. $\vec{v}-\vec{w}=$



Name: Connecting Algebra and Geometry | 7.11H

Set

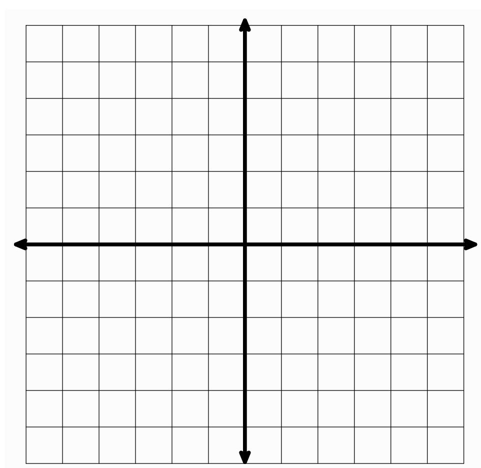
Topic: Matrices and transformations of the plane

3. List the coordinates of the four vertices of the parallelogram you drew in question 1 as a matrix. The x-values will be in the left column, and the y-values will be in the right column.

$$\begin{matrix} & \begin{matrix} x & \quad & y \end{matrix} \\ \begin{matrix} \text{Point 1} \\ \text{Point 2} \\ \text{Point 3} \\ \text{Point 4} \end{matrix} & \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \\ & & \end{bmatrix} \end{matrix}$$

4. Multiply the matrix you wrote in question 3 by the following matrix: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

5. Plot the original parallelogram. Then, using the ordered pairs from your answer in question 4, using the points from the matrix in number 4 on the following coordinate grid. Connect those 4 points. What transformation occurred between your original parallelogram and the new one?



Go

Topic: Transformations of functions

Function $f(x)$ is defined by the following table below:

x	2	4	6	8	10	12	14	16
$f(x)$	-8	-3	2	7	12	17	22	27
$g(x)$								
$h(x)$								

6. Write an equation for $f(x)$.

7a. Fill in the values for $g(x)$ assuming that $g(x) = f(x) + 3$

b. Write an equation for $g(x)$.

8a. Fill in the values for $h(x)$ assuming that $h(x) = 2f(x)$

b. Write an equation for $h(x)$.

7.12H Plane Geometry

A Practice Understanding Task

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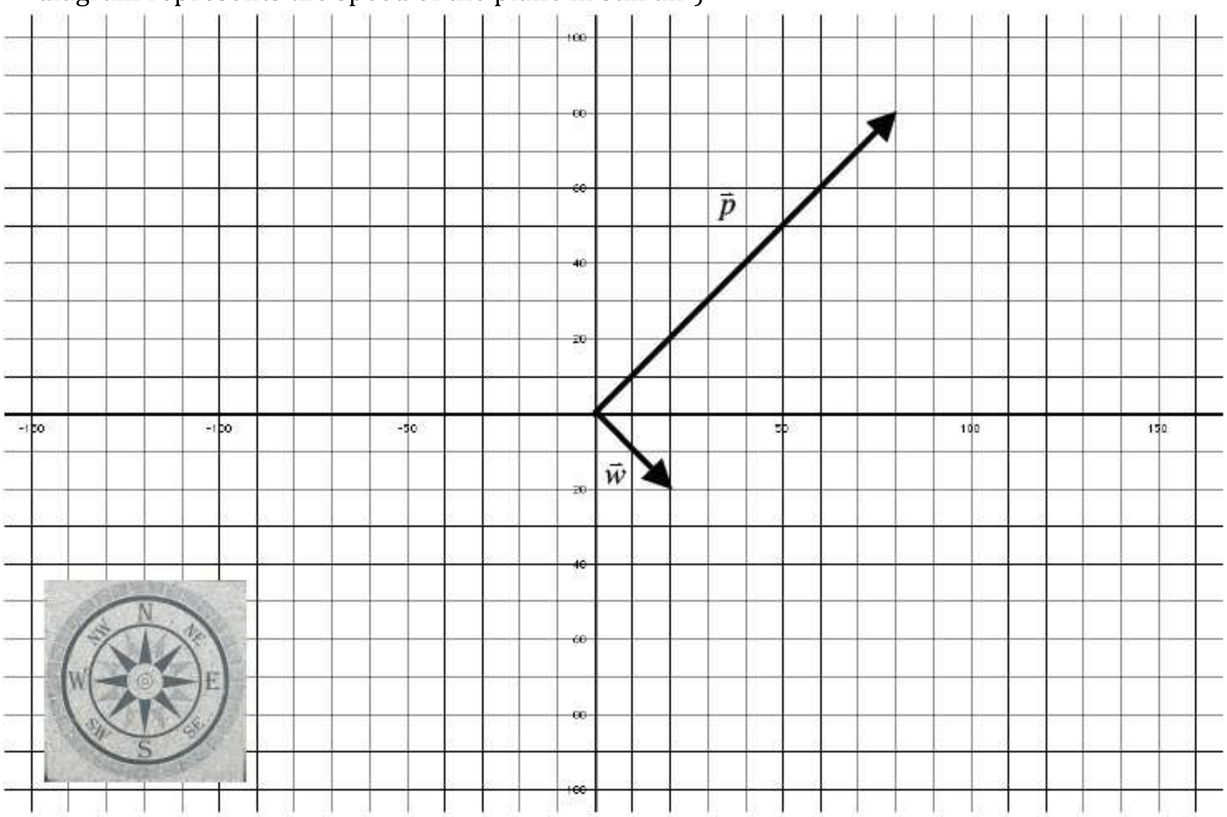
Jon's father is a pilot and he is using vector diagrams to explain some principles of flight to Jon. His father has drawn the following diagram to represent a plane that is being blown off course by a strong wind. The plane is heading northeast as represented by $\vec{p}$ and the wind is blowing towards the southeast as represented by $\vec{w}$.

1. Based on this diagram, what is the plane's speed and what is the wind's speed? (The vector diagram represents the speed of the plane in still air.)



compass rose: www.flickr.com/photos/64167416@N03/7022634029/

2. Use this diagram to find the ground speed of the plane, which will result from a combination of the plane's speed and the wind's speed. Also, indicate on the diagram the direction of motion of the plane relative to the ground.

3. Jon drew a parallelogram to determine the ground speed and direction of the plane. If you have not already done so, draw Jon's parallelogram and explain how it represents the original problem situation as well as the answers to the questions asked in problem 2.

4. Write a matrix equation that will reflect the parallelogram you drew in problem 3 over the *y*-axis. Use the solution to the matrix equation to draw the resulting parallelogram.

5. Prove that the resultant figure of the reflection performed in problem 4 is a parallelogram. That is, explain how you know opposite sides of the resulting quadrilateral are parallel.

6. Find the area of the parallelogram drawn in problem 3. Explain your method for determining the area.

Ready, Set, Go!

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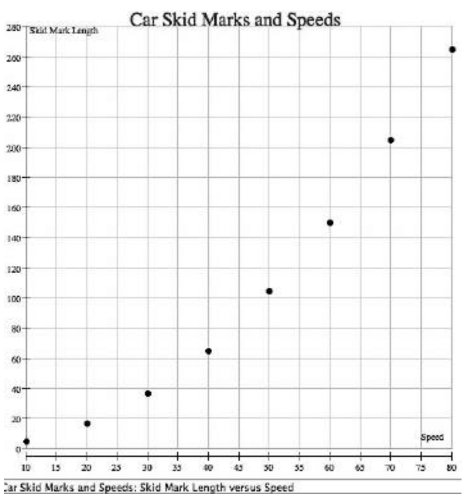
Ready

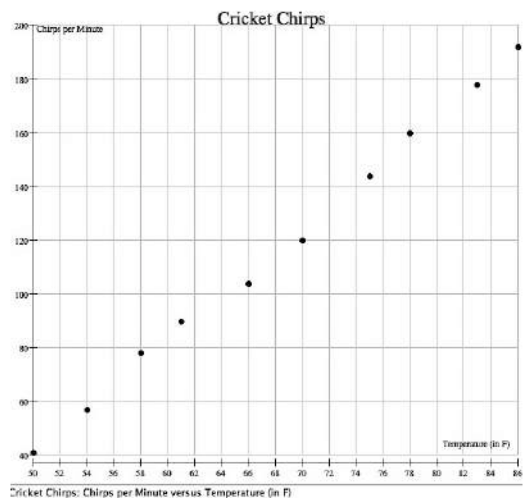
Topic: Scatterplots and trend lines

Examine each of the scatterplots shown below. If possible, make a statement about relationships between the two quantities depicted in the scatterplot.

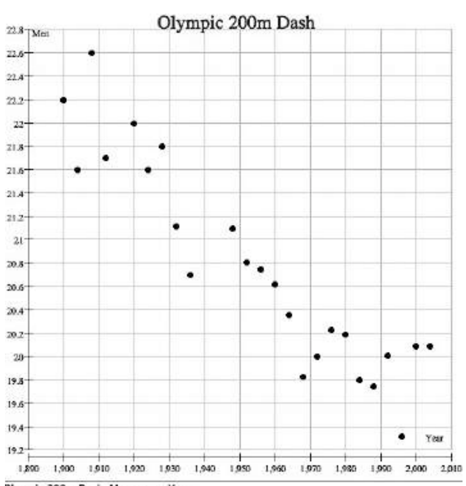
1.



2.



3.



4. For each scatterplot, write the equation of a trend line that you think best fits the data.

a. Trend line #1

b. Trend line #2

c. Trend line #3

Set

Topic: Applications of vectors

Given: $\vec{u}:\langle -5,1\rangle$, $\vec{v}:\langle 3,5\rangle$, $\vec{w}:\langle 4,-3\rangle$. **Each of these three vectors represents a force pulling on an object—such as in a three-way tug of war—with force exerted in each direction being measured in pounds.**

5. Find the magnitude of each vector. That is, how many pounds of force are being exerted on the object by each tug? (Round to the nearest hundredth)

a. $\|\vec{u}\| =$

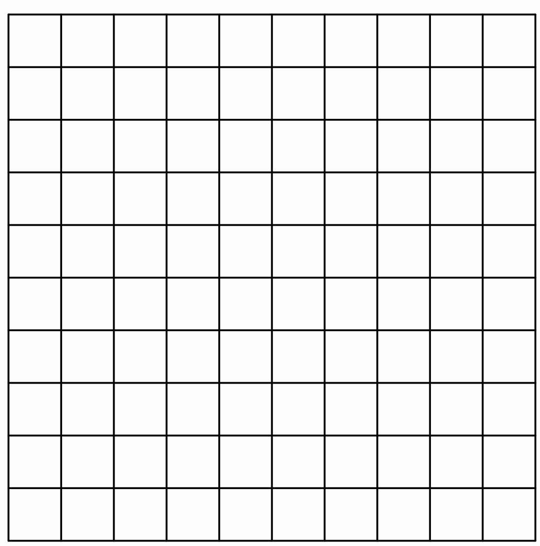
b. $\|\vec{v}\| =$

c. $\|\vec{w}\| =$

6. Find the magnitude of the sum of the three forces on the object.

$\|\vec{u}+\vec{v}+\vec{w}\| =$

7. Draw a vector diagram showing the resultant direction and magnitude of the motion resulting from this three-way tug of war.



