Mathematics Curriculum Guide

Honors Geometry

2017-18

Topic 3: Parallel and Perpendicular Lines

To begin this unit, students will use precise vocabulary and definitions to discuss angle pairs and parallel lines. They will build their reasoning from the previous unit to analyze angle-pair relationships and prove geometric statements. Students will also learn to write equations of lines and find equations of lines that are parallel or perpendicular to the given line and/or through a point on the coordinate plane. They will apply those equations to a broad understanding of the special relationships of parallel and perpendicular lines with regards to slope and apply deductive reasoning, algebra, and construction to create proofs to determine whether or not lines are parallel or perpendicular. Students will see how the concept of slope is used in both algebraic and geometric contexts. As they move on in this course they will continue to see the ideas from these two areas interweave. In addition to concepts of parallel and perpendicular lines, students will also learn to prove theorems about triangles which include the measures of interior angles and exterior angles, as well as triangle midsegments (which is supported with parallel line understandings).

Common Misconceptions and/or Errors:

- **Angle Relationships:** Students may assume that the relations of corresponding angles, same-side interior angles, and alternate interior and exterior angles do not exist at all when the two intersected lines are not parallel. Students need to understand that the angle-pair relations exist when two lines are intersected by a transversal, whether the lines are parallel or not. Likewise, they need to understand that some of the angle-pair relations can be used to find angle measures *only* when the intersected lines are parallel.
- **Slopes of Parallel and Perpendicular Lines:** Students may attempt to read the slope directly from equations not given in slope-intercept form or point-slope form. Given an equation such as $3y = 2x + 6$, for example, students may assume that the slope is 2 without first solving the equation for y. When drawing and identifying perpendicular lines, students may forget that the two slopes of perpendicular lines must be both opposite and reciprocal: they need to understand that lines with slopes of 2 and -2 are not perpendicular, nor are lines with slopes 2 and ½.
- **Special Segments in Triangles:** Special segments can often be difficult to distinguish from each other. Students should use angle and segment congruency marks and right angle marks to help make clear the purpose of a given segment.

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Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 3 weeks/15 days
Start Date: Sept. 29, 2017
Assessment Dates: Oct. 18-19, 2017

Standards

G-CO 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc.~~

G-CO 9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent…

G-GPE 5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-CO 10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; ~~base angles of isosceles triangles are congruent~~; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; ~~the medians of a triangle meet at a point.~~

Meaning-Making

Understandings

Students will understand that…

- When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs.
- The special angle pairs formed by parallel lines and a transversal are either parallel or congruent. Central angle pairs can be used to decide whether two lines are parallel.
- A line can be graphed and its equation written (in different forms) when certain facts about the line such as its slope and a point on the line are known.
- The relationship between parallel or perpendicular lines can sometimes be used to write the equation of a line.
- Comparing the slopes of two lines can show whether the lines are parallel or perpendicular.
- The sum of the angle measures of a triangle is always the same - 180°.
- Any exterior angle of a triangle has a special relationship with the two remote interior angles of a triangle.
- Logical reasoning from one step to another is essential in building proofs. Reasons in a proof include given information, definitions, properties, postulates, and previously proven theorems.
- The midsegment of a triangle is related to the third side in two ways.

Essential Questions

Students will keep considering…

- How do we name angle pairs formed when a line intersects two lines?
- How are the angles formed by a line intersecting two parallel lines related?
- How can we determine if two lines are parallel based on angle pair relationships?
- How can we write the equation of line given two points? Given a point and a line parallel? Or, a point and a line perpendicular?
- How can we tell lines are parallel just by looking at their equations?
- What is always true about the interior angles of any triangle?
- How is the exterior angle of a triangle related to the interior angles?
- What is the relationship between a midsegment and the side opposite of it in a triangle?

