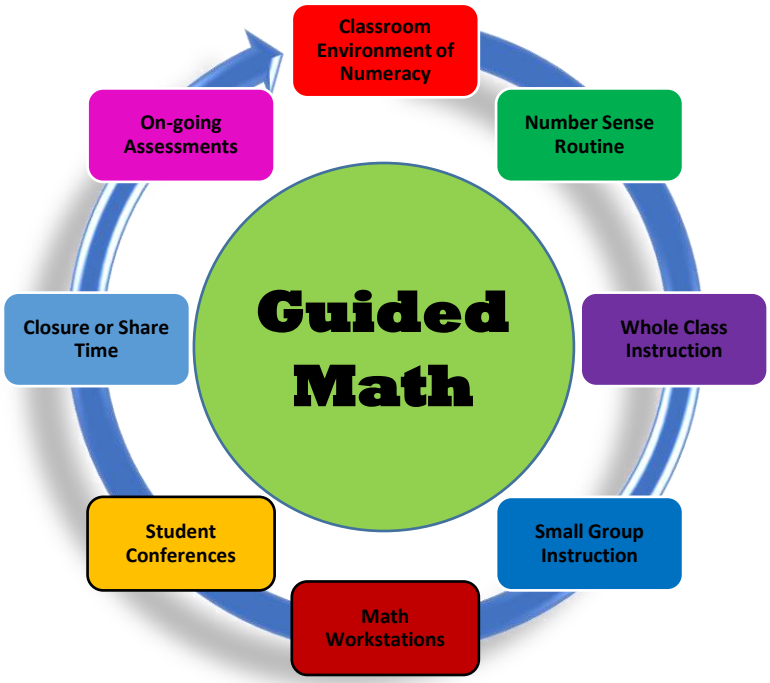
Chester County School District

K-5 Mathematics Framework
Guided Math Model



"Create a mathematically literate student."

Acknowledgements

Special thanks to the following educators who embraced this scope of work on behalf of the Chester County School District:

Mrs. Wendy Shuler……Director of Elementary Mathematics

Mrs. Rachel Owens……Math Coach | Lewisville Elementary

Mrs. Dequawnsha Worthy……..Math Coach | School of Inquiry

Mrs. Franchette Vance-Ingram……Math Coach | School of the Arts

Mrs. Amy Buffington……Math Coach | Chester Park COLT

Mrs. Tonya Weir……Math Coach | Great Falls Elementary

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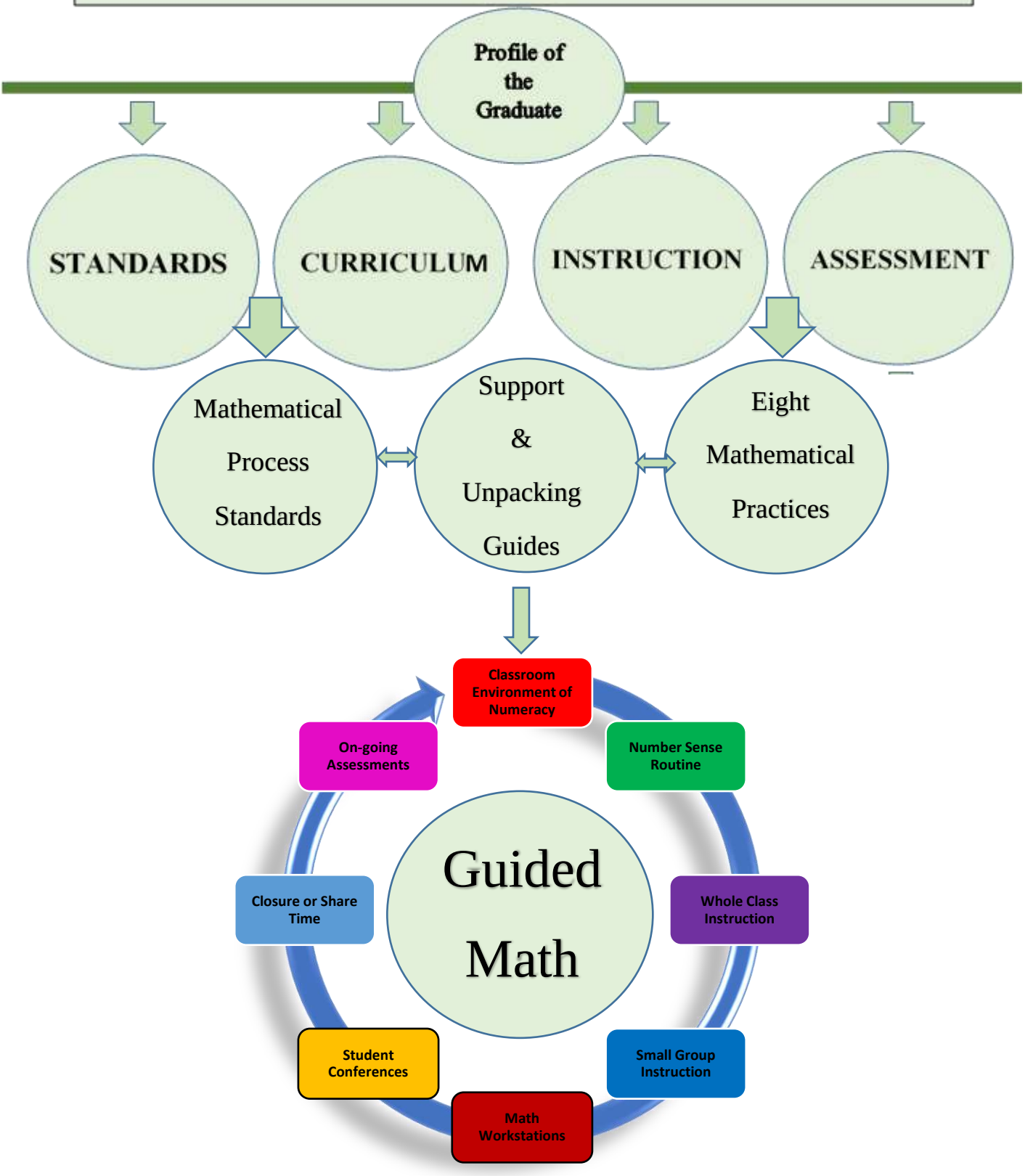
Purpose and Rationale

The Chester County School District K-5 Mathematics Framework is designed to provide all K-5 teachers in Chester County with a foundational understanding of best practices in the delivery of instruction in mathematics. The development of this framework was accomplished through the hard work of all of our math coaches and input from K-5 staff throughout the district during monthly meetings with the schools' math coaches.