Go

Topic: Solving systems

Given: $\begin{cases} 4x - 4y = 7 \\ 6x - 8y = 9 \end{cases}$

8. Solve the given system in each of the following ways.

a. By substitution

b. By elimination

c. Using matrix row reduction

d. Using an inverse matrix

Advanced Mathematics I
Module 8 Advanced Modeling Data

By

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Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

Module 8 – Modeling Data

Classroom Task: 8.1 Texting By the Numbers- A Solidify Understanding Task
Use context to describe data distribution and compare statistical representations **(S.ID.1, S.ID.3)**
Ready, Set, Go Homework: Modeling Data 8.1

Classroom Task: 8.2 Data Distributions – A Solidify/Practice Understanding Task
Describe data distributions and compare two or more data sets **(S.ID.1, S.ID.3)**
Ready, Set, Go Homework: Modeling Data 8.2

Classroom Task: 8.3 After School Activity – A Solidify Understanding Task
Interpret two way frequency tables **(S.ID.5)**
Ready, Set, Go Homework: Modeling Data 8.3

Classroom Task: 8.4 Relative Frequency– A Solidify Understanding Task
Use context to interpret and write conditional statements using relative frequency tables **(S.ID.5)**
Ready, Set, Go Homework: Modeling Data 8.4

Classroom Task: 8.5 Connect the Dots– A Develop Understanding Task
Develop an understanding of the value of the correlation co-efficient **(S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.5

Classroom Task: 8.6 Making More $ – A Solidify Understanding Task
Estimate correlation and lines of best fit. Compare to the calculated results of linear regressions and correlation the co-efficient **(S.ID.7, S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.6

Classroom Task: 8.7 Getting Schooled – A Solidify Understanding Task
Use linear models of data and interpret the slope and intercept of regression lines with various units **(S.ID.6, S.ID.7, S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.7

Classroom Task: 8.8 Rocking the Residuals – A Develop Understanding Task
Use residual plots to analyze the strength of a linear model for data **(S.ID.6)**
Ready, Set, Go Homework: Modeling Data 8.8

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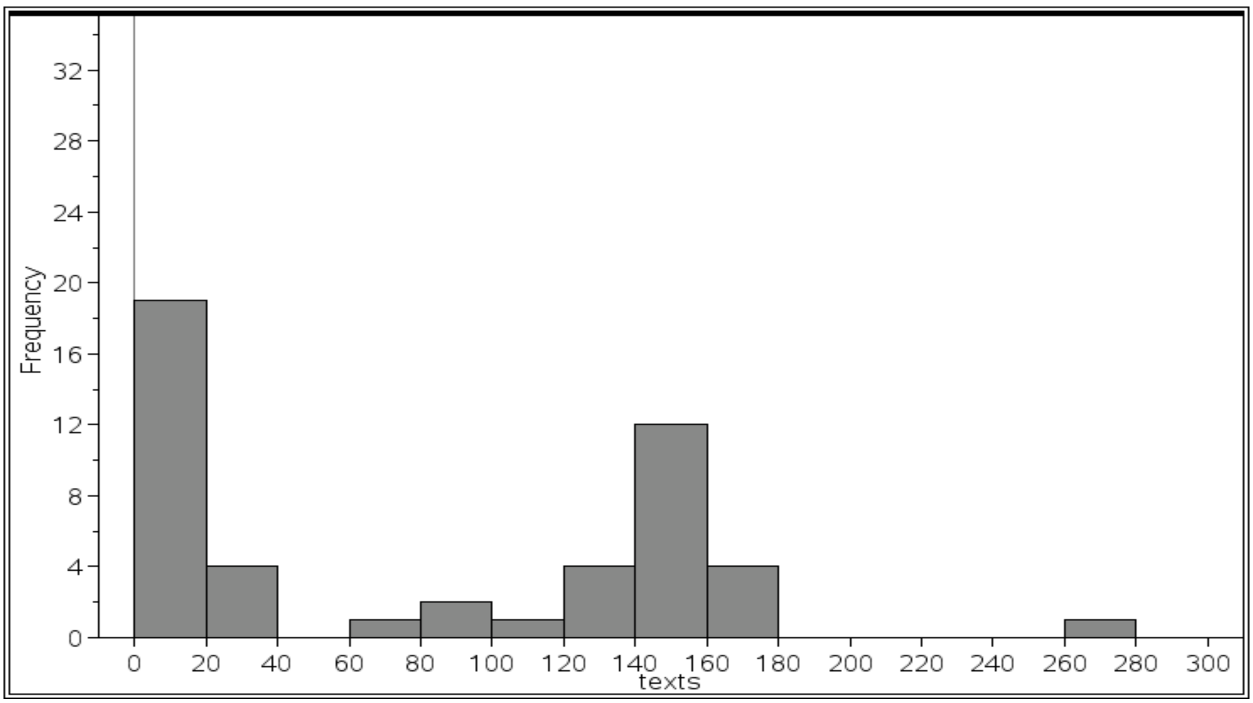
8.1 Texting by the Numbers
A Solidify Understanding Task

Technology changes quickly and yet has a large impact on our lives. Recently, Rachel was busy chatting with her friends via text message when her mom was trying to also have a conversation with her. Afterward, they had a discussion about what is an appropriate amount of texts to send each day. Since they could not agree, they decided to collect data on the number of texts people send on any given day. They each asked 24 of their friends the following question: "What is the average number of texts you SEND each day?" The data and histogram representing all 48 responses are below:

{150, 5.5, 6, 5, 3, 10, 150, 15, 20, 15, 6, 5, 3, 6, 0, 5, 12, 25, 16, 35, 5, 2, 13, 5, 130, 145, 155, 150, 162, 80, 140, 150, 165, 138, 175, 275, 85, 137, 110, 143, 138, 142, 164, 70, 150, 36, 150, 150}

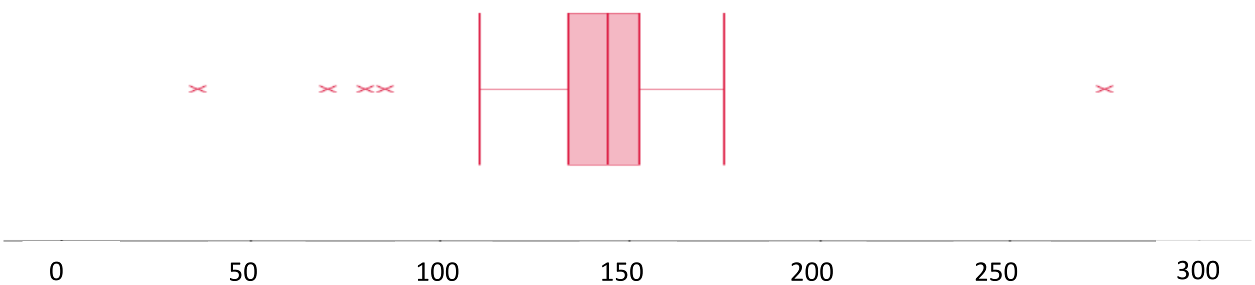


Part I: What information can you conclude based on the histogram above?

Represent the same data by creating a box plot below.

Describe the pros and cons of each representation (histogram and box plot). In other words, what information does each representation highlight? What information does each representation hide or obscure?

Part II: Prior to talking about the data with her mom, Rachel had created a box plot using her own data she collected and it looked quite different than when they combined their data.



Average number of texts sent each day

Describe the data Rachel collected from her friends. What does this information tell you?

What do you think is a reasonable number of texts Rachel can send per day?

Rachel wants to continue sending her normal number of texts (average of 100 per day) and her mom would like her to decrease this by half. Present an argument for each side, using mathematics to justify each person's request.

Name: Modeling Data | 8.1

Ready, Set, Go!

http://www.flickr.com/photos/garryknight/740038

Ready

Topic: Measures of central tendency

Sam's test scores for the term were 60, 89, 83, 99, 95, and 60.

1. Suppose that Sam's teacher decided to base the term grade on the mean.
 a. What grade would Sam receive?

 b. Do you think this is a fair grade? Explain your reasoning.

2. Suppose that Sam's teacher decided to base the term grade on his median score.
 a. What grade would Sam receive?

 b. Do you think this is a fair grade? Explain your reasoning.

3. Suppose that Sam's teacher decided to base the term grade on the mode score.
 a. What grade would Sam receive?

 b. Do you think this is a fair grade? Explain your reasoning.

4. Aiden's test scores for the same term were 30, 70, 90, 90, 91, and 99. Which measure of central tendency would Aiden want his teacher to base his grade on? Justify your thinking.

5. Most teachers base grades on the mean. Do you think this is a fair way to assign grades? Why or why not?

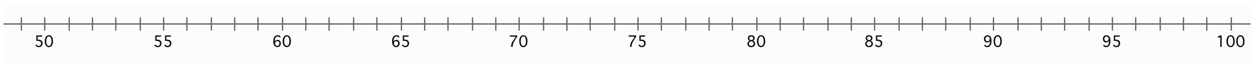
Name: Modeling Data | 8.1

Set

Topic: Examining data distributions in a box-and-whisker plot

60, 64, 68, 68, 72, 76, 76, 80, 80, 80, 84, 84, 84, 84, 88, 88, 88, 92, 92, 96, 96, 96, 96, 96, 96, 96, 100, 100

6. Make a box-and-whisker plot for the following test scores.



7a. How much of the data is represented by the box?
b. How much is represented by each whisker?

8. What does the graph tell you about student success on the test?

Go

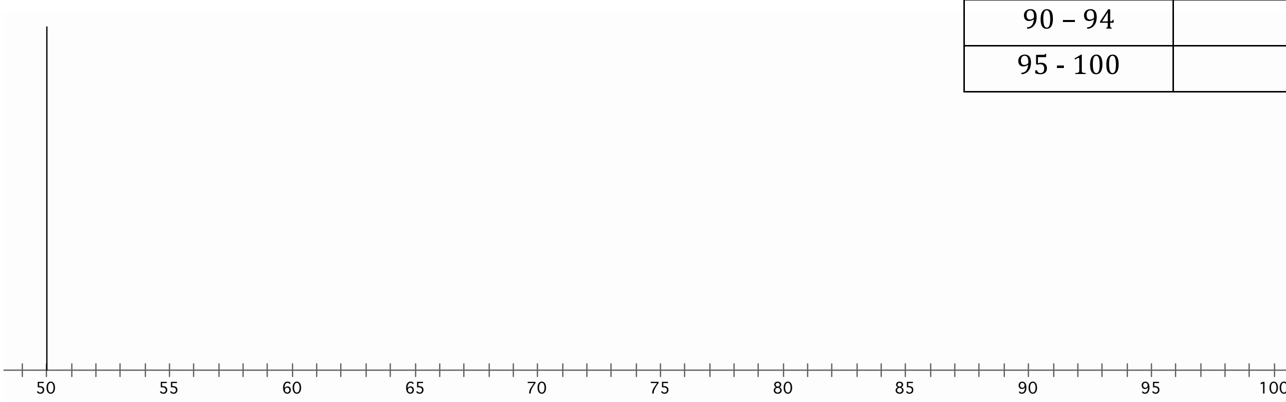
Topic: Drawing histograms.

Use the data from the SET section to answer the following questions

9. Make a frequency table with intervals. Use an interval of 5.

10. Make a histogram of the data using your intervals of 5.

Score	Frequency
60 – 64	
65 – 69	
70 – 74	
75 – 79	
80 – 84	
85 – 89	
90 – 94	
95 - 100	



Need Help? Check out these related videos:

http://www.khanacademy.org/math/statistics/e/mean_median_and_mode

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/box-and-whisker-plot

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/histograms

8.2 Data Distribution
A Practice Understanding Task

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A lot of information can be obtained from looking at data plots and their distributions. It is important when describing data that we use context to communicate the **shape, center,** and **spread.**

Shape and spread:

- Modes: uniform (evenly spread- no obvious mode), unimodal (one main peak), bimodal (two main peaks), or multimodal (multi locations where the data is relatively higher than others).
- Skewed distribution: when most data is to one side leaving the other with a 'tail'. Data is skewed to side of tail. (if tail is on left side of data, then it is skewed left).
- Outliers: values that stand away from body of distribution.
- Normal distribution: curve is unimodal and symmetric.
- Variability: values that are close together have low variability; values that are spread apart have high variability.