Acquisition

Knowledge

Students will know…

Vocabulary: parallel lines, skew lines, parallel planes, transversals, alternate interior angles, same side interior angles, corresponding angles, alternate exterior angles, flow proof, slope, slope-intercept, standard form, point-slope form, negative slope, positive slope, undefined slope, or zero slope, reflexive property, symmetric property, transitive property, algebraic properties of equality, proof, and two-column proof, midsegment of a triangle

Theorems & Postulates: Corresponding Angles Postulate, Same-Side Angles Theorem, Alternate Interior Angles Theorem, and Alternate Exterior Angles Theorem, Corresponding Angles Postulate Converse, Same-Side Interior Angles Theorem Converse, Alternate Interior Angles Theorem Converse, and Alternate Exterior Angles Theorem Converse, Parallel Postulate, Triangle Angle-Sum Theorem, Triangle Exterior Angle Theorem, Triangle Midsegment Theorem

Concepts: Forms of Linear Equations, Slopes of Parallel Lines, Slopes of Perpendicular Lines

Skills

Students will be skilled at and able to do the following…

- Name angle pairs formed from a line intersecting two lines.
- Identify parallel lines, parallel planes, and skew lines from a given figure.
- Prove theorems about parallel lines.
- Use properties of parallel lines to find angle measures.
- Use angle pair relationships to determine whether or not two lines are parallel.
- Write a linear equation in various forms given two points, a point and a line parallel, a point and a line perpendicular.
- Determine if two lines are parallel or perpendicular based of the slopes of those lines.
- Use parallel lines to prove a theorem about triangles.
- Find measures of angles of triangles.
- Use a two-column format to write deductive proofs using algebraic properties.
- Use properties of midsegments to find segment lengths, angle measures, and coordinates.

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Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

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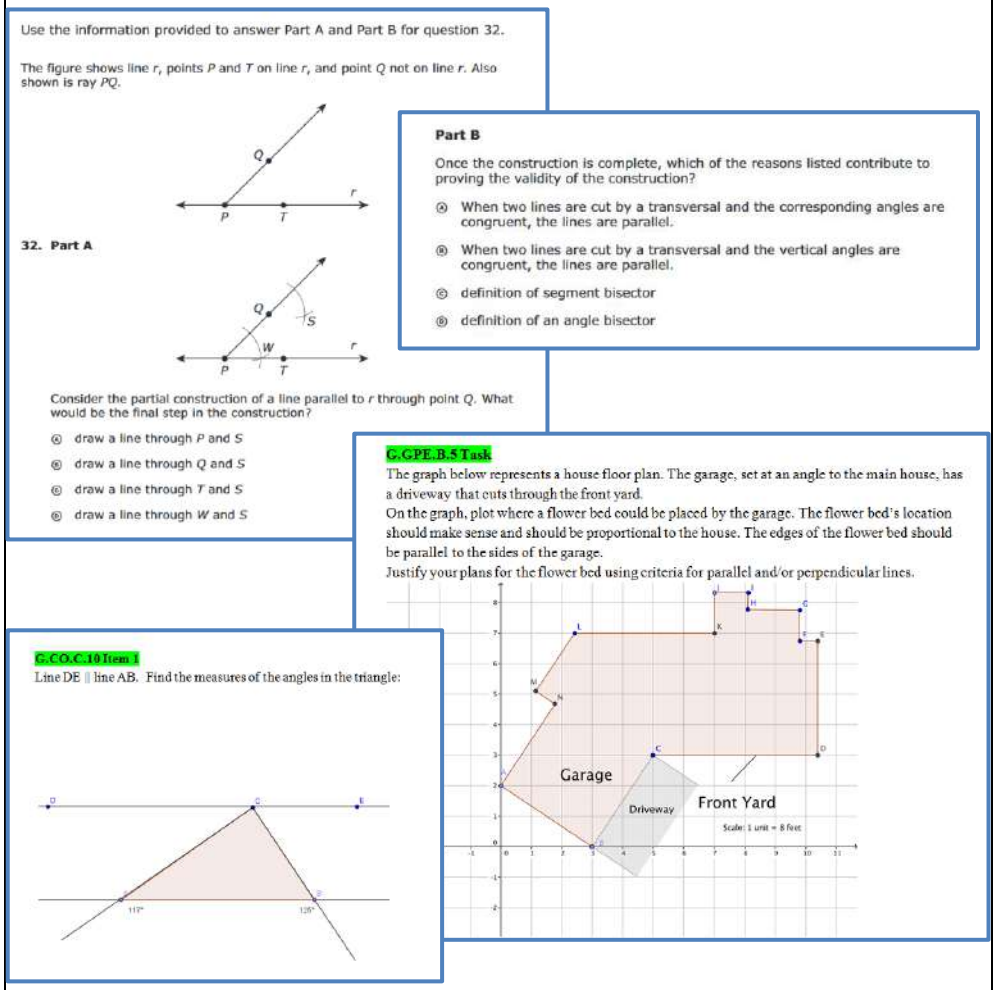
The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Name angle pairs formed from a line intersecting two lines. • Identify parallel lines, parallel planes, and skew lines from a given figure. • Prove theorems about parallel lines. • Use properties of parallel lines to find angle measures. • Use angle pair relationships to determine whether or not two lines are parallel. • Write a linear equation in various forms given two points, a point and a line parallel, a point and a line perpendicular. • Determine if two lines are parallel or perpendicular based of the slopes of those lines. • Use parallel lines to prove a theorem about triangles. • Find measures of angles of triangles. • Use a two-column format to write deductive proofs using algebraic properties. • Use properties of midsegments to find segment lengths, angle measures, and coordinates.	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Use the technique of breaking an argument into cases. (2,3) Distinguish correct logic or reasoning form that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3)	**G-CO 1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc.~~ **G-CO 9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent… **G-GPE 5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G-CO 10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; ~~base angles of isosceles triangles are congruent~~; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; ~~the medians of a triangle meet at a point.~~	

