# Assisting Students Struggling with Mathematics: RTI for Elementary Schools

# 8 Recommendations

1. Screen to identify and provide interventions to students identified.
2. Materials: K-5 should focus on whole numbers and 4-8 on rational numbers.
3. Intervention should be explicit and systematic: models for problem solving, verbalization of thought process, guided practice, corrective feedback, and frequent cumulative review.
4. Intervention on solving word problems based on common underlying structures.

# Recommendations Cont.

5. Students work with visual representations with interventionists being proficient in this.
6. 5-10 minutes should be spent building retrieval of basic facts.
7. Monitor the progress.
8. Motivational strategies should occur in Tier 2 and 3 interventions.

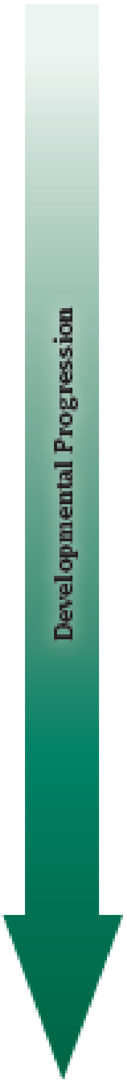
# How do we do this for K-2?

Recommendation 1: Need to give attention to counting and counting strategies

How do I do this?

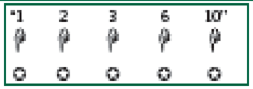
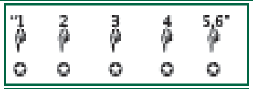
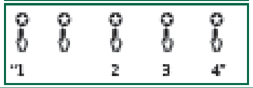
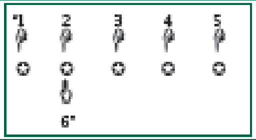
- Read: Article by Kathy Richardson
- Click on document. Scroll to page 19 (which is page 13). See next 2 slides.

**Table 3. Examples of a specific developmental progression for number knowledge**




| | |
|---|---|
| **Subitizing (small-number recognition)** | Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as non-examples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three. |
| | **A child is ready for the next step when**, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers. |
| **Meaningful object counting** | Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle). |
| | **A child is ready for the next step when**, for example, if given five blocks and asked, "How many?" he or she counts by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five." |
| **Counting-based comparisons of collections larger than three** | Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., "seven" items is more than "six" items because you have to count further). |
| | **A child is ready for the next step when** he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more). |
| **Number-after knowledge** | Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one. |
| | **A child is ready for the next step when** he or she can answer questions such as, "What comes after five?" by stating "five, six" or simply "six" instead of, say, counting "one, two, ... six." |
| **Mental comparisons of close or neighboring numbers** | Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that "nine" is larger than "eight"). |
| | **A child has this knowledge when** he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers. |
| **Number-after equals one more** | Once children can mentally compare numbers and see that "two" is one more than "one" and that "three" is one more than "two," they can conclude that any number in the counting sequence is exactly one more than the previous number. |
| | **A child is ready for the next step when** he or she recognizes, for example, that "eight" is one more than "seven." |