Center:

- Analyze the data and see if one value can be used to describe the data set. Normal distributions make this easy. If not a normal distribution, determine if there is a 'center' value that best describes the data. Bimodal or multimodal data may not have a center that would provide useful data.

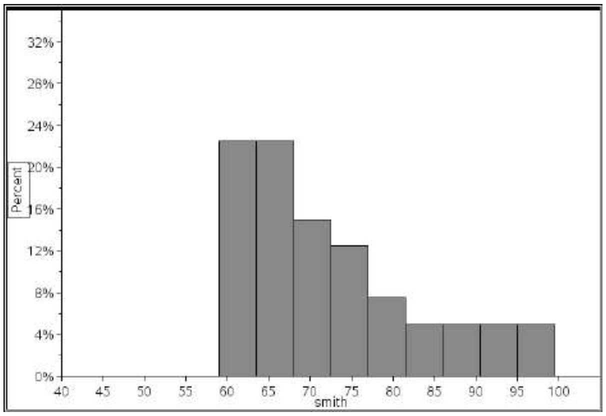
Part I: Use the *Texting By the Numbers* task to describe the shape, center, and spread.

1. Describe the distribution of the histogram that represents the data collected from Rachel and her mom (part I of *Texting by the Numbers Task*).
2. Describe the distribution of the box plot that represents the data collected from Rachel only (part II of *Texting by the Numbers Task*).

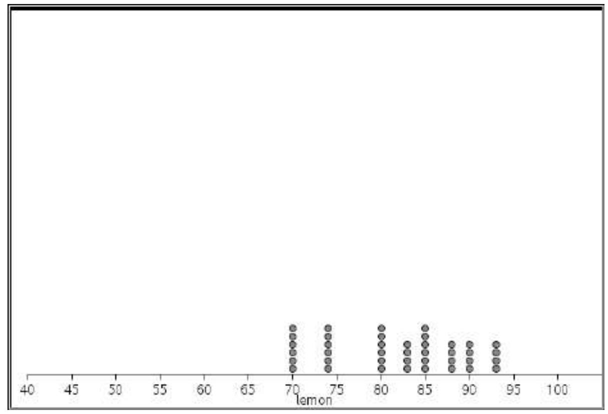
Part II: The following represents test scores from six different classes.

1. Describe the data distribution of each.
2. Compare data distributions between Adams and Smith.
3. Compare data distributions between Smith and Lemon.
4. Compare data distributions between Croft and Hurlea.
5. Compare data distributions between Jones, Adams, and Hurlea.

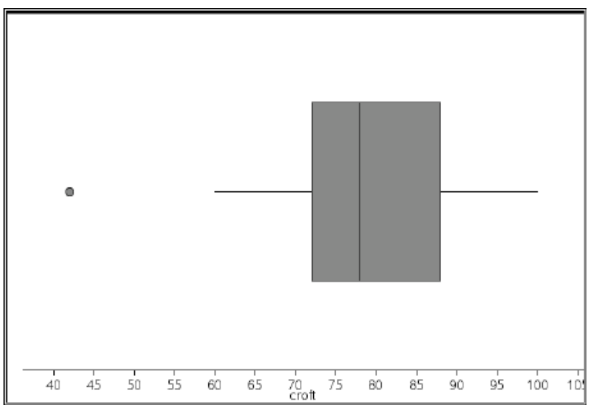
Data set I: Smith's class



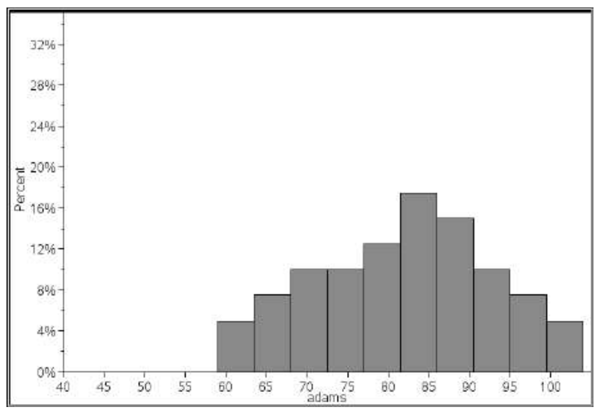
Data set II: Lemon's class



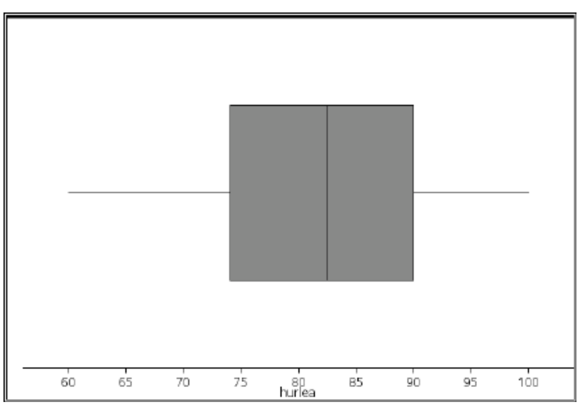
Data set III: Croft's Class



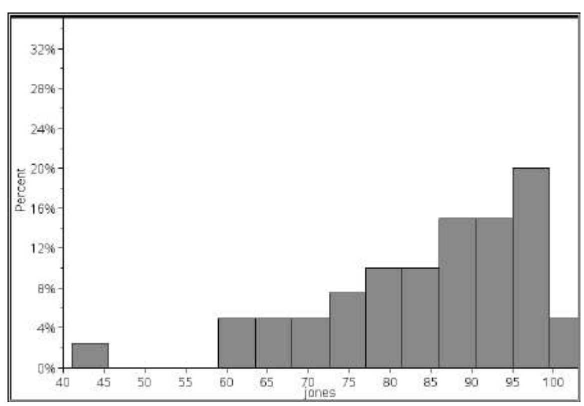
Data set IV: Adam's Class



Data set V: Hurlea's class



Data set VI: Jones' class



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Ready, Set, Go!

Ready

Topic: Sequences in statistics

In problems 1 – 4 you are to select the best answer based on the given data. Below your chosen answer is a confidence scale. Circle the statement that best describes your confidence in the correctness of the answer you chose.

1. Data: 1, 2, 4, 8, 16, 32, _______? The next number in the list will be: _______

a. larger than 32 b. positive c. exactly 64 d. about 63.89

I am certain I am correct. I am a little unsure. I had no idea so I guessed.

What about the data made you feel the way you did about the answer you marked?

2. Data: 47, -13, -8, 9, -23, 14, _________? The next number in the list will be: _______

a. positive b. negative c. less than 100 d. less than -100

I am certain I am correct. I am a little unsure. I had no idea so I guessed.

What about the data made you feel the way you did about the answer you marked?

3. Data: -10, ¾, 38, -10, ½, -81, -10, ¼, 93, -10, ______? The next number in the list will be: _____

a. more than 93 b. negative c. a fraction d. a whole number

I am certain I am correct. I am a little unsure. I had no idea so I guessed.

What about the data made you feel the way you did about the answer you marked?

4. Data: 50, -43, 36, -29, 22, -15, _______? The next number in the list will be: ______

a. odd b. less than 9 c. two-digits d. greater than -15

I am certain I am correct. I am a little unsure. I had no idea so I guessed.

What about the data made you feel the way you did about the answer you marked?

Name: Modeling Data | 8.2

Set

Topic: Drawing histograms.

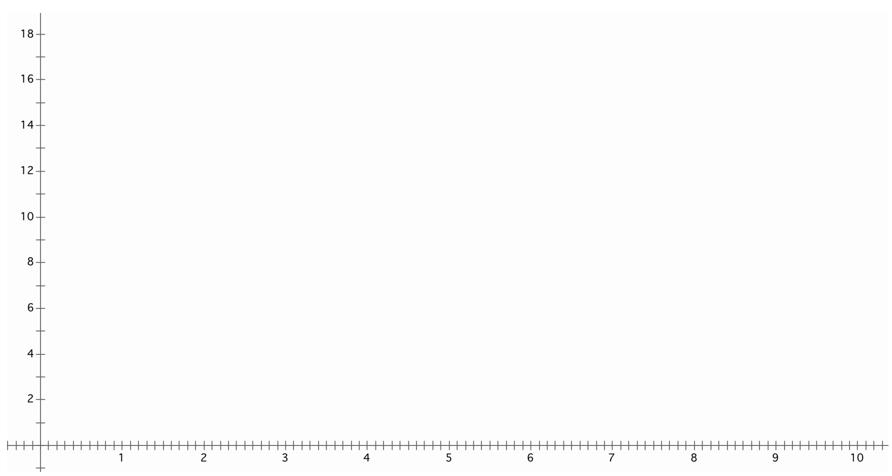
Mr. Austin gave a ten-point quiz to his 9th grade math classes. A total of 50 students took the quiz. Mr. Austin scored the quizzes and listed the scores alphabetically as follows.

1st Period Math	2nd Period Math	3rd Period Math
6, 4, 5, 7, 5, 9, 5, 4, 6, 6, 8, 5, 7, 5, 8, 1, 8, 7, 10, 9	4, 5, 8, 6, 8, 9, 5, 8, 5, 1, 5, 5, 7, 5, 7	9, 8, 10, 5, 9, 7, 8, 9, 8, 5, 8, 10, 8, 8, 5

5. Use the ALL of the quiz data to make a frequency table with intervals. Use an interval of 2.

Score	Frequency
0 - 1	
2 – 3	
4 – 5	
6 – 7	
8 - 10	

6. Use your frequency table to make a histogram for the data



7. Describe the data distribution of the histogram you created. Include words such as: *mode, skewed, outlier, normal, symmetric, center,* and *spread,* if they apply.

Go

8. What percent of 97 is 11?

9. What percent of 88 is 132?

10. What percent of 84 is 9?

11. What percent of 88.6 is 70?

12. What is 270% of 60?

13. What is 84% of 25?

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/histograms

http://www.khanacademy.org/math/statistics/v/ck12-org-normal-distribution-problems--qualitative-sense-of-normal-distributions

http://stattrek.com/statistics/two-way-table.aspx

8.3 After School Activity
A Develop Understanding Task

Part I

Rashid is in charge of determining the upcoming after school activity. To determine the type of activity, Rashid asked several students whether they prefer to have a dance or play a game of soccer. As Rashid collected preferences, he organized the data in the following two-way frequency table:

	Girls	Boys	Total
Soccer	14	40	54
Dance	46	6	52
Total	60	46	106

Rashid is feeling unsure of the activity he should choose based on the data he has collected and is asking for help. To better understand how the data is displayed, it is useful to know that the outer numbers, located in the margins of the table, represent the total frequency for each row or column of corresponding values and are called *marginal frequencies* . Values that are part of the 'inner' body of the table are created by the intersection of information from the column and the row and they are called the *joint frequencies.* Using the data in the table, construct a viable argument and explain to Rashid which after school event he should choose.

Part II: Two way frequency tables allow us to organize categorical data in order to draw conclusions. For each set of data below, create a frequency table. When each frequency table is complete, write three sentences about observations of the data, including any trends or associations in the data.

Data set 1: There are 45 total students who like to read books. Of those students, 12 of them like non-fiction and the rest like fiction. Four girls like non-fiction. Twenty boys like fiction.

	Fiction	Nonfiction	Total
Boys			
Girls			
Total			

Observation 1:

Observation 2:

Observation 3:

Data set 2: 35 seventh graders and 41 eighth graders completed a survey about the amount of time they spend on homework each night. 50 students said they spent more than an hour. 12 eighth graders said they spend less than an hour each night.

			Total
More than one hour			
Less than one hour			
Total			

Observation 1:

Observation 2:

Observation 3:

Name: Modeling Data | 8.3

Ready, Set, Go!