Topic 3: Parallel and Perpendicular Lines

Transfer Goals
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Essential Questions:

- How do we name angle pairs formed when a line intersects two lines?
- How are the angles formed by a line intersecting two parallel lines related?
- How can we determine if two lines are parallel based on angle pair relationships?
- How can we write the equation of line given two points? Given a point and a line parallel? Or, a point and a line perpendicular?
- How can we tell lines are parallel just by looking at their equations?
- What is always true about the interior angles of any triangle?
- How is the exterior angle of a triangle related to the interior angles?
- What is the relationship between a midsegment and the side opposite of it in a triangle?

Standards: G-CO 1, G-CO 9, G-GPE 5, G-CO 10

Timeframe: 3 weeks/15 days
Start Date: September 29, 2017
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Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 Day (Sept 29th)	**Topic Opener** pg. 139 *Planning the Paths for a Park*					
2 Days (Sept. 30th & Oct. 3rd)	**Lesson 3-1: Lines and Angles** SMP: 1,3,6 (pp. 140-146) **G-CO 1, G-CO 9** **Lesson 3-2: Properties of Parallel Lines** SMP: 1,3,4,6 (pp. 148-155) **G-CO 9**	**Focus Question:** • How do we name angle pairs formed when a line intersects two lines? • How are the angles formed by a line intersecting two parallel lines related? **Inquiry Question:** Pg. 140 Solve It! Pg. 141 Problem 1 Pg. 148 Solve it	• Not all lines and not all planes intersect. • When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs. • The special angle pairs formed by parallel lines and a transversal are either parallel or congruent. • Geometric postulates and theorems can be combined to find some angle measures.	**Vocabulary:** Parallel lines, skew lines, parallel planes, transversals, alternate interior angles, same side interior angles, corresponding angles, alternate exterior angles, Corresponding Angles Postulate, Same-Side angles Theorem, Alternate Interior Angles Theorem, and Alternate Exterior Angles Theorem	• Name angle pairs formed from a line intersecting two lines. • Identify parallel lines, parallel planes, and skew lines from a given figure. • Prove theorems about parallel lines. • Use properties of parallel lines to find angle measures.	**3-1 CC Problems:** #8, 9, 10, 24, 29, 36, 43, 44, 45, 47, 48 **3-2 CC Problems:** #5,6, 10, 21, 22, 23, 24, 25, 26, 28 **3-2:** p. 152 Problem 4 **Thinking Map:** Tree Map with different Angle Pairs formed by Transversals. **Constructing parallel lines:** www.youtube.com/watch?v=EKL_3xppHcQ

