Mathematics Curriculum Guide

Honors Geometry

2017-18

Topic 5: Polygons and Quadrilaterals

To begin this unit, students will build on their understanding of congruent triangles and parallel lines and transversal by learning about properties of parallelograms. They will further develop conclusions by proving the converses of these properties and theorems. Students will also learn precise definitions for special parallelograms and develop ideas to prove theorems about their properties as well as the converses of those theorems. As students apply these new learnings to coordinate geometry, they will move from the quantitative to the abstract.

Common Misconceptions and/or Errors:

- **Properties of Parallelograms:** If it is known that two sides of a quadrilateral are parallel, but it is not known if they are congruent, then it cannot be determined from that information alone whether the quadrilateral is a parallelogram. For example, if it is given that $\overline{PQ} \parallel \overline{SR}$, but it is not known if $\overline{PQ} = \overline{SR}$, it would be incorrect to assume that the quadrilateral is a parallelogram.
- **Special Parallelograms:** Students may get confused about all the properties of special parallelograms. One way to help them see the relationship between the figures is with a Tree Map or a Double-bubble Map (when comparing two).
- **Coordinate Geometry:** Figure $PQRS$ is a parallelogram. Students might make the mistake of classifying it as a rhombus, either because its shape is close to that of a rhombus or because it has been shown that one pair of sides is parallel and congruent. However, proving one pair of sides is congruent and parallel is not sufficient to classify the figure as a rhombus. The other two sides need to be proved congruent.

Topic 5: Polygons and Quadrilaterals

Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 4 weeks/17 days
Start Date: November 16, 2017
Assessment Dates: December 14-15, 2017

Standards

G-SRT 5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-CO 11 Prove theorems about parallelograms.

G-GPE 7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Meaning-Making

Understandings

Students will understand that…

- The sum of the angle measures of a polygon depends on the number of sides the polygon has.
- Parallelograms have special properties regarding their sides, angles, and diagonals.
- Parallelograms can be used to prove theorems about parallel lines and their transversals.
- In a parallelogram: consecutive angles are supplementary, opposite angles are congruent, opposite sides are congruent, and the diagonals bisect each other.
- If a quadrilateral's sides, angles, and diagonals have certain properties, it can be shown that the quadrilateral is a parallelogram.
- The properties of parallelograms and algebra can be used to find the lengths of some sides and the measures of some angles of some parallelograms.
- The special parallelograms, rhombus, rectangle, and square, have basic properties about their sides, angles, and diagonals that help identify them.
- The angles, sides, and diagonals of a trapezoid have certain properties.
- The formulas for slope, distance and midpoint can be used to classify and to prove geometric relationships for figures in the coordinate plane.

Essential Questions

Students will keep considering…

- How can you determine the sum of the interior angles of an n-sided polygon?
- How do we know if a quadrilateral is a parallelogram?
- Are all parallelograms alike? Do they all share exactly the same properties?
- What properties do special parallelograms have? What makes them "special"?
- How can we use the coordinate plane to determine what kind of polygon we have?

Acquisition

Knowledge

Students will know…

Vocabulary: equilateral polygon, equiangular polygon, regular polygon, parallelogram, opposite sides, opposite angles, consecutive angles, rhombus, rectangle, square, trapezoid, base, leg, base angle, isosceles trapezoid, midsegment of a trapezoid, kite, coordinate plane, slope, distance, midpoint

- **Theorems:** Polygon Angle-Sum Theorem, Corollary to the Polygon Angle-Sum Theorem (Interior Angle Theorem), Polygon Exterior Angle-Sum Theorem, Theorems 6-1 thru 6-22
- **Formulas:** Distance Formula, Midpoint Formula, Slope Formula
- Special Parallelograms: rhombus, rectangle, square

Skills

Students will be skilled at and able to do the following…

- Use properties of polygons and regular polygons to find the measures of angles, the lengths of segments, and perimeter.
- Use the properties of parallelograms to find the measure of angles, the lengths of segments, and to prove triangles congruent.
- Use one of five methods/theorems to establish that a quadrilateral is a parallelogram.
- Use properties of rhombus, rectangle, and square to find segment lengths and measures.
- Use properties of kite, trapezoid, and isosceles trapezoid to find angle measures and segment lengths.