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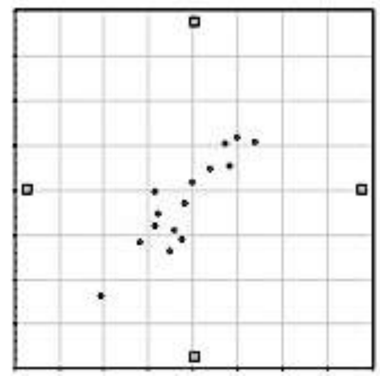
Ready

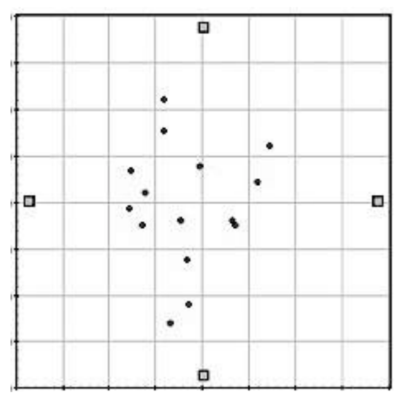
Topic: Interpreting data from a scatter plot

1. The scatter plot compares shoe size and height in adult males. Based on the graph, do you think there is a relationship between a man's shoe size and his height?



Explain your answer.

2. The scatter plot compares left-handedness to birth weight. Based on the graph, do you think being left-handed is related to a person's birth weight?



Explain your answer.

Name: Modeling Data | 8.3

Set

Topic: Two-way frequency tables.

Here is the data from Mr. Austin's ten-point quiz. Students needed to score a 6 or better to pass the quiz.

1st Period Math	2nd Period Math	3rd Period Math
6, 4, 3, 7, 5, 9, 5, 4, 6, 6, 8, 5, 7, 3, 6, 2, 8, 7, 10, 9	3, 3, 8, 6, 6, 9, 5, 8, 5, 3, 5, 5, 7, 5, 7	9, 8, 10, 5, 9, 7, 8, 9, 8, 3, 8, 10, 8, 7, 5

3. Make a two-way frequency table showing how many students passed the quiz and how many failed in each class.

	1st Period	2nd Period	3rd Period	Total
Passed				
Failed				
Total				

4. Use a colored pencil to lightly shade the cells containing the *joint frequency* numbers in the table. The un-shaded numbers are the *marginal frequencies.* (Use these terms to answer the following questions.)

5. If Mr. Austin wanted to see how many students in all 3 classes combined passed the quiz, where would he look?

6. If Mr. Austin wanted to write a ratio of the number of passing students compared to the number of failing students for each class, where would he find the numbers he would need to do this?

7. Make a two-way frequency table that gives the *relative frequencies* of the quiz scores for each class.

	1st Period	2nd Period	3rd Period	Total
Passed				
Failed				
Total				

Name: Modeling Data | 8.3

Go

8. Sophie surveyed all of the 6th grade students at Reagan Elementary School to find out which TV Network was their favorite. She thought that it would be important to know whether the respondent was a boy or a girl so she recorded her information this way.

Animal Planet	Cartoon Network	Disney	Nickelodeon
GGBBBB BGBBBGBBB GGBB BBBBBB	BBBBBBB BBGGGBBBG BGBGGGBGG	GGGGGGBBBBBB GBGBGG BBBGGBGG GGGBBBGGGGGB	BBBBGGGGGGGG GGGGGGBB GGGGBGGGGGGGGGBBBB BGGGGGGGG

Sophie planned to use her data to answer the following questions:

I. Are there more girls or boys in the 6th grade?

II. Which network was the boys' favorite?

III. Was there a network that was favored by more than 50% of one gender?

But when she looked at her chart, she realized that the data wasn't telling her what she wanted to know. Her teacher suggested that her data would be easier to analyze if she could organize it into a two-way frequency chart. Help Sophie out by putting the frequencies into the correct cells.

Favorite TV Networks	*Girls*	*Boys*	*Totals*
Animal Planet			
Cartoon Network			
Disney			
Nickelodeon			
Totals			

Now that Sophie has her data organized, use the two-way frequency chart to answer her 3 questions.

a. Are there more girls or boys in the 6th grade?

b. Which network was the boys' favorite?

c. Was there a network that was favored by more than 50% of one gender

Need Help? Check out these related links:

http://stattrek.com/statistics/two-way-table.aspx

8.4 Relative Frequency

A Solidify Understanding Task

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Rachel is thinking about the data she and her mom collected for the average number of texts a person sends each day and started thinking that perhaps a two-way table of the data they collected would help convince her mom that she does not send an excessive amount of texts for a teenager. The table separates each data point by age (teenager and adult) and by the average number of texts sent (more than 100 per day or less than 100 per day).

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenager	20	4	24
Adult	2	22	24
Total	22	26	48

Write two observation statements of this two way table.

To further provide evidence, Rachel decided to do some research. She found that only 43% of people with phones send over 100 texts per day. She was disappointed that the data did not support her case and confused because it did not seem to match what she found in her survey. What questions do these statistic raise for you? What data should Rachel look for to support her case?

After looking more closely at the data, Rachel found other percentages within the same data that seemed more accurate with the data she collected from her teenage friends. How might Rachel use the data in the two way table to find percentages that would be useful for her case?

Part II: Once Rachel realized there are a lot of ways to look at a set of data in a two way table, she was self-motivated to learn about *relative frequency tables* and conditional frequencies. When the data is written as a percent, this is called a *relative frequency table*. In this situation, the 'inner' values represent a percent and are called conditional frequencies. The conditional values in a *relative frequency table* can be calculated as percentages of one of the following:

- the whole table (relative frequency of table)
- the rows (relative frequency of rows)
- the columns (relative frequency of column)

Since Rachel wants to emphasize that a person's age makes a difference in the number of texts sent, the first thing she decided to do is focus on the ROW of values so she could write conditional statements about the number of texts a person is likely to send based on their age. This is called a *relative frequency of row* table. Fill in the percentage of teenagers for each of the conditional frequencies in the highlighted row below:

Row →

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenager % of teenagers	20 ___ %	4 __%	24 100%
% of Adults	**2** 8%	**22** 92%	**24** 100%
% of People	**22** 46%	**26** 54%	**48** 100%

Since the PERCENTAGES created focus on ROW values, all conditional observations are specific to the information in the row. Complete the following sentence for the *relative frequency of row:*

Of all teenagers in the survey, _______ % average more than 100 texts per day.

Write another statement based on the *relative frequency of row:*

Below is the *relative frequency of column* using the same data. This time, all of the percentages are calculated using the data in the column.

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenagers	**20** 91%	**4** 15%	**24** 50%
Adults	**2** 9%	**22** 85%	**24** 50%
Total	**22** 100%	**26** 100%	**48** 100%

Write two conditional statements using the *relative frequency of column*.

This data represents the *relative frequency of whole table*:

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
% of Teenagers	**20** 42%	**4** 8%	**24** 50%
% of Adults	**2** 4%	**22** 46%	**24** 50%
% of Total	**22** 46%	**26** 54%	**48** 100%

Create two conditional distribution statements for the *relative frequency of whole table*

How do *relative frequency tables* impact the way you look at data in two way tables?

Name: Modeling Data | 8.4

Ready, Set, Go!

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Ready

Topic: Linear functions and relationships

Write the explicit linear function for the given information below.

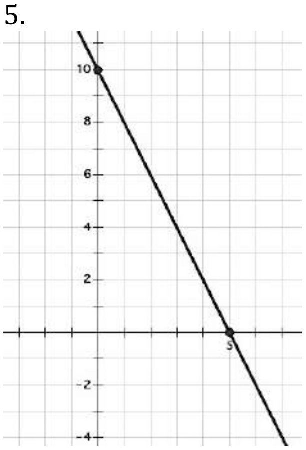
1. (3, 7) (5, 13)

2. Mike earns $11.50 an hour

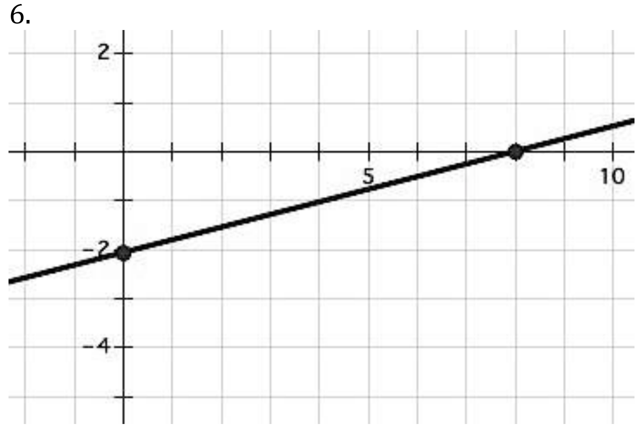
3. (-5, -2) (1, 10)

4. (-2, 12) (6, 8)

5.



6.



Set

Topic: Relative Frequency Tables

For each two-way table below, create the indicated relative frequency table and also provide two observations with regard to the data.

7. This table represents survey results from a sample of students regarding mode of transportation to and from school.

	Walk	Bike	Car Pool	Bus	Total
Boys	37	47	27	122	233
Girls	38	22	53	79	192
Total	75	69	80	201	425

Create the *relative frequency of row table*. Then provide two observation statements.

	Walk	Bike	Car Pool	Bus	Total
Boys					
Girls					
Total	100%	100%	100%	100%	100%

8. The two-way table contains survey data regarding family size and pet ownership.

	No Pets	Own one Pet	More than one pet	Total
Families of 4 or less	35	52	85	172
Families of 5 or more	15	18	10	43
Total	50	70	95	215

Create the *relative frequency of column table*. Then provide two observation statements.

	No Pets	Own one Pet	More than one pet	Total
Families of 4 or less				100%
Families of 5 or more				100%
Total				100%

9. The two-way table below contains survey data about boys and girls shoes.

	Athletic shoes	Boots	Dress Shoe	Total
Girls	21	35	60	116
Boys	50	16	10	76
Total	71	51	70	192

Create the *relative frequency of whole table*. Then provide two observation statements.

	Athletic shoes	Boots	Dress Shoe	Total
Girls				
Boys				
Total				100%

Go

Topic: One variable statistical measures and comparisons

For each set of data determine the mean, median, mode and range. Then create either a box-and-whisker plot or a histogram.

10. 23, 24, 25, 20, 25, 29, 24, 25, 30

11. 20, 24, 10, 35, 25, 29, 24, 25, 33

12. How do the data sets in problems 10 and 11 compare to one another?

13. 2, 3, 4, 5, 3, 4, 7, 4, 4

14. 1, 1, 3, 5, 5, 10, 5, 1, 14

15. How do the data sets in problems 13 and 14 compare to one another?

8.5 Connect the Dots

A Develop Understanding Task

1. For each set of data:
- Graph on a scatter plot.
- Use technology (graphing calculator or computer) to calculate the correlation coefficient.

Set A

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
1	1.5	2.5	1.9	2.8	3.2	4.5	3.7	1.7	4.8	2.7	2.3

Set B

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
1	1.5	2.5	1.9	2.8	3.2	4.5	3.7	4	4.8	5	4.6

Set C

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4.9	4.2	3.9	3.5	3.2	3.1	2.6	3.2	2.1	1.3	0.8

Set D

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4.9	3.6	3.9	2.1	4.5	3.1	1.7	3.7	2.1	1.3	1.8

Set E

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4	4.2	3.9	2.8	3.2	4.5	3.7	3.2	4.8	5	4.4

Set F

2	2.3	3.3	3.7	4.2	4.6	4.5	5
1.8	2.22	3.62	4.18	4.88	5.44	5.3	6

Set G

2	2.3	3.3	3.7	4.2	4.6	4.5	5
4.4	4.01	2.71	2.19	1.54	1.02	1.15	0.5

2. Put the scatter plots in order based upon the correlation coefficients.

3. Compare each scatter plot with its correlation coefficient. What patterns do you see?

4. Use the data in Set A as a starting point. Keeping the same *x*-values, modify the y-values to obtain a correlation coefficient as close to 0.75 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a greater correlation coefficient?