Our Instructional Framework is grounded in math instructional practices that promote students' development of conceptual understanding and proficiency in the **Standards for Mathematical Practice.** We believe instruction should be authentic, engaging, and differentiated to meet the academic needs and interests of students through the mathematical processes that develop students' abilities to think critically, problem solve, make connections, communicate their ideas, represent what they have learned in various ways, make reasonable estimations, and provide proof of solutions. All students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary to graduate college or become career ready. In order to do this, teachers must speak a common academic language with clear, consistent definitions in order to provide the most effective Tier 1 instruction possible.

We support the **Guided Math Model** in teaching mathematics, which emphasizes both conceptual understanding and procedural fluency, which leads to increased mathematical proficiency for all students in Chester County. We believe students must have a comprehension of mathematical concepts, operations and relations as well as skills in carrying out procedures flexibly, accurately, efficiently, and appropriately. This is best accomplished through a well-articulated, comprehensive, coherent and consistent mathematics curriculum in all Chester County Schools.

K-5 Mathematics Framework



CCSD Mathematics Expectations

Every Component. Every Day

1. All K-5 teachers will teach mathematics with high expectations and rigor.
    a. *recognize that each and every student is able to solve challenging mathematical tasks successfully*
    b. *build in each student a positive growth mindset in math*
    c. *design instruction that builds on students' prior knowledge and experiences*
    d. *teach in ways that ensure that each and every student is reasoning and making sense of mathematics on a daily basis*
    e. *reflect on ways that tasks and teaching can be improved to provide greater access, challenge, and support for every learner.*
2. All K-5 teachers will employ the Guided Math Workshop Model as their core, Tier 1 Math instructional program for 90 minutes daily.
3. All K-5 teachers are expected to teach **every component - every day**: Number Sense Routine, Whole Group (Mini-lesson/Focus lesson), Guided Small Group, Math Workstations, Teacher/Student Conferring, Closure/Share Time, and On-going Assessments.
4. All K-5 teachers will use the SCCCRS and the CCSD Pacing Guide as their curriculum.
5. All K-5 teachers will have access to the math curriculum notebook in TEAMS with resources and lesson plan samples for each standard.
6. All K-5 lesson plans must reflect every component.
7. Calendar Math (Everyday Counts) will also be taught daily in grades K5-1st to support the development of students' number sense, flexibility, and fluency, which will support students' performances on any mathematical task.
    a. *Kindergarten must do Calendar Math all year long*
    b. *1st grade must do Calendar Math daily first semester and then up to twice a week second semester*
8. Number Talks will be taught daily in grades K5-5th grade based on Sherry Parrish's number strings to support the development of students' number sense, flexibility, and fluency. Teachers will follow CCSD district pacing guide for Number Talks.
    a. *Kindergarten must do Number Talks and Calendar Math daily*
    b. *1st grade must do Number Talks daily and follow the Calendar Math schedule above.*
9. All K-5 lesson plans must be aligned to the South Carolina College and Career-Ready State Standards – with standards noted.
10. All K-5 teachers will utilize the following: student math folder (K5-1st) and/or notebooks (2nd – 5th), math toolkits and/or manipulatives, anchor charts, and the CRA (Concrete-Representational-Abstract) model.
11. In all K-5 classrooms, vocabulary will be taught to build mathematical content knowledge. Illustrated math word walls are a non-negotiable and must be interactive. Words should be added depending on topics being addressed.
12. Formative (CFAs) and summative (MAP & Benchmarks) assessments will be the progress monitoring tools utilized by teachers to determine instructional strategies and/or interventions needed to enhance student learning.
13. K-5 Math Coaches will provide professional development and coaching cycles to include modeling of the Guided Math Workshop Model, Cognitive Guided Instruction, Mathematical Process Standards, CRA Model, DOK Questioning, and best practices in mathematics to support math instruction in the classroom.
14. All teachers must adhere to the K-5 Math Homework Policy listed in framework on page 63.

CCSD Mathematics Intervention Expectations

Every Component. Every Day

1. All K5-5th grade teachers will employ Bridges Intervention as their Tier II intervention program every two weeks (AA/BB schedule) for 30-minutes daily with all students identified as a Tier II. Teachers are required to progress monitor weekly during the cycle. A typical intervention window lasts between 8-12 weeks and is decided upon by the RTI Team.

2. Each elementary school will employ a math interventionist who will focus on Tier III students using Do the Math Intervention. Do the Math will be employed with program fidelity. **Tier III students should not be pulled during their Math block because students need double the instruction in order to catch up to grade level. Preferably student should be pulled during Science, Social Studies, or Health times.**

3. Intervention may not begin prior to 8:30am and must commence by 2pm. Many students who could benefit from either intervention or enrichment arrive late and/or are signed out early.
    a. *Per the South Carolina Department of Education… "Intervention must be scheduled strategically for student equitability."*

4. Students not receiving intervention will engage in math tasks, problem solving, fluency, and/or other enrichment activities.
    a. *Technology programs should not be the primary task for students not involved in intervention. Teachers may rotate students between enrichment activities and instructional technology programs such as MAP Skills/Math Seeds.*
    b. *The classroom teacher must use the instructional technology placement with intentionality and purpose. Teachers must also keep track of student data within the programs.*

5. All teachers are required to use the tiered paperwork found within the CCSD RTI Packet when meeting with their school-level RTI teams.