Topic 5: Polygons and Quadrilaterals

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

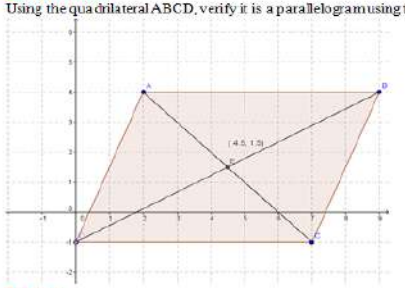
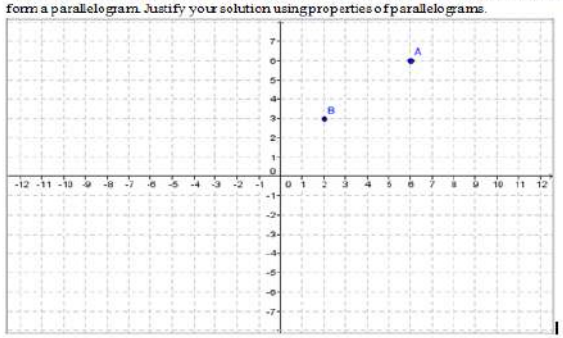
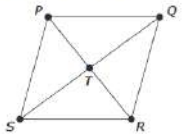
1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 5: Polygons and Quadrilaterals

The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Use properties of polygons and regular polygons to find the measures of angles, the lengths of segments, and perimeter. • Use the properties of parallelograms to find the measure of angles, the lengths of segments, and to prove triangles congruent. • Use one of five methods/theorems to establish that a quadrilateral is a parallelogram. • Use properties of rhombus, rectangle, and square to find segment lengths and measures. • Use properties of kite, trapezoid, and isosceles trapezoid to find angle measures and segment lengths.	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Use the technique of breaking an argument into cases. (2,3) Distinguish correct logic or reasoning form that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3)	**G-SRT 5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G-CO 11** Prove theorems about parallelograms. **G-GPE 7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	G.CO.C.11 Item Using the quadrilateral ABCD, verify it is a parallelogram using the properties of parallelogram  Solution: Opposite sides congruent: $AB = 7, CD = 7, AD = 5.39, BC = 5.39$ Diagonals bisect each other: $DE \approx 5.15, EB \approx 5.15, AE \approx 3.54, EC \approx 3.54$ G.CO.C.11 Item 2 Use the points shown in the coordinate plane pictured to determine two other points that can be form a parallelogram. Justify your solution using properties of parallelograms.  Use the information provided to answer Part A through Part D for question 26. One method that can be used to prove that the diagonals of a parallelogram bisect each other is shown in the given partial proof.  Given: Quadrilateral PQRS is a parallelogram Prove: $\overline{PT} = \overline{RT}$ $\overline{ST} = \overline{QT}$ Statements / Reasons 1. Quadrilateral $PQRS$ is a parallelogram — 1. Given 2. $\overline{PQ} \parallel \overline{SR}$, $\overline{PS} \parallel \overline{QR}$ — 2. Definition of parallelogram 3. $\angle PQS \cong \angle RSQ$, $\angle QPR \cong \angle SRP$ — 3. ? 4. ? — 4. Opposite sides of a parallelogram are congruent 5. $\triangle SRT \cong \triangle QPT$ — 5. ? 6. $\overline{PT} \cong \overline{RT}$, $\overline{ST} \cong \overline{QT}$ — 6. Corresponding parts of congruent triangles are congruent 7. $\overline{PT} = \overline{RT}$, $\overline{ST} = \overline{QT}$ — 7. Definition of congruent line segments