**Table 4. Common counting errors**

| Type of Counting Error | Example | Remedy |
|---|---|---|
| **SEQUENCE ERROR** | | |
| Saying the number sequence out of order, skipping numbers, or using the same number more than once. |  "1 2 3 6 10" | Practice reciting (or singing) the single-digit sequence, first focusing on one to ten, then later moving on to numbers greater than ten. |
| Struggling with the count sequence past twelve. | Skips 15:<br>"1…13, 14, 16, 17, 18."<br>Uses incorrect words:<br>"1…13, 14, fiveteen."<br>"1…18, 19, 10-teen" or<br>"1…29, 20-ten, 20-eleven."<br>Stops at a certain number:<br>"1…20" (stops)<br>"1…20" (starts from 1 again) | Highlight and practice exceptions, such as *fif* + *teen*. Fifteen and thirteen are commonly skipped because they are irregular.<br>Recognize that a nine signals the end of a series and that a new one needs to begin (e.g., nineteen marks the end of the teens).<br>Recognize that each new series (decade) involves combining a decade and the single-digit sequence, such as twenty, twenty plus one, twenty plus two, etc.<br>Recognize the decade term that begins each new series (e.g., twenty follows nineteen, thirty follows twenty-nine, and so forth). This involves both memorizing terms such as ten, twenty, and thirty by rote and recognizing a pattern: "add -*ty* to the single-digit sequence" (e.g., *six* + *ty*, *seven* + *ty*, *eight* + *ty*, *nine* + *ty*). |
| **COORDINATION ERROR** | | |
| Labeling an object with more than one number word. |  "1 2 3 4 5, 6" | Encourage the child to slow down and count carefully. Underscore that each item needs to be tagged only once with each number word. |
| Pointing to an object but not counting it. |  "1 2 3 4" | Same as above. |
| **KEEPING TRACK ERROR** | | |
| Recounting an item counted earlier. |  "1 2 3 4 5 6" | Help the child devise strategies for sorting counted items from uncounted items. For movable objects, for instance, have the child place counted items aside in a pile clearly separated from uncounted items. For pictured objects, have him or her cross off items as counted. |
| **SKIM** | | |
| No effort at one-to-one counting or keeping track. | Waves finger over the collection like a wand (or jabs randomly at the collection) while citing the counting sequence (e.g., "1, 2, 3…9, 10"). | Underscore that each item needs to be tagged with one and only one number word and help the child to learn processes for keeping track. Model the counting. |
| **NO CARDINALITY RULE** | | |
| Not recognizing that the last number word used in the counting process indicates the total. | Asked how many, the child tries to recount the collection or simply guesses. | Play *Hidden Stars* with small collections of one to three items first and then somewhat larger collections of items. |

# How do we do this for K-2?

Recommendation 2: Number Composition and Decomposition to understand the place value and make 10.

How do I do this?

- Greg Tang games and worksheets
- Teacher tube short clip

# How do we do this for K-2?

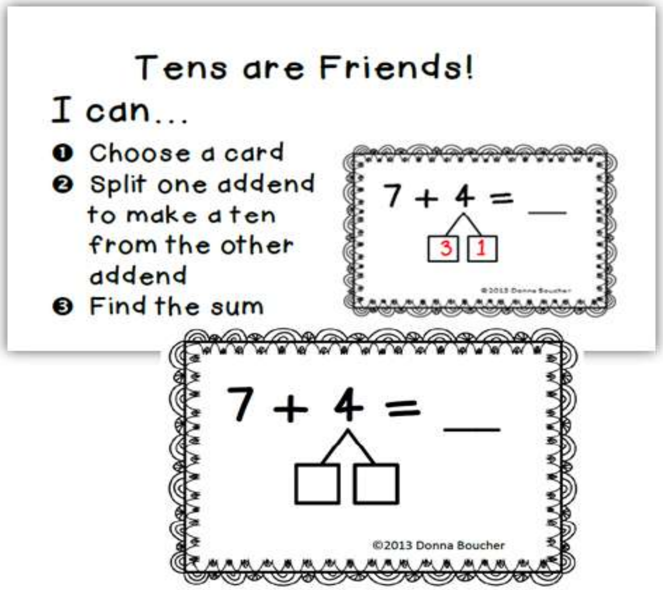
Recommendation 3: Meaning of addition and subtraction and the reasoning behind the algorithms

- Greg Tang K-2 handouts
- Kathy Richardson Book 2
- Unpacking documents

# How do we do this for K-2?

Recommendation 4: Build fact fluency with strategy cards

- Website for strategy cards.
- Another site for strategy card site



# Access Number Knowledge Test

1. Click on Access Number Knowledge Test

2. Scroll down to Unit 5 Participant Handouts. This is what you will see:

**Unit 5 Participant Handouts**

Number Knowledge Test Levels 0, 1, 2 and 3 (Dec 2013)
Number Knowledge Test Props Full Size (Dec 2013)
Number Knowledge Test Recording Form (Dec 2013)
The Number Knowledge Test Admin and Scoring (Dec 2013)