6. RTI Guidelines can be found with the CCSD RTI Handbook.

Tier II Intervention AA/BB Rotation Schedule
2019-2020

August 19-September 6 (Fall MAP and F&P Assessments)	
September 9-13	Math Intervention / Enrichment
September 16-20	Math Intervention / Enrichment /Progress Monitoring
September 23-27	Reading Intervention / Enrichment
September 30- October 4	Reading Intervention / Enrichment / Progress Monitoring
October 7-11	Math Intervention / Enrichment
October 14-18	Math Intervention / Enrichment /Progress Monitoring
October 21-25	Reading Intervention / Enrichment
October 28-November 1	Reading Intervention / Enrichment / Progress Monitoring
November 4-8	Math Intervention / Enrichment
November 11-15	Math Intervention / Enrichment /Progress Monitoring
November 18-22	Reading Intervention / Enrichment
November 25-29	Reading Intervention / Enrichment / Progress Monitoring
December 2-6	Math Intervention / Enrichment
December 9-13	Math Intervention / Enrichment /Progress Monitoring
December 16-20	Reading Intervention / Enrichment
December 23-January 3 (Winter Break)	
January 6-10	Math Intervention / Enrichment
January 13-17	Math Intervention / Enrichment /Progress Monitoring
January 20-24	Reading Intervention / Enrichment
January 27-31	Reading Intervention / Enrichment / Progress Monitoring
February 3-7	Math Intervention / Enrichment
February 10-14	Math Intervention / Enrichment /Progress Monitoring
February 17-21	Reading Intervention / Enrichment
February 24-28	Reading Intervention / Enrichment / Progress Monitoring
March 2-6	Math Intervention / Enrichment
March 9-13	Math Intervention / Enrichment /Progress Monitoring
March 16-20	Reading Intervention / Enrichment
March 23-27	Reading Intervention / Enrichment / Progress Monitoring
March 30-April 3	Math Intervention / Enrichment
April 6-10 (Spring Break)	
April 13-17	Reading Intervention / Enrichment
April 20-24	Reading Intervention / Enrichment / Progress Monitoring
April 27-May 1	Math Intervention / Enrichment
May 4-8	Math Intervention / Enrichment /Progress Monitoring
May 11-15	Reading Intervention / Enrichment
May 18-22	Reading Intervention / Enrichment / Progress Monitoring
May 25-29 (Last Week of School)	

South Carolina College- and Career-Ready Mathematical Process Standards

The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, **the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.**

Students who are college- and career-ready take a productive and confident approach to mathematics. They are able to recognize that mathematics is achievable, sensible, useful, doable, and worthwhile. They also perceive themselves as effective learners and practitioners of mathematics and understand that a consistent effort in learning mathematics is beneficial.

A mathematically literate student can:

1. Make sense of problems and persevere in solving them.

a. Relate a problem to prior knowledge.
b. Recognize there may be multiple entry points to a problem and more than one path to a solution.
c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem.
d. Evaluate the success of an approach to solve a problem and refine it if necessary.

2. Reason both contextually and abstractly.

a. Make sense of quantities and their relationships in mathematical and real-world situations.
b. Describe a given situation using multiple mathematical representations.
c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
d. Connect the meaning of mathematical operations to the context of a given situation.

3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

a. Construct and justify a solution to a problem.
b. Compare and discuss the validity of various reasoning strategies.
c. Make conjectures and explore their validity.
d. Reflect on and provide thoughtful responses to the reasoning of others.

4. Connect mathematical ideas and real-world situations through modeling.

a. Identify relevant quantities and develop a model to describe their relationships.
b. Interpret mathematical models in the context of the situation.
c. Make assumptions and estimates to simplify complicated situations.
d. Evaluate the reasonableness of a model and refine if necessary.

5. Use a variety of mathematical tools effectively and strategically.

a. Select and use appropriate tools when solving a mathematical problem.
b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.

6. Communicate mathematically and approach mathematical situations with precision.

a. Express numerical answers with the degree of precision appropriate for the context of a situation.
b. Represent numbers in an appropriate form according to the context of the situation.
c. Use appropriate and precise mathematical language.
d. Use appropriate units, scales, and labels.

7. Identify and utilize structure and patterns.

a. Recognize complex mathematical objects as being composed of more than one simple object.
b. Recognize mathematical repetition in order to make generalizations.
c. Look for structures to interpret meaning and develop solution strategies.

*Below is a friendly guide to the Mathematical Process Standards for K-1, 2-3, and 4-5.

Grades K-1 Mathematical Process Standards

MPS1 Make sense of problems and persevere in solving them.

Definition	Student Friendly
Students understand and look for a way to solve the problem. They can explain their process, monitor their work, and prove their solutions. "Do I understand the problem?" "Did I persevere?" "Does my solution make sense?"	• I can explain/understand what the problem is asking. • I can make a plan. • I can get unstuck when I am stuck. • I can change my plan if it isn't working. • I monitor my work. • I can prove that my answer makes sense. • I can clearly explain my answers.

Examples:

Teachers	Students
• provide open-ended rich problems • ask probing questions • model solving multi-step problems • scaffold instruction • provide a safe environment for learning from mistakes and collaborating together • model how to find key information • model a variety of strategies • allow time for student-led discussions • probe for a variety of different strategies	• share and discuss strategies • kids work in groups collaboratively • persevere through frustrations • use a variety of strategies and methods to solve a problem • monitor their problem solving process • prove their solutions in more than one way • re-read and retell the problem in their own words • find key information • check to see if their answer is reasonable

Non-examples

Teachers	Students
• teach isolated skills not connected to other learning • do not access or build on prior knowledge • do not create a safe learning environment for kids to make mistakes. • ask closed questions versus open-ended	• do not explain solutions or problem solving process • quit due to frustration • do not make connections or use their prior knowledge and experiences • do not evaluate if their solution is a reasonable answer

MPS2 Reason both contextually and abstractly.

Definition	**Student Friendly**
Students can visualize problems. They can share their thinking with others to solve problems and explain their solution. Students can create multiple representations to explain their thinking (using pictures, words, number sentences, tables, graphs).	• I can make sense of the problem. • I can use numbers, pictures, words, equations to show my thinking. • I can prove that my answer makes sense.

Examples

Teachers	**Students**
• develop opportunities for problem solving strategies • model how to use problem solving strategies • provide real world situations • value feedback given for invented strategies/representations used • teach students to focus on steps taken to solve the problem, not necessarily focusing on the answer	• create multiple representations to explain their thinking (pictures, words, number sentences, tables, & graphs) • learn to self-check - Does my answer make sense? (i.e. 5-3 ≠ 8, because my answer should be less than 5) • visualize problems

Non-examples

Teachers	**Students**
• do not model or allow time to practice • provide rigid teacher centered environment • limit variety of questions and provide no real-world connection • do not provide a safe environment for students to share invented strategies/representations • have a lot of emphasis on the answer • only teach by teacher led discussion	• lack the understanding that math relates to and is used in everyday life • use paper/pencil tasks only • are unaware of strategies and relationships • are confused with tasks and strategies • are unable to explain their thinking • give unreasonable answers

MPS3 Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

Definition	**Student Friendly**
Students can explain to others how they solved the problem. Students can listen and respond and critique the thinking and reasoning of others.	• I can decide if my peers' answers make sense. • I can construct, explain and prove my answer.