5. This time, again start with the data in Set A. Keep the same x-values, but this time, modify the y values to obtain a correlation coefficient as close to 0.25 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a correlation coefficient that is closer to 0?

6. One more time: start with the data in Set A. Keep the same x-values, modify the y-values to obtain a correlation coefficient as close to -0.5 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a correlation coefficient that is negative?

7. What aspects of the data does the correlation coefficient appear to describe?

Name: Modeling Data | 8.5

Ready, Set, Go!

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Ready

Topic: Estimating the line of best fit

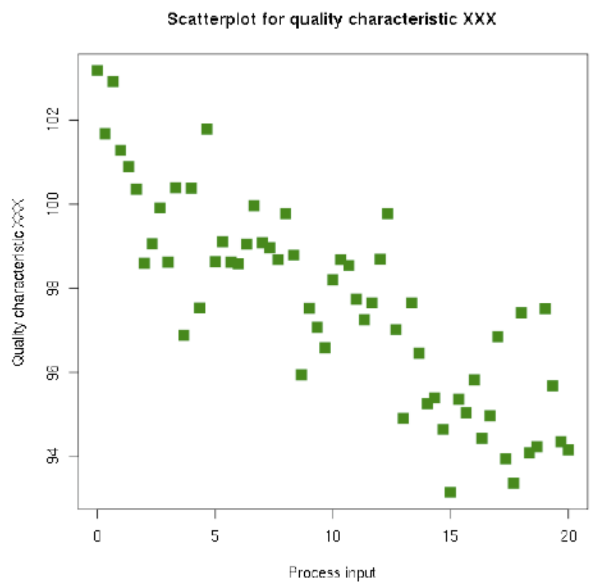
Examine the scatterplot below. Imagine that you drew a straight line through the general pattern of the points, keeping as close as possible to all points with as many points above the line as below.

1. Predict a possible y-intercept and slope for that line.

   a. y-intercept: ________________

   b. slope: ______________________

2. Sketch the line that you imagined for question #1 and write an equation for that line.



© 2012 http://en.wikipedia.org/wiki/File:Scatter_diagram_for_quality_characteristic_XXX.svg

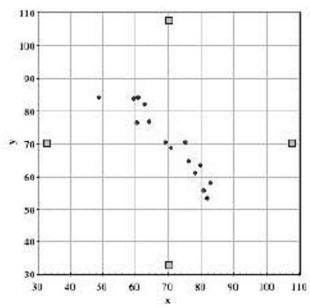
Name: Modeling Data | 8.5

Set

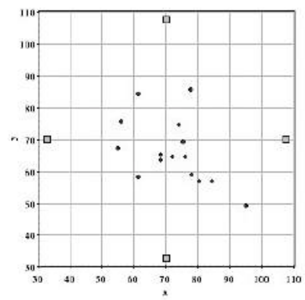
Topic: Estimating the correlation coefficient

Match the scatterplot with its correlation coefficient.

___ 3.



___ 4.



Possible
Correlation Coefficients

a. 0.05

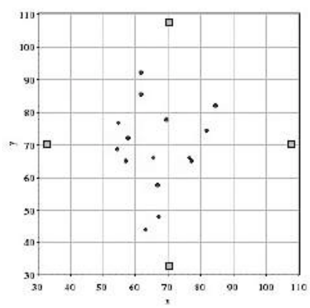
b. 0.97

c. -0.94

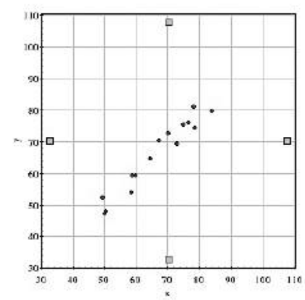
d. -0.49

e. 0.68

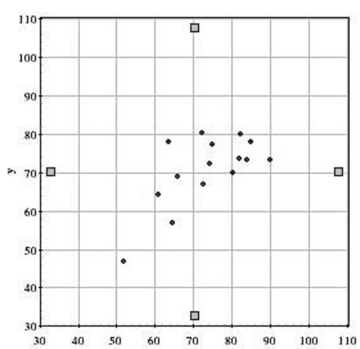
___ 5.



___ 6.



___ 7.



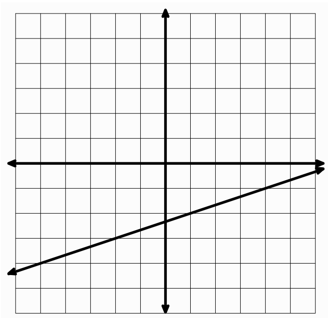
Name: Modeling Data 8.5

Go

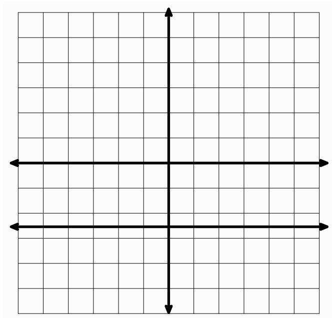
Topic: Visually comparing slopes of lines

Follow the prompt to sketch the graph of a line on the same grid with the given characteristics.

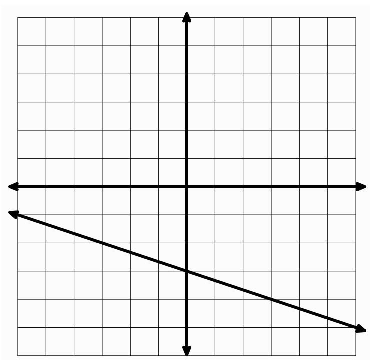
8. A larger slope



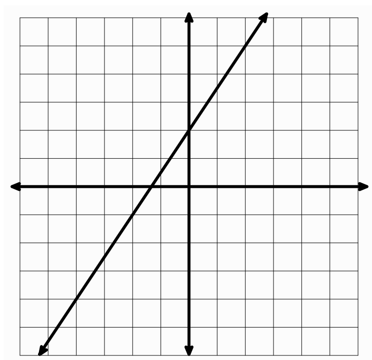
9. A smaller slope



10. A larger y-intercept and a smaller slope

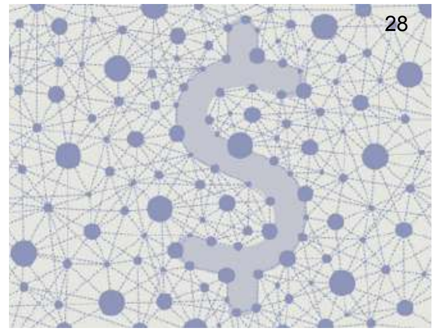


11. Slope is the negative reciprocal



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/fitting-a-line-to-data



8.6 Making More $

A Solidify Understanding Task

Each year the U.S. Census Bureau provides income statistics for the United States. In the years from 1990 to 2005, they provided the data in the tables below. (All dollar amounts have been adjusted for the rate of inflation so that they are comparable from year-to-year.)

Year	Median Income for All Men
2005	41196
2004	41464
2003	40987
2002	40595
2001	41280
2000	41996
1999	42580
1998	42240
1997	40406
1996	38894
1995	38607
1994	38215
1993	37712
1992	37528
1991	38145

Year	Median Income for All Women
2005	23970
2004	23989
2003	24065
2002	23710
2001	23564
2000	23551
1999	22977
1998	22403
1997	21759
1996	20957
1995	20253
1994	19158
1993	18751
1992	18725
1991	18649

1. Create a scatter plot of the data for men. What is your estimate of the correlation coefficient for these data?
   What is the actual correlation coefficient?
   What does it tell you about the relationship between income and years for men?

2. On a separate graph, create a scatter plot of the data for women. What is your estimate of the correlation coefficient for these data?
   What is the actual correlation coefficient?
   What does it tell you about the relationship between income and years for women?

   How does that compare to the data for men?

3. Estimate and draw a line of best fit for each set of data.
   a. Describe how you estimated the line for men. If you chose to run the line directly through any particular points, describe why you selected them.

   b. Describe how you estimated the line for women. If you chose to run the line directly through any particular points, describe why you selected them.

4. Write the equation for each of the two lines in slope intercept form.
   a. Equation for men:

   b. Equation for women:

5. Use technology to calculate a linear regression for each set of data. Add the regression lines to your scatter plots.
   a. Linear regression equation for men:

   b. Linear regression equation for women:

6. Compare your estimated line of best fit to the regression line for men. What does the slope mean in each case? (Include units in your answer.)

7. Compare your estimated line of best fit to the regression line for women. What does the y-intercept mean in each case? (Include units in your answer.)

8. Compare the regression lines for men and women. What do the lines tell us about the income of men vs women in the years from 1991-2005?

9. What do you estimate will be the median income for men and women in 2015?

10. The Census Bureau provided the following statistics for the years from 2006-2011.

Year	Median Income for All Men
2011	37653
2010	38014
2009	38588
2008	39134
2007	41033
2006	41103

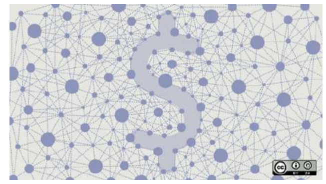
Year	Median Income for All Women
2011	23395
2010	23657
2009	24284
2008	23967
2007	25005
2006	24429

With the addition of these data, what would you now estimate the median income of men in 2015 to be? Why?

11. How appropriate is a linear model for men's and women's income from 1991-2011? Justify your answer.

Name: Modeling Data | 8.6

Ready, Set, Go!



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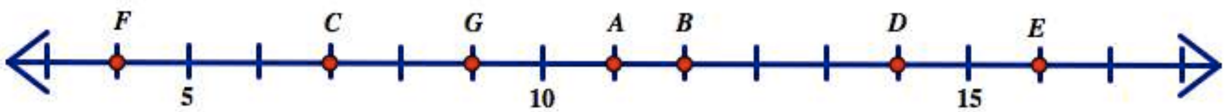
Ready

Topic: Finding distance and averages.

Use the number line below to answer the questions.



1. How far away is each of the points on the number line from *point A*?
   (You need to list each point and its distance from *point A*.)

2. What is the total of all the distances from *point A* that you found in exercise number one?

3. What is the average distance that any of the given *points B through G* are from *point A*?

4. Which point on the number line is located the average distance away from *point A*?

5. Label another location on the number line that is the average distance away from *point A.*
   (Call it *point X*)

6. How far away is each of the points on the number line from *point D*?
   (You need to list each point and its distance from *point D*.)

7. What is the total of all the distances from *point D* that you found in exercise number six?

8. What is the average distance that any of the six other points are from *point D*?

9. Is there a point on the number line located the average distance away from *point D*?

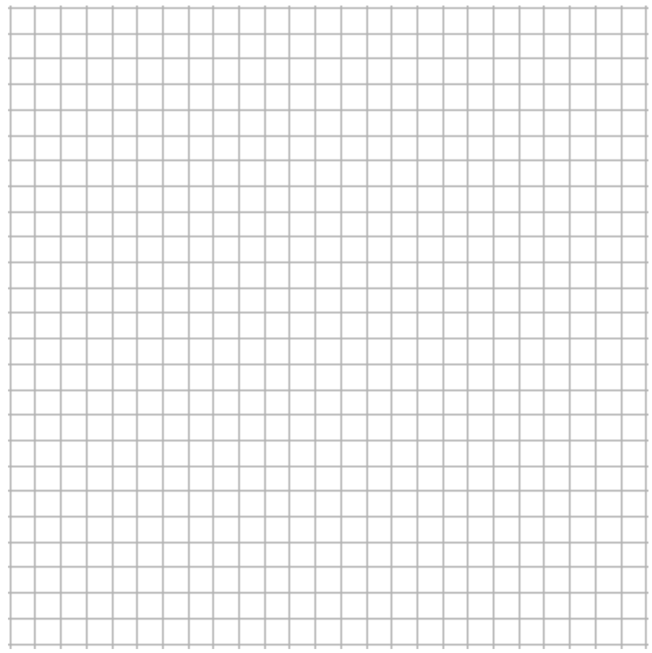
10. Label another location on the number line that is the average distance away from *point D*?
    (Call it *point Y*)

Set

Topic: Scatter Plots and line of best fit or trend lines.