1. Scroll down to Unit 6 Participant Handout. This is what you will see.

**Unit 6 Participant Handouts**

Math Instruction Checklist (Dec 2013)

# How do we do this for 3-5?

Recommendation 1: Address any whole number issues, but at the same time begin the work on rational numbers. The emphasis should be on fractions.

- Fraction Video
- Teacher Video
- Series of Videos
- Site for more videos

## Grade 1

- Introduction to Fractions
- Equal shares- halves and fourths

## Grade 2

- Fractional Language
- Equal shares- halves, fourths, thirds
- Foundation for equivalency

## Grade 3

- The Meaning of Fractions
- The Number Line and Number Line Diagrams
- Equivalent Fractions
- Comparing Fractions

## Grade 4

- Equivalent Fractions
- Comparing Fractions
- Adding and Subtracting Fractions- Like Denominators
- Multiplication of a Fraction by a Whole Number

## Grade 5

- Adding and Subtractracting Fractions- Unlike Denominators
- Multiplying Fractions
- Multiplication as Scaling
- Dividing Fractions- Whole Number by a Fraction and Fraction by a Whole Whole Number

## Grade 6

- Dividing Fractions- Fraction by a fraction

http://ime.math.arizona.edu/progressions/

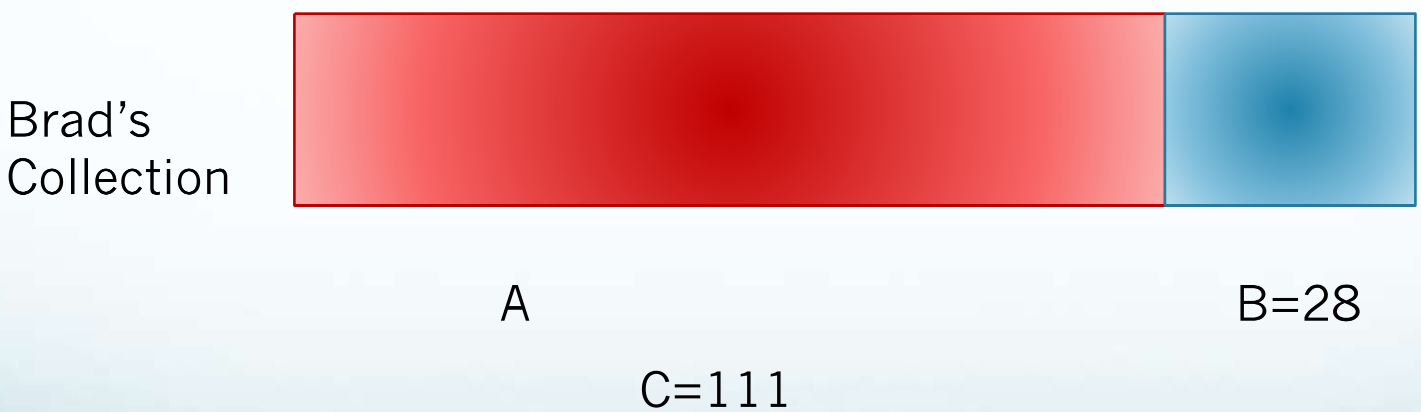
# How do we do this for 3-5?

Recommendation 2: Visual models should be used with an emphasis on using bar or strip models with movement to the number line. At the same time, working on problem solving and putting fractions into context.

- Greg Tang Worksheets
- Model Drawing websites

# Problem Types: Use Simple Bar Diagrams to Solve

- Change problem example: Brad has a bottle cap collection. After Madhavi gave Brad 28 more bottle caps, Brad had 111 bottle caps. How many bottle caps did Brad have before Madhavi gave him more?