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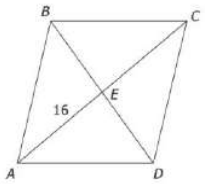
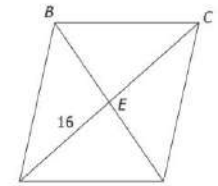
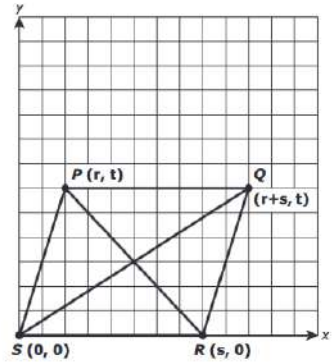
• Use properties of polygons and regular polygons to find the measures of angles, the lengths of segments, and perimeter. • Use the properties of parallelograms to find the measure of angles, the lengths of segments, and to prove triangles congruent. • Use one of five methods/theorems to establish that a quadrilateral is a parallelogram. • Use properties of rhombus, rectangle, and square to find segment lengths and measures. • Use properties of kite, trapezoid, and isosceles trapezoid to find angle measures and segment lengths.	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Use the technique of breaking an argument into cases. (2,3) Distinguish correct logic or reasoning form that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3)	**G-SRT 5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G-CO 11** Prove theorems about parallelograms. **G-GPE 7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	Use the information provided to answer Part A and Part B for question 14. The figure shows parallelogram *ABCD* with $AE = 16$.  not drawn to scale **14. Part A** Let $BE = x^2 - 48$ and let $DE = 2x$. What are the lengths of $\overline{BE}$ and $\overline{DE}$? Justify your answer. **Part B** What conclusion can be made regarding the specific classification of parallelogram *ABCD*? Justify your answer. Use the information provided to answer Part A and Part B for question 14. The figure shows parallelogram *ABCD* with $AE = 16$.  not drawn to scale **14. Part A** Let $BE = x^2 - 48$ and let $DE = 2x$. What are the lengths of $\overline{BE}$ and $\overline{DE}$? Justify your answer. **Part B** What conclusion can be made regarding the specific classification of parallelogram *ABCD*? Justify your answer. Enter your answer and your justification in the space provided. **Part D** Another method of proving diagonals of a parallelogram bisect each other uses a coordinate grid.  What could be shown about the diagonals of parallelogram *PQRS* to complete the proof? Ⓐ $\overline{PR}$ and $\overline{SQ}$ have the same length. Ⓑ $\overline{PR}$ is a perpendicular bisector of $\overline{SQ}$. Ⓒ $\overline{PR}$ and $\overline{SQ}$ have the same midpoint. Ⓓ Angles formed by the intersection of $\overline{PR}$ and $\overline{SQ}$ each measure 90°.

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Transfer Goals
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Essential Questions:	**Standards:** G-SRT 5, G-CO 11, G-GPE 7 **Timeframe:** 4 weeks/17 days **Start Date:** November 16, 2017 **Assessment Dates:** December 14-15, 2017
• How can you determine the sum of the interior angles of an n-sided polygon? • How do we know if a quadrilateral is a parallelogram? • Are all parallelograms alike? Do they all share exactly the same properties? • What properties do special parallelograms have? What makes them "special"? • How can we use the coordinate plane to determine what kind of polygon we have?	

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 Day (Nov. 16th)	**Topic Opener** pg. 351 *Building a Kite*					
2 Days (Nov. 17-18)	**Lesson 6-1: Polygon Angle Sum Theorems** SMP: 1,3 (pp. 353-358) **G-SRT 5**	**Focus Question:** • How can you determine the sum of the interior angles of an n-sided polygon? **Inquiry Question:** In a pentagon, what is the sum of all the interior angles?	• The sum of the angle measures of a polygon depends on the number of sides the polygon has.	**Vocabulary:** equilateral polygon, equiangular polygon, regular polygon **Concepts:** • Polygon Angle-Sum Theorem • Corollary to the Polygon Angle-Sum Theorem (Interior Angle Theorem) • Polygon Exterior Angle-Sum Theorem	• Use properties of polygons and regular polygons to find the measures of angles, the lengths of segments, and perimeter.	**CC Problems:** #4, 5,6, **26,** 27, **28, 38**, 37, **40,** 45, 47 **Thinking Map:** Tree Map with Theorems (include formulas, drawings, and examples).