11. Create a scatter plot for the data in the table.

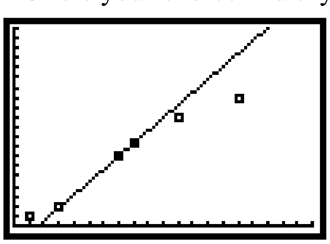
English Score	History Score
60	65
53	59
44	57
61	61
70	67

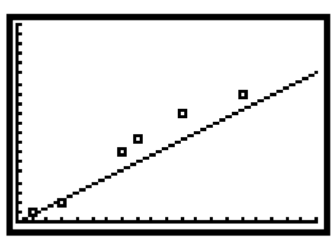


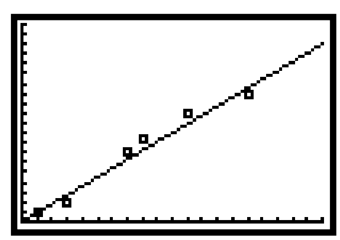
12. Do the English and History scores have a positive or negative correlation?

13. Do English and History scores have a strong or weak correlation?

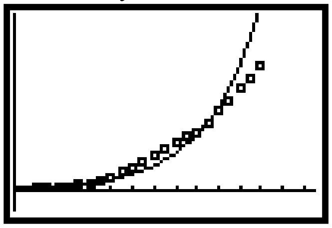
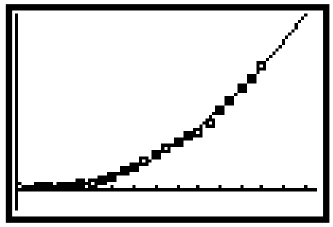
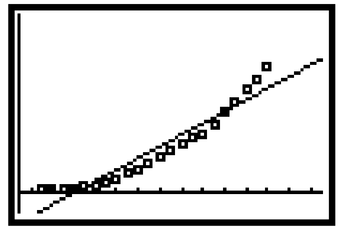
14. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.



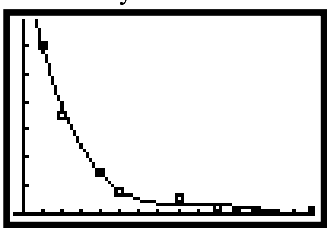
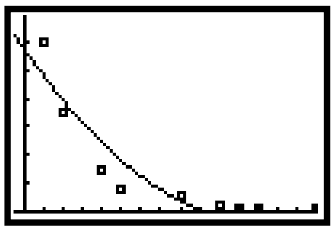
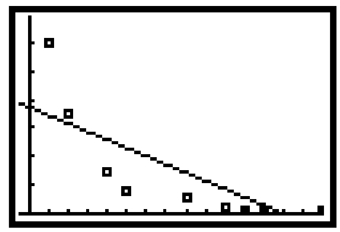




15. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.

16. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.

Go

Topic: Creating explicit functions for arithmetic and geometric sequences.

In each problem below an input connected output are given along with either the common difference or the common ratio. Use this information to create an explicit function for the sequence.

17. $f(2)=7$, common difference = 3

18. $g(1)=8$, common ratio = 2

19. $h(6)=3$, common ratio = -3

20. $r(5)=-3$, common difference = 7

21. $g(7)=1$, common difference = -9

22. $g(1)=5$, common ratio = $\frac{1}{2}$

8.7 Getting Schooled

A Solidify Understanding Task

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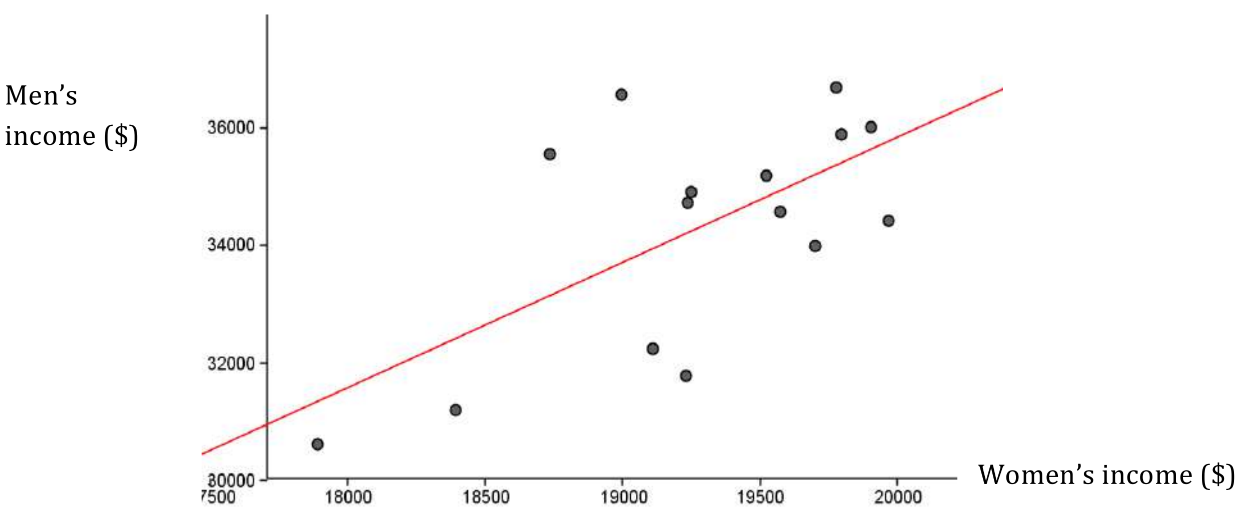
In *Getting More $,* Leo and Araceli noticed a difference in men's and women's salaries. Araceli thought that it was unfair that women were paid less than men. Leo thought that there must be some good reason for the discrepancy, so they decided to dig deeper into the Census Bureau's income data to see if they could understand more about these differences.

First, they decided to compare the income of men and women that graduated from high school (or equivalent), but did not pursue further schooling. They created the scatter plot below, with the x value of a point representing the average woman's salary for some year and the y value representing the average man's salary for the same year. For instance, the year 2011 is represented on the graph by the point (17887, 30616). You can find this point on the graph in the bottom left corner.



1. Based upon the graph, estimate the correlation coefficient.

2. Estimate the average income for men in this time period. Describe how you used the graph to find it.

3. What is the average income for women in this time period? Describe how you used the graph to find it.

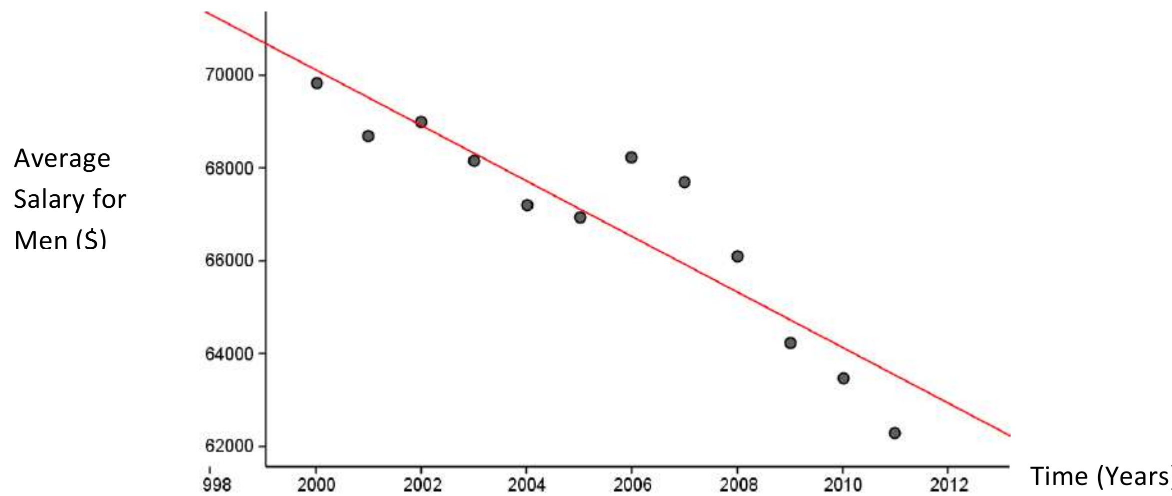
4. Leo and Araceli calculated the linear regression for these data to be $y = 2.189x - 6731.8$. What does the slope of this regression line mean about the income of men compared to women? Use precise units and language.

"Hmmmm," said Araceli, "It's just as I suspected. The whole system is unfair to women." "No, wait," said Leo, "Let's look at incomes for men and women with bachelor's degrees or more. Maybe it has something to do with levels of education."

5. Leo and Araceli started with the data for men with bachelor's degrees or more. They found the correlation coefficient for the average salary vs year from 2000-2011 was r = -.9145.

   Predict what the graph might look like and draw it here. Be sure to scale and label the axes and put 12 points on your graph.

The actual scatter plot for salaries for men with bachelor's degrees from 2000-2011 is below. How did you do?



6. Both Leo and Araceli were surprised at this graph. They calculated the regression line and got $y = -598.25x + 1266626.34$. What does this equation say about the income of men with bachelor's degrees from 2000-2011?

7. Leo wondered why the y-intercept in the equation was $1,266,626.34 and yet the graph seems to cross the y axis around $72,000. What would you tell Leo to resolve his concern?

Next, they turned their attention to the data for women with bachelor's degrees or more from 2000-2011. Here's the data:

Year	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000
Income for Women ($)	41338	42409	42746	42620	44161	44007	42690	42539	42954	42871	42992	43293

Analyze these data by creating a scatter plot, interpreting the correlation coefficient and the regression line. Draw the graph and report the results of your analysis below:

Now that you have analyzed the results for women, compare the results for men and women with bachelor's degrees and more over the period from 2000-2011.

Leo believes that the difference in income between men and women may be explained by differences in education, but Araceli believes there must be other factors such as discrimination. Based on the data in this task and *Getting More $,* make a convincing case to support either Leo or Araceli.

What other data that would be useful in making your case? Explain what you would look for and why.

Ready, Set, Go!

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Ready

Topic: Finding distances and averages.

The graph below has several points and shows the line ***y=x*** use this graph to answer each question.

1. The vertical distance between *point N* and the line ***y=x*** is labeled on the graph. Find all of the vertical distances between the points and the line $y = x$.