Examples

Teachers	**Students**
• create a safe environment for risk taking and critiquing with respect • use feedback to model desired student discourse • provide complex, rigorous tasks that foster deep thinking • plan effective questions, student grouping and time for student discourse • probe students to explain their thinking • model how to talk about and share solutions • model how to be a respectful participant (speaker or listener) when others are sharing • encourage the use of mathematical	• ask questions for clarification of another student's thinking • analyze others arguments • use examples and counterexamples • observe mathematical patterns and make reasonable arguments • use objects, drawings, diagrams, and actions to support their thinking • use mathematical vocabulary

Non-examples

Teachers	**Students**
• do not promote discourse or conversation among students through feedback • lead discourse and not students • do not foster risk taking and respect for the thinking of others in their classroom • ask only simple questions • attend only to the answer not the process • use language that is not precise (not mathematical)	• accept answers of others without questioning • respond inappropriately to the thinking of others • are unaware of mathematical patterns and unable to support their thinking • use only one method to support their thinking • use vague and imprecise mathematical language

MPS4 Connect mathematical ideas and real-world situations through modeling.

Definition	**Student Friendly**
Students can find key information and use it to make a representation to draw conclusions and determine if it makes sense. "Does my model/representation match the problem?" "Does my answer make sense?"	• I can use objects or pictures to show my thinking. • I can use equations to represent my thinking.

Examples

Teachers	Students
• allow time for the process to take place • model the process not the product • make appropriate tools available • create a safe environment where risk taken is valued • provide meaningful, real-world, authentic, performance-based tasks • allow for discourse and investigation	• realize they use mathematics to solve real-world situations • use number relationships to draw conclusions (if 5+5=10 then 6+4=10) • determine if the solution makes sense and make corrections if needed • understand the tasks being asked

Non-examples

Teachers	Students
• provide lack of modeling or time to practice • limit tools • use rigid teacher centered environment • use limited questions and provide no real-world connection • only teach through teacher led discussion	• lack understanding that math relates and is used in everyday life • use paper/pencil tasks only • are unaware of strategies and relationships • are confused with the tasks and strategies • are unable to explain their thinking

MPS5 Use a variety of mathematical tools effectively and strategically.

Definition	**Student Friendly**
Students can choose the appropriate tools to solve the given problem. They can use the appropriate tool for the situation at hand.	• I can choose the appropriate math tool to solve a problem. • I can use math tools to show how I solved a problem. • I can use math tools to help me solve a problem.

Examples

Teachers	Students
• model use of appropriate tools • provide opportunities for students to discover which tool is appropriate and why • compare/contrast the effectiveness of the tools • have a variety of tools available and encourage use of these tools (number lines, pictures, counting jars, tiles, number writing practices) • facilitate discussion with student experiences using tools (the why or why not of each tool)	• choose appropriate tools to solve a given problem • recognize the usefulness and limitations of different tools (number lines, pictures, counting jars, tiles, number writing practices)

Non-examples

Teachers	Students
• only provide modeling for one tool • provide limited time for discovery of tools • provide little or no discussion regarding efficiency of tools • do not provide instruction on how to use each tool (tape measurer) • do not refer to manipulatives as tools (instead of toys)	• use math tools as toys • do not pick the tool that is most appropriate • do not verbalize their rationale for use of the tool chosen

MPS6 Communicate mathematically and approach mathematical situations with precision.

Definition	**Student Friendly**
Student can understand and use math vocabulary. Student can solve problems accurately and talk about their thinking. They can work carefully and check their work. "Is my answer correct?" "How can I prove it?"	• I can work carefully and check my work. • I can understand and use math vocabulary. • I can solve problems accurately and talk about my thinking.

Examples

Teachers	**Students**
• use mathematical language, and or charts, labeling shelves with pictures and words • link new vocabulary to common vocabulary • model using manipulatives, pictures, symbols or verbally • explain/think aloud how to check work • ask open ended questions • encourage fruitful mistakes • are open to divergent thinking • value the process, not just the product	• use math language • show thinking with manipulatives, pictures, symbols, verbally • double check work • show thinking in more than one way • partner chat to think about their work • work toward efficient thinking

Non-examples

Teachers	**Students**
• tell how to solve the problem • show only one way • do not question students reasoning • teach skills in isolation • do not provide opportunity for classroom discourse	• only try one strategy • only try one tool • show only product, not the process • work in isolation

MPS7 Identify and utilize structure and patterns.

Definition	**Student Friendly**
Students use prior knowledge (what they already know) to solve new problems. They can break down complex problems into simpler, more manageable chunks. Students can recognize and understand the patterns they see in problem situations.	• I can find patterns in numbers. • I can use patterns in numbers to solve problems. • I can take apart and put numbers back together. • I can break down problems into easier parts.

Examples

Teachers	Students
• use open ended questioning • are quiet and allow time for student discussion • foster persistence/stamina in problem solving by modeling and practice • provide students with tasks where they can look for structures and patterns	• look for and identify patterns in numbers • use skills previously learned to solve new problems • break down complex problems into simpler tasks • understand how others solved the problem differently

Non-examples

Teachers	Students
• lead lessons without student discussion or questioning • provide one type of problem repeatedly • explain without modeling • provide clarification and explanations quickly	• do not see the patterns in numbers • use a skill in isolation only • do not transfer a skill to a new situation • attempt to solve complex problem all at once • do not understand a different explanation from your own

*Below is a friendly guide to the Mathematical Process Standards for K-1, 2-3, and 4-5.

Grades 2-3 Mathematical Process Standards

MPS1 Make sense of problems and persevere in solving them.

Definition	Student Friendly
Students can explain to themselves the meaning of a problem and look for entry points to a solution They can plan a solution pathway- not jump into a solution attempt. Students can use prior knowledge to plan and solve, continually asking, "Does it make sense?" They will monitor and evaluate progress and change course if necessary. Students will check work using different methods/strategies and explain relationships between equations, graphs, diagrams, etc.	• I can explain/understand what the problem is asking. • I can make a plan. • I can make a plan using what I already know. • I can check to be sure that my answer makes sense. • I can change my plan if it isn't working out. • I can solve my problem in different ways. • I can use representations to support my solution.