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days (Oct. 4-5)	**Lesson 3-3: Proving Lines Parallel** SMP: 1,3,7 (pp. 156-163) **G-CO 9**	**Focus Questions:** • How can we determine if two lines are parallel based on angle pair relationships? **Inquiry Questions:** Pg. 156 Solve It!	• Central angle pairs can be used to decide whether two lines are parallel. • Paragraph, two-column, and flow proofs are three forms of proof.	**Vocabulary:** flow proof, Corresponding Angles Postulate Converse, Same-Side Interior Angles Theorem Converse, Alternate Interior Angles Theorem Converse, and Alternate Exterior Angles Theorem Converse	• Use angle pair relationships to determine whether or not two lines are parallel.	**Common Core Problems:** #5, 6, 11, 12, 17-26, 29, 30, 31-34, 35-38, 39, 40, 41, 42-45, 46 **3-3:** p. 157-159 Problems 1-4 **Thinking Map:** Flow Map to demonstrate a proof.
3 Days (Oct. 6, 7, 10)	**Lesson 3-7: Equations of Lines in the Coordinate Plane** SMP: 1,3,4 (pp. 189-196) **G-GPE 5** **Lesson 3-8: Slopes of Parallel and Perpendicular Lines** SMP: 1,3,4 (pp. 197-204) **G-GPE 5**	**Focus Question:** • How can we write the equation of line given two points? Given a point and a line parallel? Or, a point and a line perpendicular? • How can we tell lines are parallel just by looking at their equations? **Inquiry Question:** Pg. 189 Solve It!	• A line can be graphed and its equation written when certain facts about the line such as its slope and a point on the line are known. • The equation of a line can be written in various forms. • Comparing the slopes of two lines can show whether the lines are parallel or perpendicular. • The relationship between parallel or perpendicular lines can sometimes be used to write the equation of a line.	**Vocabulary:** slope, slope-intercept, standard form, point-slope form, negative slope, positive slope, undefined slope, or zero slope **Concepts:** • Forms of Linear Equations • Slopes of Parallel Lines • Slopes of Perpendicular Lines	• Write a linear equation in various forms given two points, a point and a line parallel, a point and a line perpendicular. • Determine if two lines are parallel or perpendicular based of the slopes of those lines.	**3-7 CC Problems:** #5,6,7,42, 43, 48, 53, 56, 57, 58-60 **3-8 CC Problems:** #5,6,22, 27, 28, 29, 30-33, 34, 39, 40, 41, 42, 44, 45, 46, 47 **Thinking Map:** Double-bubble Map showing the similarities and differences between slope-intercept form and point-slope form. **For constructions:** https://www.mathsisfun.com/geometry/constructions.html

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day (Oct. 12th)	**Lesson 3-5: Parallel Lines and Triangles** SMP: 1,3,6 (pp. 171-178) **G-CO 10**	**Focus Questions:** • What is always true about the interior angles of any triangle? • How is the exterior angle of a triangle related to the interior angles? **Inquiry Questions:** Pg. 171 Solve It!	• The sum of the angle measures of a triangle is always the same. • Any exterior angle of a triangle has a special relationship with the two remote interior angles of a triangle. • Logical reasoning from one step to another is essential in building proofs. • Reasons in a proof include given information, definitions, properties, postulates, and previously proven theorems.	**Vocabulary:** reflexive property, symmetric property, transitive property, algebraic properties of equality, proof, and two-column proof **Concepts:** • Parallel Postulate • Triangle Angle-Sum Theorem • Triangle Exterior Angle Theorem	• Use parallel lines to prove a theorem about triangles. • Find measures of angles of triangles. • Use a two-column format to write deductive proofs using algebraic properties.	**Common Core Problems:** #7, 8, 25, 26, 27, 28, 33, 34, 36-40, 41, 42 **Proof 3-11:** p. 172 **Thinking Map:** Bridge Map to show the relationships between the Triangle Angle-Sum Theorem and the Triangle Exterior Angle Theorem.
2 Days (Oct. 13-14)	**Lesson 5-1: Midsegments of Triangles** SMP: 1,3,4,5 (pp. 285-291) **G-CO 10**	**Focus Questions:** • What is the relationship between a midsegment and the side opposite of it in a triangle? **Inquiry Questions:** Pg. 285 Solve It!	• To draw a midsegment, students must find the midpoint of two sides of a triangle and draw the segment joining the midpoints. • The midsegment of a triangle is related to the third side in two ways.	**Vocabulary:** midsegment of a triangle **Concepts:** • Triangle Midsegment Theorem	• Use properties of midsegments to find segment lengths, angle measures, and coordinates.	**Common Core Problems:** #4,5,6,25, 26, 27, 28, 29, 30, 37, 38-41, 42-44, 45, 46, 47, 48 **5-1:** p. 287 Problem 3
1 Day	**Lesson 3-6: Constructing Parallel and Perpendicular Lines** SMP: 1,3,4 **G-CO.D.12** **G-CO.D.13**	**Constructions with GeoGebra**				
1 day	**Review Topic 3 Concepts & Skills**					
2 days	**Topic 3 Assessment**					

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