B:

D:

E:

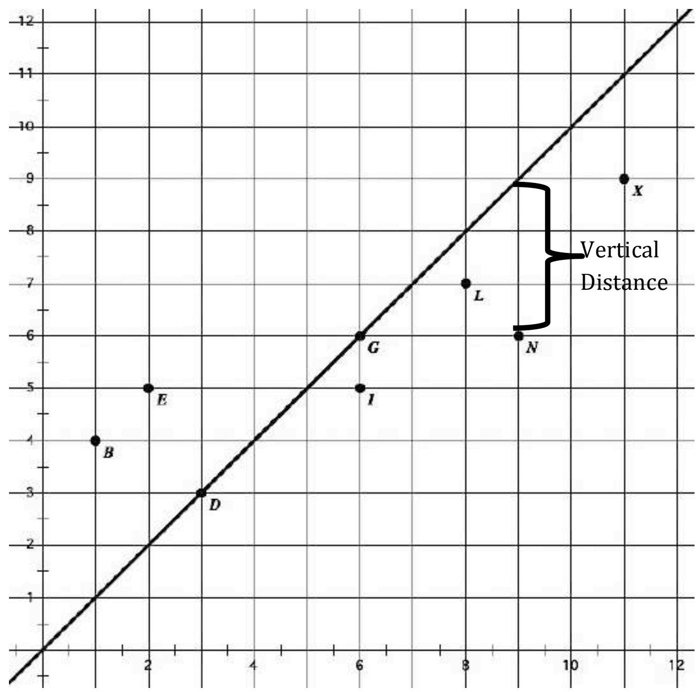
G:

I:

L:

N:

X:



2. What is the *sum of all the distances* that the points are away from the line ***y=x***?

3. What is the *average vertical distance* that any of the points are away form the line **y=x**?

4. Is the line on the graph the line of best fit? Explain why or why not. If it is not the best then draw a line that is better fit to the data.

5. Estimate the correlation coefficient for this set of data points.
If you have a way to calculate it exactly then do so. (Using a graphing calculator or data software.)

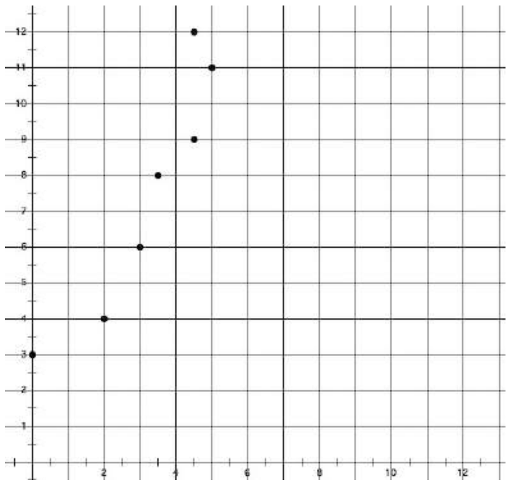
Name: Modeling Data | 8.7

Set

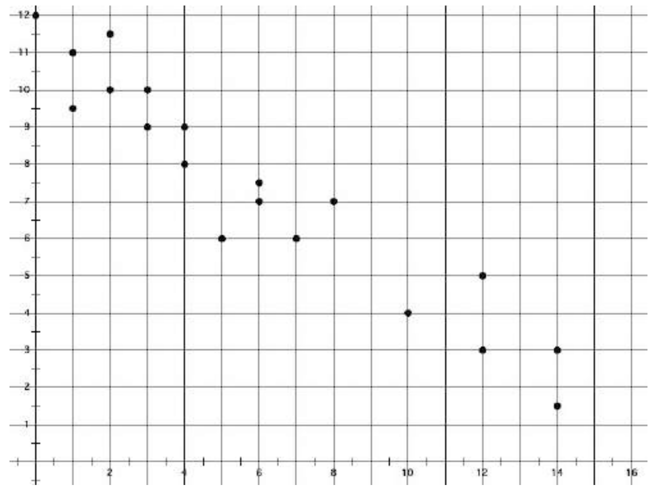
Topic: Creating and analyzing scatter plots.

Determine whether a linear or an exponential model would be best for the given scatter plots. Then sketch a model on the graph that could be used to make predictions.

6.

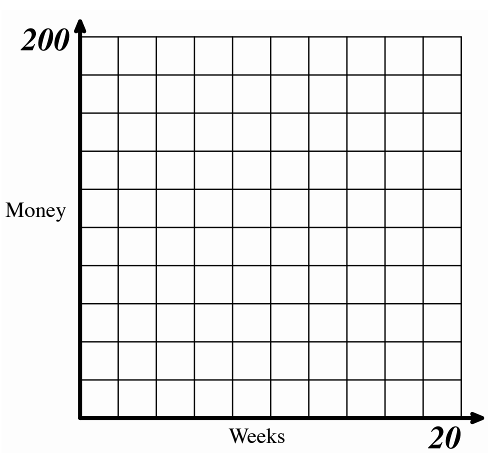


7.



8a. Use the data to make a scatter plot.

Weeks since school started	Money in savings
1	200
3	175
4	162
7	120
10	87
13	57
20	5



b. Is the correlation of the graph positive or negative? Why?

c. What would you estimate the correlation coefficient to be? Why? (If you have a calculator or software that can calculate it precisely then do so.)

d. Create a regression line and find the regression equation. What is the regression equation?

e. What does the slope of the regression equation mean in terms of the variables?

f. Most school years are 36 weeks. If the rate of spending is kept the same how much more money needs to be saved during the summer in order for there to be money to last all 36 weeks.

Name: Modeling Data | 8.7

Go

Topic: Data and statistics, when to use two way tables when to use scatter plots.

9. In what situations does it make the most sense to use a two-way table and look at residual frequencies to make decisions or conclusions?

10. In what situations does it make the most sense to use a scatter plot and a linear or exponential model to analyze and make decisions or draw conclusions?

For each of the representations below label as a *function, not a function.* If, not a function say why. If it is a function then label as *linear, exponential or neither*.

11.

x	f(x)
0	5
1	169
2	333
3	497

12.

X	Y
1	15
2	25
3	15
2	30

13.

x	h(x)
2	5
3	10
4	20
5	40

14. $g(x) = 4 - 12x$

15. $s(t) = 3 \cdot 4^{t-1}$

16. The amount of medicine in the blood stream of a cat as time passes. The initial dose of medicine is 80mm and the medicine brakes down at 35% each hour.

17.

Time	0	1	2	3	4
Money in Bank	250	337.50	455.63	615.09	830.38

8.8 Rockin' the Residuals

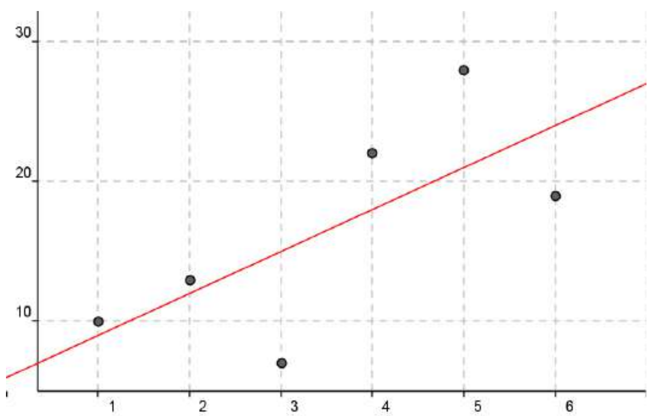
A Solidify Understanding Task

The correlation coefficient is not the only tool that statisticians use to analyze whether or not a line is a good model for the data. They also consider the residuals, which is to look at the difference between the observed value (the data) and the predicted value (the y-value on the regression line). This sounds a little complicated, but it's not really. The residuals are just a way of thinking about how far away the actual data is from the regression line.

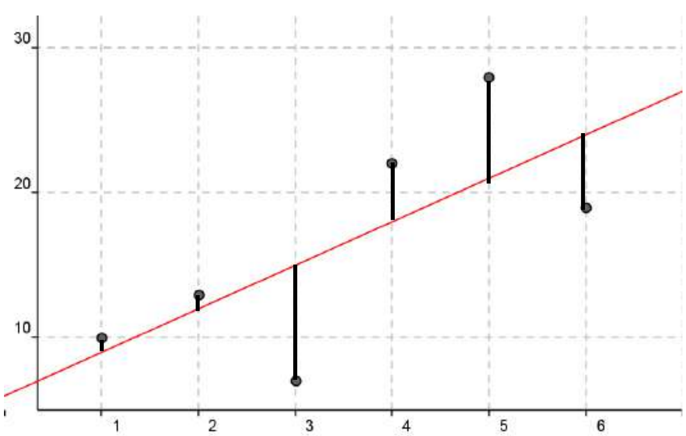
Start with some data:

x	1	2	3	4	5	6
y	10	13	7	22	28	19

Create a scatter plot and graph the regression line. In, this case the line is $y = 3x + 6$.



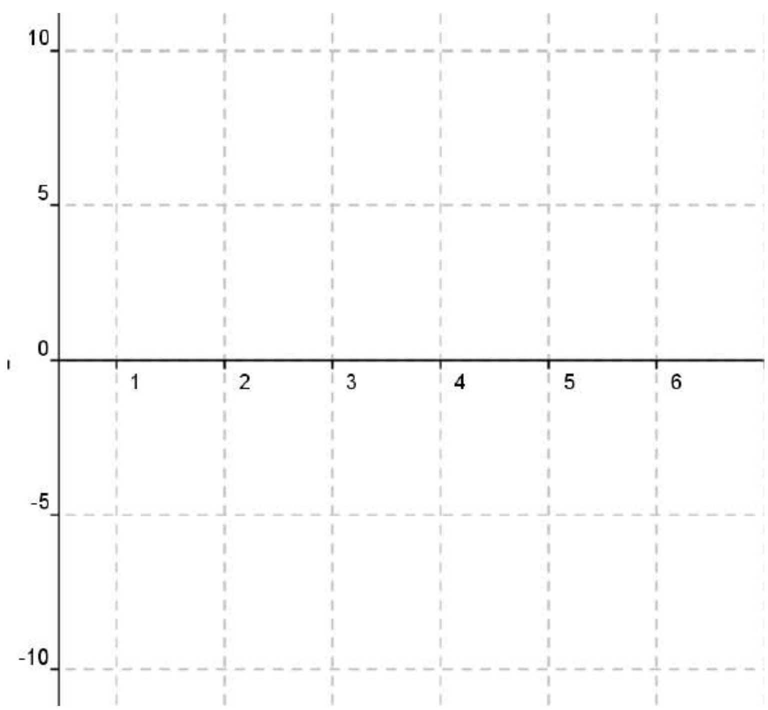
Draw a line from each data point to the regression line, like the segments drawn from each point below.



1. The residuals are the lengths of the segments. How can you calculate the length of each segment to get the residuals?

2. Generally, if the data point is above the regression line the residual is positive, if the data point is below the line, the residual is negative. Knowing this, use your plan from #1 to create a table of residual values using each data point.

3. Statisticians like to look at graphs of the residuals to judge their regression lines. So, you get your chance to do it. Graph the residuals here.



Now, that you have constructed a residual plot, think about what the residuals mean and answer the following questions.

4. If a residual is large and negative, what does it mean?

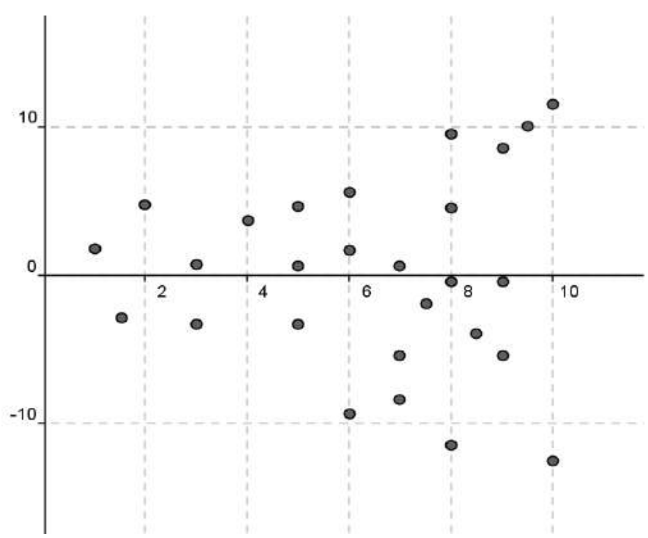
5. What does it mean if a residual is equal to 0?

6. If someone told you that they estimated a line of best fit for a set of data points and all of the residuals were positive, what would you say?