Examples

Teachers	Students
• provide open-ended, rich problems • ask probing questions • provide opportunities for reflection (i.e. math journals) • provide rubrics for assessment and self-reflection • use manipulatives and technology to support strategies • display and discuss student work samples to show a variety of strategies • provide opportunities for student discourse and collaboration (i.e. Think-Pair-Share) • promote a safe environment to learn where students feel comfortable taking risks	• thoughtfully read information given - What do I notice? • highlight key words, numbers, and phrases within the question to help make sense of the problem • ask themselves if they need other information to solve • choose efficient strategies (plan) to solve • draw pictures, use representations, use manipulatives to create a plan • reflect to see the reasonableness of answers • collaborate by talking with peers about math • communicate their thinking orally and in writing

Non-examples

Teachers	Students
• use limited math and language vocabulary • have students working only independently • teach a single algorithm as the "way" to solve • always have a quiet classroom • don't provide time to share	• work with homework and classwork in a "drill and kill" fashion • do not use math manipulatives to demonstrate understanding • don't share their thinking • give only one strategy to solve a problem • are required to solve problems a certain way

MPS2 Reason both contextually and abstractly.

Definition	**Student Friendly**
Students make sense of the quantities and use reasoning skills to understand relationship to the problem at hand. Students reason and recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. Students can solve problems in context. They can also pull out of a context to solve or represent with a number pattern.	• I can make sense of the problem. • I can write an equation that matches the problem. • I can use numbers, pictures, words, equations to show my thinking. • I can prove that my answer makes sense.

Examples

Teachers	Students
• encourage and model reasoning and making sense of problems • use concrete models to foster habits of reasoning and create representations • focus on the meaning of quantities - pairing visual images to representations and symbols • connect meaning of quantities to computations and properties of operations	• develop habits of reasoning and creating of representations of the problem at hand • consider the units involved • attend to the meaning of quantities • make sense of quantities and their relationship in problem solving • understand the meaning of quantities and are flexible in the use of operations and their properties • THINK - Engage - What makes sense?

Non-examples

Teachers	Students
• use a lot of drill and practice worksheets • focus on mostly on memorization rather than reasoning skills • don't allow talking • promote the philosophy, "just do the work..... quietly" • have an attitude of "my way or the highway"	• rush through problems with little time spent in discussion or questioning • don't make sense of quantities • don't try to contextualize or decontextualize to solve problems • do not represent math symbolically

MPS3 Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

Definition	**Student Friendly**
Students need to articulate their reasoning and identify what works and what doesn't in the reasoning of others. Their conversations should show understanding of the concept to build a logical progression of statements, as well as recognizing and using counterexamples. Students should justify their conclusions, communicate them to others and respond to the arguments of others.	• I can explain my thinking and support it with math words, symbols and visuals. • I can listen to the reasoning of others and decide if it is reasonable. • I can apply the reasoning of others to a math problem.

Examples

Teachers	**Students**
• create a safe environment for risk-taking and critique with respect • model desired student discourse • provide complex, rigorous tasks that foster understanding/opportunities for discussion • plan and use effective questions • use effective grouping of students • provide time for student discourse	• ask questions to clarify misconceptions • use examples and nonexamples • make sense in their reasoning and explain their thinking clearly • compare two plausible arguments, distinguish correct reasoning from flawed reasoning, and explain why an argument is flawed

Non-examples

Teachers	**Students**
• is seen as the sage on the stage • dictates one way to find a solution	• learn and work silently • focus on one person's work and one way to solve a problem • hurt others feelings due to personal attack • argue inappropriately about work samples

MPS4 Connect mathematical ideas and real-world situations through modeling.

Definition	**Student Friendly**
Students can apply the mathematics they know to solve real world problems. This includes writing an equation to describe and solve a situation.	• I can use geometric figures, pictures, or physical objects or diagrams such as a number line, a table or graph to represent a real world problem. • I can show my work in many ways. • I can use expressions or equations to represent my thinking.

Examples

Teachers	Students
• allow time for the process to take place (model, make graphs…) • model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • make appropriate tools available for students to select from • create an emotionally safe environment where risk taking is valued • provide meaningful, real world, authentic, performance-based tasks (non-traditional word problems) • encourage and model discourse • encouraging students to investigate their own mathematical curiosities	• realize they use mathematics (numbers and symbols) to solve/work out real-life situations • analyze relationships to draw conclusions • interpret mathematical results in context • show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information • make sense of the mathematics • simplify a complicated situation • identify important quantities

Non-examples

Teachers	Students
• limit time for students to process • provide a lack of opportunity for discourse • limit or allow no tools to be available for students to demonstrate understanding • do not model • demonstrate lack of planning • provide students with low level traditional word problems	• work completely independent of others often • do not provide an explanation with their answer • do not use models or equations to represent their thinking • write simple answers • do not see mathematics as a tool to solve real world problems

MPS5 Use a variety of mathematical tools effectively and strategically.

Definition	**Student Friendly**
Students consider the available tools when solving a mathematical problem. Tools should be a variety of manipulatives and technology. Students are sufficiently familiar with tools appropriate for the task.	• I can choose and use the most appropriate tool for solving a problem. • I am familiar with lots of different tools I can use to solve math problems.

Examples

Teachers	Students
• maintain knowledge of appropriate tools • effectively model use of the tools available, their benefits and limitations • model a situation where the decision needs to be made as to which tool should be used • compare/contrast effectiveness of tools • make available and encourage use of a variety of tools	• choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor) • choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry, software, calculator, web 2.0 tools) • compare the efficiency of different tools • recognize the usefulness and limitations of different tools

Non-examples

Teachers	Students
• do not show awareness of effective tools • limit the availability of tools to what they would choose	• relies solely on paper and pencil as a tool • relies solely on mental math when the task warrants more

MPS6 Communicate mathematically and approach mathematical situations with precision.

Definition	**Student Friendly**
Precise clear communication of mathematical process and reasoning; includes work, algorithms, language, etc.	• I can work carefully and check my work. • I can understand and use the correct math vocabulary. • I can solve problems accurately and talk or write about my thinking • I can use appropriate labels with my answers.

Examples

Teachers	Students
• understand precise definitions and context • use appropriate vocabulary throughout instruction • correct miscues • clarify vocabulary • display appropriate math language	• communicate with precision • calculate efficiently and accurately • explain mathematical reasoning using precise language, tools, pictures, labels, etc. • organize thinking • ask purposeful questions • check answers for accuracy • work towards clarity in discussions by carefully formulating an explanation

Non-examples

Teachers	Students
• do not understand the precise meaning of important mathematical vocabulary • rush thinking • limit resources • do not support and provide opportunities for student engagement	• have sloppy/incomplete labeling • show inaccurate computation • overuse the words "it" or "thing" • use lots of everyday language • lack explanation

MPS7 Identify and utilize structure and patterns.