Brad had _____ bottle caps before Madhavi gave him more.

# Compare problems

- There are 21 hamsters and 32 kittens at the pet store. How many more kittens are at the pet store than hamsters?


There are ____ more kittens at the pet store than hamsters.

# Bar Diagrams help make sense of fractions.

- Shauntay spent 2/3 of the money she had on a book that cost $26. How much money did Shauntay have before she bought the book?

Shauntay's money at first


2 parts = $26 $26 book

1 part = 26/2= $13

3 parts = 3 * $13 = $39 Shauntay had $39 before she bought the book.

Shauntay had _____ amount of money before she bought the book.

# How do we do this for 3-5?

Recommendation 3: Building fact fluency with strategy cards.

- Website for strategy cards.
- Another site for strategy card site

# 4 Criteria for Intervention Materials

1. How well do the materials integrate computation with solving problems and pictorial representations rather than teaching computation apart from problem solving.
2. The materials stress the reasoning underlying calculation methods and focus students attention on making sense of mathematics.
3. Materials ensure that students build algorithmic proficiency.
4. Materials include frequent review for both consolidating and understanding the links of the mathematical principles.

# Focus of Interventions

Needs to be Explicit and Systematic

- Clear models of problem solving with all problem types represented
- Think aloud modeled by teachers as well as the students solving the problems
- Guided practice
- Extensive corrective feedback
- Frequent cumulative review
- Use of correct vocabulary
- Language support

# How do we instruct on word problems?

- Look at the underlying structures.
- Explicitly teach students about the structure of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.
- Teaching part-part whole structure

Glossary

Table 1 Common addition and subtraction situations[1]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| Add to | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? (K) | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 (1st) | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 One-Step Problem (2nd) |
| Take from | Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? (K) | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3 (1st) | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 One-Step Problem (2nd) |
| | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown[2]** |
| Put Together/ Take Apart[2] | Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? (K) | Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ? (1st) | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 (K) |
| | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| Compare[3] | ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1st) | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? One-Step Problem (1st) | (Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? ? + 3 = 5 One-Step Problem (2nd) |
| | ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ? (1st) | (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ? One-Step Problem (2nd) | (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? One-Step Problem (1st) |