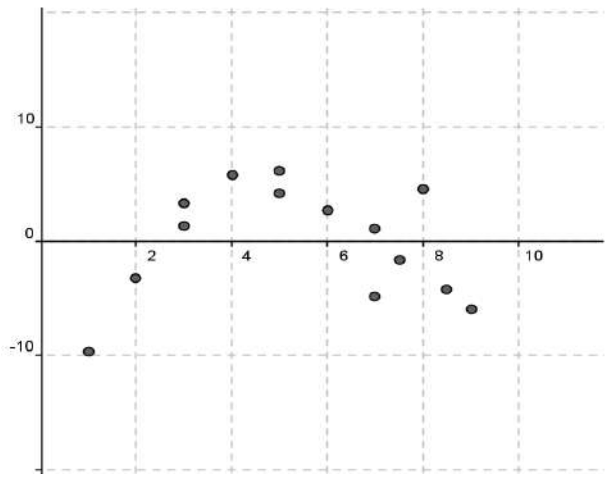
7. If the correlation coefficient for a data set is equal to 1, what will the residual plot look like?

Statisticians use residual plots to see if there are patterns in the data that are not predicted by their model. What patterns can you identify in the following residual plots that might indicate that the regression line is not a good model for the data? Based on the residual plot are there any points that may be considered outliers?

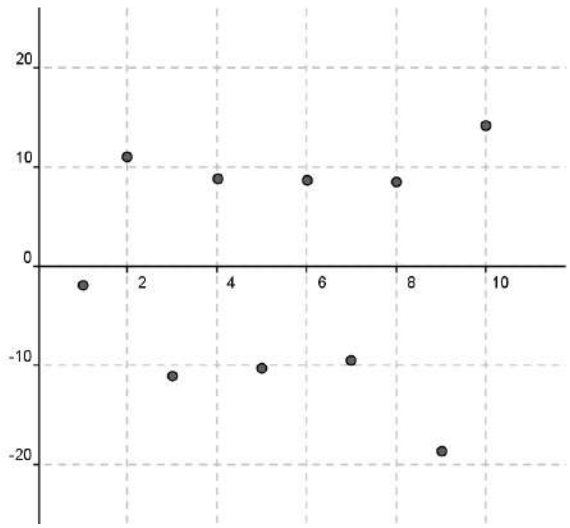
8.



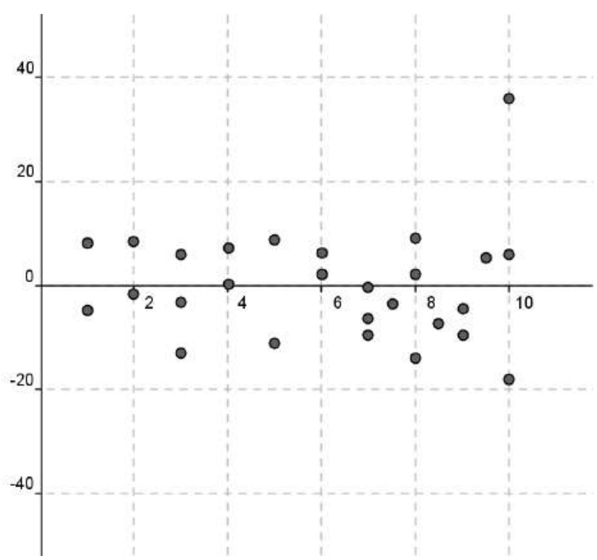
9.



10.



11.



Name: Modeling Data | 8.8

Ready, Set, Go!

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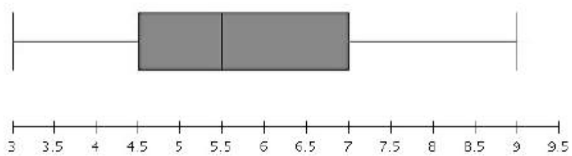
Ready

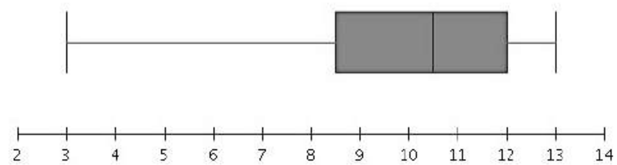
Topic: Describe the spread of the data.

Given the box-and-whisker plots describe the spread of the data set. Provide specifics about the median, range, interquartile range and so forth.

1.

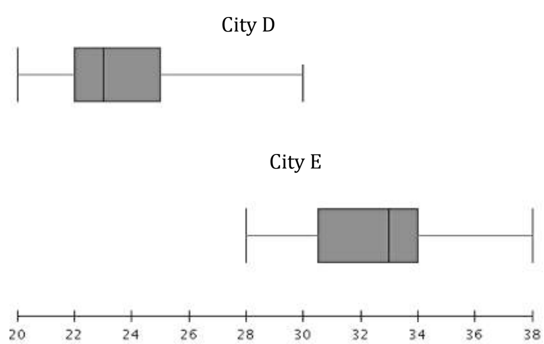


2.



3. If the box-and-whisker plots above represent the results of two different classes on the same assessment, which class did better? Why?

4. The two box-and-whisker plots below show the low temperatures for two cities in the United States.



a. Which city would be considered the coldest City D or City E? Why?

b. Do these cities ever experience the same temperature? How do you know?

c. Is there any way to know the exact temperature for any given day from the box and whisker plots?

d. What advantage if any could a scatter plot of temperature data have over a box and whisker plot?

Name: Modeling Data 8.8

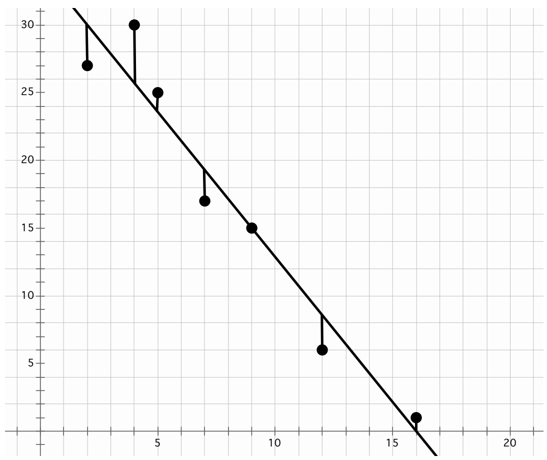
Set

Topic: Residuals, residual plots and correlation coefficients.

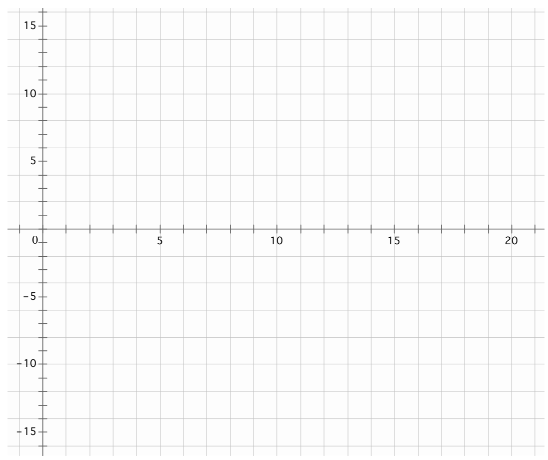
The Data Sheets below are scatter plots that have the regression line and the residuals indicated.

5a. Mark on the graph where $(\overline{x}, \overline{y})$ would be located.
 b. Use this given plot to create a residual plot.
 c. What would you predict the correlation coefficient to be?

Data Sheet 1

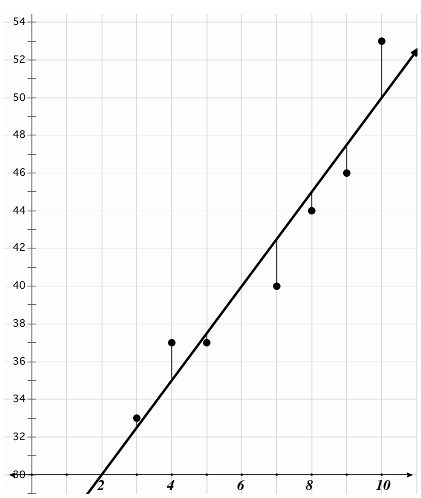


Residual Plot 1

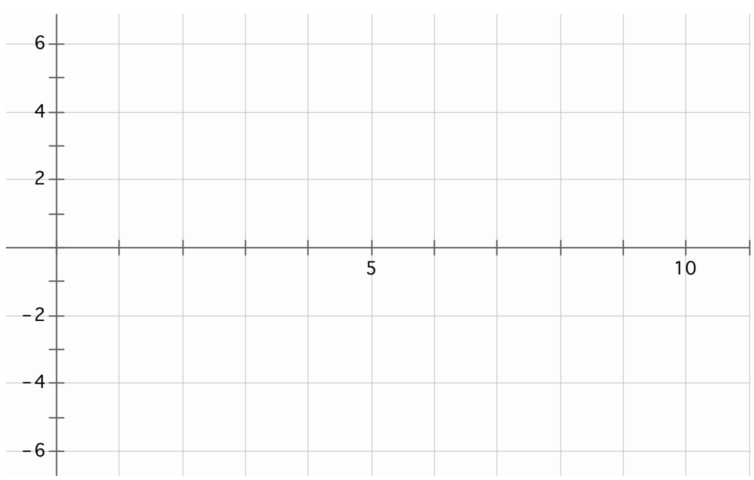


6a. Mark on the graph where $(\overline{x}, \overline{y})$ would be located.
 b. Use this given plot to create a residual plot.
 c. What would you predict the correlation coefficient to be?

Data Sheet 2



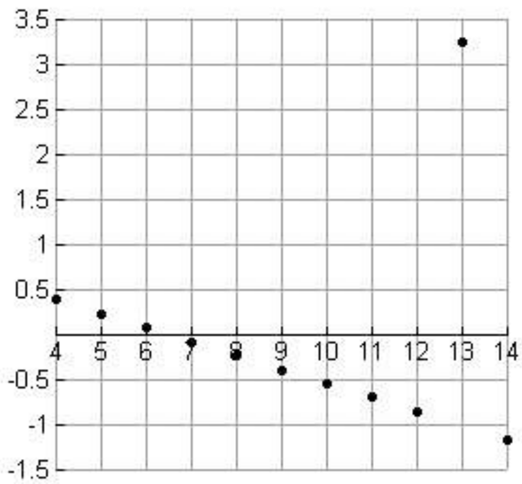
Residual Plot 2



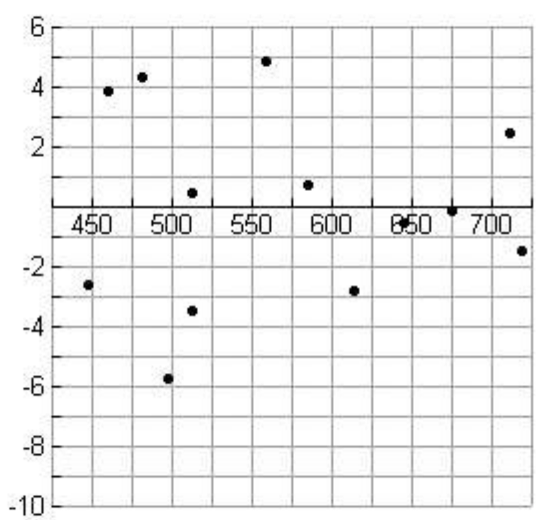
Name: Modeling Data 8.8

The following graphs are residual plots. Analyze the residual plots to determin how well the prediction line (line of best fit) describes the data.

7. Plot 1 Analysis



8. Plot 2 Analysis



Name: Modeling Data | 8.8

Go

Topic: Geometric constructions.

9. Construct an isosceles triangle with a compass and straight edge.

10. Construct a square using compass and straight edge..

11. Use a compass and straight edge to construct a hexagon inscribed in a circle.