Definition	**Student Friendly**
Students look closely to discern a pattern or structure. Examples would be commutative property, associative property, decomposing a number with place value within addition and subtraction strategies, mentally adding/subtracting 10 and 100.	• I can identify patterns within a problem and use efficient strategies to solve. • I can see and understand how numbers and shapes are organized and put together as parts and wholes. • I can break down complex problems into smaller chunks to solve accurately.

Examples

Teachers	**Students**
• facilitate learning by using open-ended questioning to assist students in exploration • carefully select tasks to promote opportunities to look for and identify patterns • guide students to use patterns to make generalizations • provide time for student discussion and processing (i.e. wait time) • foster persistence and stamina in problem solving through practice and modeling	• look closely to discern patterns and structure • recognize, reflect on and interpret patterns and structures using number grid, number line, number bonds, place value chart and ten-frames • use previously learned skills and strategies to solve new problems and tasks • decompose numbers into more workable numbers such as hundreds, tens and ones • decompose shapes into more workable shapes

Non-examples

Teachers	**Students**
• do not provide mathematical structures to help students understand the mathematics • use a lot of low level problem types • are not thoughtful about the types of relevant practices provided to students to identify patterns • teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry • limit student participation	• are not engaged or involved • do not make connections or use previously learned strategies • do not use mathematical patterns or structures to make sense of the problem • cannot apply mathematical methods/algorithms to not traditional problems

*Below is a friendly guide to the Mathematical Process Standards for K-1, 2-3, and 4-5.

Grades 4-5 Mathematical Process Standards

MPS1 Make sense of problems and persevere in solving them.

Definition	**Student Friendly**
Students will interpret and analyze to find the meaning of the problem/make sense of the problem. They will create a plan to solve the problem, monitor their progress and change the approach if necessary.	• I can explain/understand what the problem is asking. • I can make a plan. • I can get unstuck when I am stuck. • I can change my plan if it isn't working out. • I can monitor my work. • I can prove that my answer makes sense. • I can show that my representations in my work support my solution.

Examples

Teachers	**Students**
• provide open ended and rich problems • ask probing questions • model multiple problem-solving strategies through *think aloud* • promote and value disclosure and collaboration • examine student responses (correct or incorrect) for understanding and multiple representations • promote a safe environment to learn in • view mistakes as learning opportunities • differentiate based on student needs • encourage self-monitoring	• thoughtfully read information given - What do I notice? • underline the question being asked • make sense of the problem by double underlining key direction words, circling information/numbers needed to solve • highlight key math vocabulary or terms in the problem • reread thoughtfully • ask themselves if they need other information to solve the problem • choose efficient strategies (Plan) to solve • draw math pictures, use representations, manipulatives to create a plan • check to see - is my answer reasonable? does it make sense of the question asked? • change plan if needed

Non-examples

Teachers	**Students**
• focus on telling the students "how to solve" rather than building problem solving strategies • simplify language so that students don't have to think to solve the problem • use limited mathematical language	• work alone • have not developed a clear path for solving problem • are more interested in the "how to" than in understanding the bigger concepts and understanding why • do not show representations of their understandings

MPS2 Reason both contextually and abstractly.

Definitions	**Student Friendly**
Students understand what the numbers represent. They show how to represent the problems using symbols and numbers. Students can develop more than one strategy or solution to a problem. Learners understand the relationships between problem scenarios and mathematical representation.	• I can make sense of the problem. • I can show my process to get a solution. • I can check my answer to see if it makes sense. • I can use numbers, words and reasoning to help me make sense of problems. • I understand what my solution means.

Examples

Teachers	**Students**
• develop opportunities for and model problem solving strategies • provide real world situations • makes connections between content areas • value invented strategies and representations • give less emphasis to the answer • help their learners understand the relationships between problem scenarios and mathematical representation	• estimate first to make more sense of the answer • make sure the answer is reasonable • create multiple ways to represent their problem solving (tables, pictures, words, symbols) • represent the unit

Non-examples

Teachers	**Students**
• expect no explanation with the answer • do not connect math to the real world or other content areas	• provide answers that are not labeled or do not make sense • do not pull out numbers/information to effectively to solve the problem

MPS3 Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

Definitions	**Student Friendly**
Students engage in active mathematical discourse, this might involve having students explain and discuss their thinking processes aloud or signaling agreement/disagreement with a hand signal. A teacher might post multiple approaches to a problem and ask students to identify plausible rationales for each approach.	• I can make conjectures and critique of the mathematical thinking of others. • I can construct, justify and communicate arguments by… 1. considering context 2. using examples and non-examples 3. using objects, drawings, diagrams and actions • I can critique the reasoning of others by… 1. listening 2. comparing arguments 3. identifying flawed logic 4. asking questions to clarify or improve arguments

Examples

Teachers	**Students**
• create a safe environment for risk-taking and critiquing with respect • use feedback to model desired student discourse • provide complex, rigorous tasks that foster deep thinking • plan effective questions, student grouping and time for student discourse • probe students' thinking to gain insight into their understanding	• ask questions • reason inductively and make plausible arguments • use examples and counterexamples to prove their understanding or arguments • analyze others arguments

Non-examples

Teachers	**Students**
• focus on just the answer • teach procedural methods without understanding • use a lot of "drill and kill" problem practice • provide feedback that does not provide any room for further discourse (i.e. right or wrong)	• don't generate the questions • completely quiet classroom • memorize procedures without reasoning • do not share thinking between themselves in whole group and small group settings

MPS4 Connect mathematical ideas and real-world situations through modeling.

Definition	**Student Friendly**
Students construct visual evidence using symbolic and graphical representations. They use pictures, numbers and words appropriate to the real world context of the problem.	• I can use geometric figures, pictures, or physical objects or diagrams such as a number line, tape diagram, table or graph to represent the problem. • I can show my work in multiple ways. • I can use expressions or equations to represent my thinking.

Examples

Teachers	Students
• provide open ended questions/problems that are meaningful, real world, authentic, performance-based tasks • demonstrate the process of modeling • share students' examples • allow time for the process to take place (model, make graphs…) • provide many opportunities to connect and explain the different relationship between the different representations	• represent work in pictures (models), numbers and words • need many opportunities to connect and explain the different relationship between the different representations • show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information

Non-examples

Teachers	Students
• provide lower level multiple choice questions with no room to solve problems • focus instruction on procedural method	• perform using algorithms only • use limited representation to explain reasoning • can't explain, justify or model

MPS5 Use a variety of mathematical tools effectively and strategically.