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
3 Days (Nov. 28-30)	**Lesson 6-2: Properties of Parallelograms** SMP: 1,3,4 (pp. 359-366) **G-CO 11**	**Focus Question:** • If we know that the opposite sides of a quadrilateral are parallel, then what other properties might be true? **Inquiry Question:** Pg. 359 Solve It!	• Parallelograms have special properties regarding their sides, angles, and diagonals. • Parallelograms can be used to prove theorems about parallel lines and their transversals. • In a parallelogram: consecutive angles are supplementary, opposite angles are congruent, opposite sides are congruent, and the diagonals bisect each other.	**Vocabulary:** parallelogram, opposite sides, opposite angles, consecutive angles **Concepts:** • Theorems 6-3 thru 6-7	• Use the properties of parallelograms to find the measure of angles, the lengths of segments, and to prove triangles congruent.	**CC Problems:** #6,7, 8, **13**, 14-16, **28**, 31, 32, 33, 34, 35, 36, 37, 43 **Thinking Map:** Bridge Map with Theorems.
3 Days (Dec. 1, 2, & 5)	**Lesson 6-3: Proving That a Quadrilateral Is a Parallelogram** SMP: 1,3 (pp. 367-374) **G-CO 11**	**Focus Question:** • How do we know if a quadrilateral is a parallelogram? **Inquiry Question:** Pg. 367 Solve It!	• If a quadrilateral's sides, angles, and diagonals have certain properties, it can be shown that the quadrilateral is a parallelogram. • The properties of parallelograms and algebra can be used to find the lengths of some sides and the measures of some angles of some parallelograms.	**Vocabulary:** (words from previous lessons in this unit) **Concepts:** • Theorems 6-8 thru 6-12	• Use one of five methods/theorems to establish that a quadrilateral is a parallelogram.	**CC Problems:** #4,5,6, 16, **17**, **18**, **19**, 20, 21, 25, **26**, 27, 30, 31 **Prob. #3 on p. 371** **Thinking Map:** Bridge Map with Theorems (p. 371).
2 Days (Dec. 6-7)	**Lesson 6-4: Properties of Rhombus, Rectangle and Square** SMP: 1,3 (pp. 375-382) **G-CO 11**, **G-SRT 5**	**Focus Question:** • Are all parallelograms alike? Do they all share exactly the same properties? **Inquiry Question:** Pg. 375 Solve It!	• The special parallelograms, rhombus, rectangle, and square, have basic properties about their sides, angles, and diagonals that help identify them.	**Vocabulary:** rhombus, rectangle, square **Concepts:** • Special Parallelograms: rhombus, rectangle, square • Theorems 6-13 thru 6-15	• Use properties of rhombus, rectangle, and square to find segment lengths and measures.	**CC Problems:** #5, 6, 18-23, **40**, **41**, 45, **46**, **48**, 49, 50 **Thinking Map:** Tree Map with information on rhombus, rectangle, and square.

Common Core Practices

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- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days (Dec. 8-9)	**Lesson 6-6: Trapezoids and Kites** SMP: 1,3,4,6 (pp. 389-397) **G-SRT 5**	**Focus Questions:** • If a quadrilateral has only two parallel sides, then what other properties does it possess? **Inquiry Questions:** Pg. 389 Solve It!	• The angles, sides, and diagonals of a trapezoid have certain properties.	**Vocabulary:** trapezoid, base, leg, base angle, isosceles trapezoid, midsegment of a trapezoid, kite **Concepts:** • Theorems 6-19 thru 6-22	• Use properties of kite, trapezoid, and isosceles trapezoid to find angle measures and segment lengths.	**Common Core Problems:** #4,5,6, 25, **26**, **27**, 39-44 **45**, 46, 47-52, 53, 54, 55, 56, 57-62, 63 **STEM:** #37, 38 **Thinking Map:** Tree Map that classifies quadrilaterals (see p. 393).
1 Day (Dec. 12th)	**Topic 5 Performance Task (p. 419)**					
1 Day (Dec. 13th)	**Lesson 6-7: Polygons in the Coordinate Plane** SMP: 1,3,4,8 (pp. 400-405) **G-GPE 7**	**Focus Questions:** • How can we use the coordinate plane to determine what kind of polygon we have? **Inquiry Questions:** Pg. 400 Solve It!	• The formulas for slope, distance and midpoint can be used to classify and to prove geometric relationships for figures in the coordinate plane.	**Vocabulary:** coordinate plane, slope, distance, midpoint **Concepts:** • Distance Formula • Midpoint Formula • Slope Formula	• Use the formulas of slope, midpoint, distance, and the properties of special quadrilaterals to prove that a quadrilateral is special. • Verify properties of special quadrilaterals.	**Common Core Problems:** #3,4, 8-13, **31, 32,** 35, **36**, 37, 38, 39 **Thinking Map:** Bridge Map that shows the relationship between the distance, midpoint, and slope formulas.
1 Day (Dec. 14th)	**Review Topic 5 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
2 days (Dec. 15-16)	**Topic 5 Assessment** (Created and provided by PUSD)					

Common Core Practices

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