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

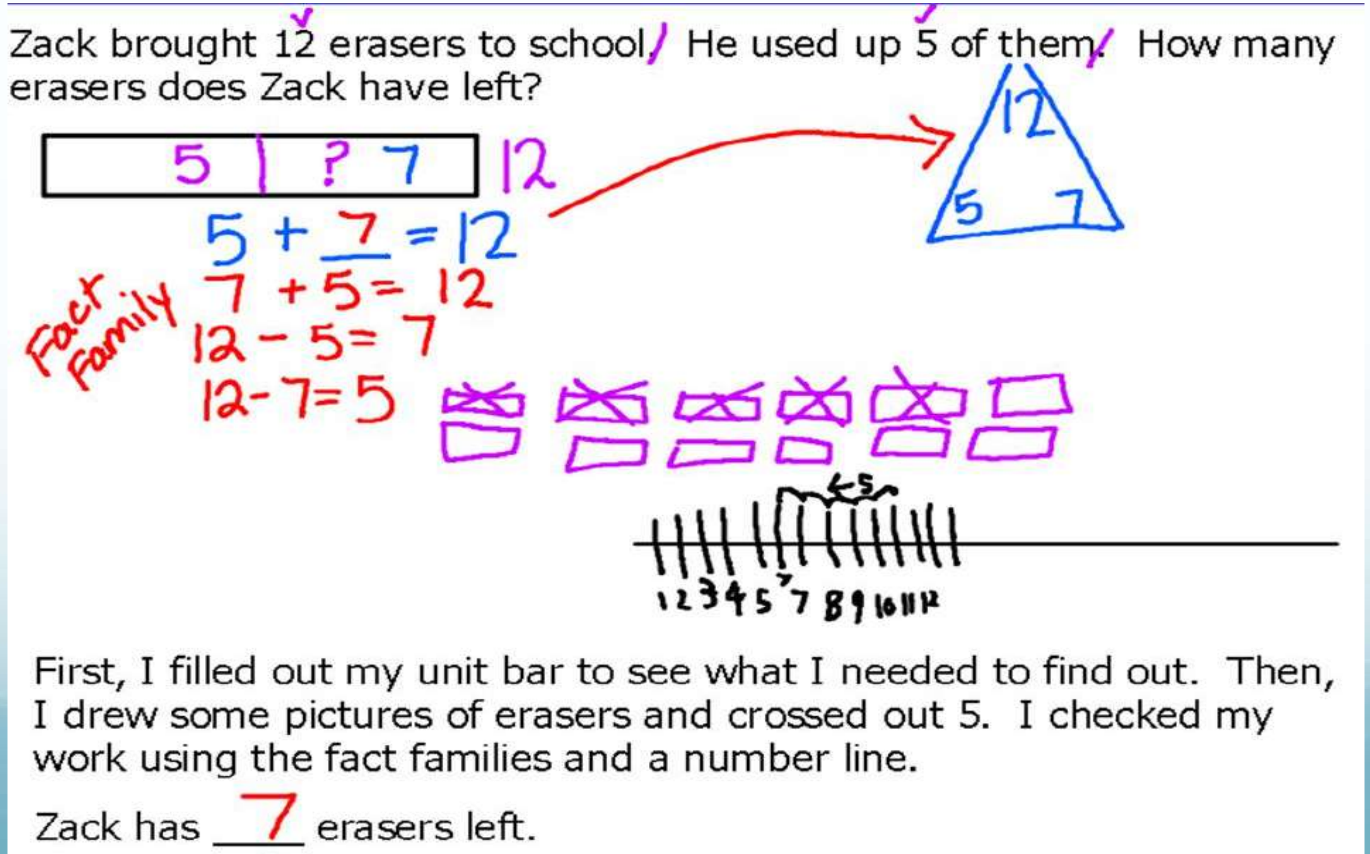
# How do we instruct on word problems?

Use guided questions.

- So what type of problem is it?
- Is it a part-part whole problem or a comparative problem?
- What in the story made you think that?
- What is it asking you?
- Have you chunked the problem?
- What are the relationships that are important in this problem?
- Does your answer make sense?
- Justify your answer for us?

# How do we instruct on word problems?

Partially work examples followed by students practicing individually or in pairs with visual representation such as a bar model.



# How do we instruct on word problems?

Look for relevant and irrelevant information and discuss how to determine the difference.

- For example: A square garden has a walking track that is eight feet wide along its sides. If one side of the garden is ten meters long, find the distance traveled by Hamid if he walks around the garden twice.

# Concrete Models are used first in Problem Solving

Primary:

- Use concrete objects more extensively in the initial stages of learning to reinforce the understanding of basic concepts and operations.
  - Unifix cubes for boxes and Base 10 for computation.
- Part/Part/Whole boxes
- Number lines with counting up and counting down
- Goal is for the student to develop a mental number line.
- Consistent language is important across representational systems.

# Extensive use of Visual Representations

Upper

- Use concrete when visual isn't enough to help with understanding.
  - Unifix cubes to represent bars, place value disks for computation.
- Diagram and pictorial representations to teach fractions such as bar diagrams/model drawing
- Focus on fading away to eventually reach the abstract.
- Consistent language is important across representational systems.

# How do we make this work for our county?

- Lots of high quality PD with emphasis on content and model drawing.
- Starting with concrete, moving to pictorial, and finally making that connection to abstract.
- Understanding how to teach mathematical content through problem solving.
- Understanding how to connect mathematical ideas to one another.


QUESTIONS