Definition	**Student Friendly**
Students identify and use tools to scaffold their learning and increase understanding of concepts, problem-solving strategies, and to further exploration of possible solutions. They make good decisions about the appropriateness of the specific tool or tools to be used.	• I can choose the most appropriate tool for each given problem. • I can use math tools to show how I solved a problem. • I can use math tools to help me solve a problem. • I know HOW to use math tools.

Examples

Teachers	**Students**
• model use of tools - ex. when is an appropriate time to use a calculator; when is a ruler needed? use square tiles to model arrays • build understanding of concepts through the use of models and manipulatives - moving from concrete to representation to abstract • model use of resources - how to use the mathematical dictionaries, online tools - to help develop math vocabulary and broaden understanding of math concepts • identify and model use of online tools	• use manipulatives to build representations and problem-solving strategies • identify math vocabulary - use math resources to determine meaning of unknown math terms • will choose the appropriate tool to help them problem solve in an efficient/strategic manner • strategically and thoughtfully use tools when appropriate in problem solving situation • demonstrate when and how to use math tools

Non-examples

Teachers	**Students**
• use the same tool over and over and over again without utilizing a variety of different tools and/or teaching methods • allowing the use of prohibited tools (unless accommodations are needed) • do not allow students to make sense of needed tools based on the mathematics • do not have tools available for easy access for students to use when they need to use them	• repeated use of tool without understanding • use the tool when other methods are more efficient and appropriate (99-97 does not need a calculator) • do not make sense of the tool and if it is the best tool for the mathematics at hand

MPS6 Communicate mathematically and approach mathematical situations with precision.

Definition	**Student Friendly**
Students calculate accurately and efficiently. They use clear and concise communication, written & oral to explain their understanding and thinking. Students use correct mathematical vocabulary and symbols to communicate their thinking.	• I can work carefully and check my work. • I can understand and use math vocabulary. • I can solve problems accurately and talk and write about my strategies and solutions.

Examples

Teachers	**Students**
• create a classroom environment that is safe for risk taking, communication and evaluation of each other's thinking • ask probing questions that require students to analyze their thinking and critically evaluate their reasoning • model mathematical language, model use of resources • value the process, not just the product • model/encourage think aloud & double checking of work	• represent their work to match what the problem is asking • talk to one another about their mathematical thinking, share ideas with one another, ask questions of one another • demonstrate mathematical fluency by efficiently, accurately and fluently adding, subtracting, multiplying and dividing • use math language/vocabulary appropriately • use resources to support understanding of math vocabulary

Non-examples

Teachers	**Students**
• do not provide problems that allow for extended responses • focus on only one path to get an answer • do not understand the precise meaning of important mathematical vocabulary • rush thinking • limit resources • do not provide opportunity for student engagement	• work in isolation • are not discussing strategies and solutions with peers • are sloppy with their representations and work is incomplete missing labeling that shows understanding • have inaccurate computation • overuse of the words "it" and "thing" • lots of everyday language • lack explanation in their work

MPS7 Identify and utilize structure and patterns.

Definition	**Student Friendly**
Students look for, interpret and identify patterns and structures. They see complicated things as single objects or being composed of several objects. Students make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers.	• I can notice when calculations are repeated. Then, I can find more efficient methods. • I can see and understand how numbers and shapes are organized and put together as parts and wholes. • I can take complex problems into simpler, more manageable chunks. • I can use the structure of mathematics to make sense of my thinking.

Examples

Teachers	**Students**
• understand the properties of operations so they can help students make connections • show connections between types of numbers; fractions, decimals • provide rich problems to look for repeated reasoning and connections	• show how patterns emerge in rich problems • use repeated patterns to find a more efficient way of solving • use repeated patterns to show understanding of mathematics

Non-examples

Teachers	**Students**
• do not provide mathematical structures to help students understand the mathematics • use a lot of low level problem types • are not thoughtful about the types of relevant practice provided to students to identify patterns • teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry • limit student participation	• do not use mathematical patterns or structures to make sense of the problem • are not able to make connections or use previous strategies. • cannot apply mathematical methods/algorithms to non-traditional problems • are sitting for long time periods • are not engaged or show little involvement

Eight Mathematical Teaching Practices

The National Council of Teachers of Mathematics (NCTM) introduces Principles to Actions: Ensuring Mathematical Success for All, setting forth a set of strongly recommended, research-informed actions, based on the Council's core principles and intended for all educational leaders and policymakers, all school and district administrators, and all teachers, coaches, and specialists of mathematics.

Principles to Actions devotes the largest section to Teaching and Learning, the First Guiding Principle, and describes and illustrates **Eight Mathematical Teaching Practices** that research indicates need to be consistent components of every mathematics lesson. These habits represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics.

1. Establish mathematics goals to focus learning. (I Can Statements)
2. Implement tasks that promote reasoning and problem solving. (Number Talks)
3. Use and connect mathematical representations. (Word Wall/Anchor Charts)
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions. (DOK Questions/Prompts for Rigor)
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

1. Establish Mathematics Goals to Focus Learning

Teachers must utilize daily student friendly learning targets in the form of "I Can Statements".

Effective teaching of mathematics

- establishes clear goals for the mathematics that students are learning
- situates goals within learning progressions
- uses the goals to guide instructional decisions

Teacher Actions	Student Actions
Establish clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identify how goals fit within a mathematics learning progression. Discuss and refer to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Use the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engage in discussions of the mathematical purpose and goals related to their current work in the mathematical classroom (What are we learning? Why are we learning it?) Use the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connect their current work with the mathematics that they studied previously and see where the mathematics is going. Assess and monitor toward the mathematics learning goals.

Instructional Research Based Strategies

- "formulating clear, explicit learning goals sets the stage for everything else" (Heibert)
- "goals should be situated within mathematical learning progressions and connected to the big ideas" (Daro, Mosher and Corcoran)
- "classrooms where students understand the learning expectations for their work perform at higher levels than those classrooms where it is unclear"(Marzano, Haystead, Hattie)
- "student friendly goals can be discussed within a lesson so that students see value in and understand the purpose of their work" (Black and William, Marzano)
- "as teachers establish specific goals and consider the connections to broader math topics, they become more prepared to use goals to make decisions during instruction" (Hiebert)

Important Instructional Considerations

- goals communicate what students will understand based on instruction
- goals identify mathematical practices that students are learning to use more proficiently
- goals should not just be reiteration of standards but should be linked to curriculum and student learning
- goals should be situated within the mathematical landscape to support opportunities to build connections so that students see how ideas build
- mathematical purpose of the lesson should not be a mystery to students

2. Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics

- engages students in solving and discussing tasks that promote mathematical reasoning and problem solving
- allows for multiple entry points and varied solution strategies

Teacher Actions	Student Actions
Motivate students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Select tasks that provide multiple entry points through the use of varied tools and representations. Pose tasks on a regular basis that require a high level of cognitive demand. Support students in exploring tasks without taking over student thinking. Encourage students to use varied approaches and strategies to make sense of and solve tasks.	Persevere in exploring and reasoning through tasks. Take responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Use tools and representations as needed to support their thinking and problem solving. Accept and expect that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Instructional Research Based Strategies

- "...mathematical tasks are viewed as placing *higher-level cognitive demands* on students when they allow students to engage in active inquiry and exploration or encourage students to use procedures in ways that are meaningfully connected with concepts or understanding." (Smith and Stein, 1998)
- "Tasks that encourage students to use procedures, formulas, or algorithms in ways that are not actively linked to meaning, or that consist primarily of memorization or the reproduction of previously memorized facts, are viewed as placing *lower-level cognitive demands* on students." (Smith and Stein, 1998)
- a task should provide students with the opportunity to engage actively in reasoning, sense making, and problem solving so that they develop a deep understanding of mathematics (NCTM)

Important Instructional Considerations

- Provide rich, open ended tasks that allow for multiple solutions
- Model and use visual supports
- Use Webb's Depths of Knowledge to guide high-level of task selection

3. Use and Connect Mathematical Representations

Effective teaching of mathematics

- engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tool for problem solving

Teacher Actions	Actions of Student
Select tasks that allow students to decide which representations to use in making sense of the problems. Allocate substantial instructional time for students to use, discuss, and make connections among representations. Introduce forms of representations that can be useful to students. Ask students to make math drawings or use other visual supports to explain and justify their reasoning. Focus students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Design ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Use and explore multiple forms of representation to make sense of and understand mathematics. Describe and justify mathematical understanding and reasoning using effective representations / drawings, diagrams, words, …. Make choices about which forms or representations to use as tools for solving problems. Make sense of problems through use of tables, drawings, diagrams and other representations. Connect mathematical ideas and concepts to real-world situations and contexts. Consider advantages or suitability of using alternate representations when problem solving.

Instructional Research Based Strategies

- When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate a deeper understanding and enhanced problem-solving abilities. (Fuson, Kalchman, & Bransford 2005; Lesh, Post and Behr 1987)
- The depth of understanding is related to the strength of connections among mathematical representations that students have internalized. (Pape and Tchoshanov 2001; Webb, Boswinkel and Dekker 2008)
- Visual representations are of particular importance in the mathematics classroom, helping students to advance their understanding of mathematical concepts and procedures, make sense of problems and engage in mathematical discourse. (Arcavi 2003; Stylianou and Silver 2004)
- Success in solving problems is also related to students' ability to move flexibly among representations. (Huinker 2013; Stylianou and Silver 2004)

Important Instructional Considerations

- Math drawings and other visual supports are of particular importance for English language learners, learners with special needs, or struggling learners, because they allow more students to participate meaningfully in the mathematical discourse in the classroom (Fuson and Murata 2007).

- Help students to see the connection between the different representations. It isn't just about showing multiple ways but seeing that the representations are connected to each other mathematically.

4. Facilitate meaningful mathematical discourse.

Effective teaching of mathematics

- facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments

Teacher Actions	Student Actions
Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Scaffold student approaches and solution strategies for whole-class analysis and discussion. Facilitate discourse among students by having them justify and explain their reasoning for their answer and approach. Ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Present and explain ideas, reasoning, and representations to one another in pairs, small-group, and whole-class discourse. Listen carefully to and critique the reasoning of peers, use examples to support or counterexamples to refute arguments. Seek to understand the approaches used by peers by asking clarifying questions, try out others' strategies, and describe the approaches used by others. Identify how approaches to solving a task are the same and how they are different.

Instructional Research Based Strategies

Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics.

Students who learn to articulate and justify their own mathematical ideas, reason through their own and others mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields.

Whole class discussions

- anticipating student responses prior to the lesson
- monitoring students work and engagement with the tasks
- selecting particular students' responses in a specific order to promote a variety of strategies for conceptual understanding
- connecting different students' responses to key mathematical ideas

Mathematical discourse includes the purposeful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication.

The discourse gives students opportunities to share ideas and clarify understandings, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives.

Teachers and students proceed through the levels in shifting from a classroom in which teachers play the leading role to one where they facilitate students' mathematical thinking.

5. Pose purposeful questions

Effective teaching of mathematics

- Uses purposeful questions to assess and advance students' ability to reason and make sense of important mathematical ideas & relationships

Teacher Actions	Student Actions
Advance student understanding by asking questions that build on student thinking, but do not take over or funnel ideas to lead students to a desired conclusion. Ask questions that go beyond gathering information. The questions should probe students' thinking and require explanation and justification. Ask intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allow sufficient wait time so that more students can formulate and offer responses.	Expect to be asked to explain, clarify, elaborate on their thinking verbally and in written format. Think carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflect on and justify their reasoning, not simply provide answers. Listen to, comment on, and question the contributions of their classmates.

Important Instructional Considerations

- Teacher Role - Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for the students to clarify thinking of others.
- Questioning - Student to student talk is student initiated. Students ask questions and listen to responses. Many questions ask why and call for justification. Teacher questions may still guide discourse.
- Explaining Mathematical Reasoning - Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.
- Mathematical Representations - Students follow and help shape the descriptions of others' math thinking through math representations and may suggest edits in others' representation.
- Building student responsibility within the community - Students believe that they are math leaders and can help shape the thinking of others. They help shape others math thinking in supportive, collegial ways and accept the same support from others.

Instructional Research Based Strategies

- Depth of Knowledge Question Stems (Webb, Norman)
- Focusing Pattern of Questioning (Wood, Terry)
- http://www.svmimac.org/images/SVMIPD.091312.Questioning_our_Patterns.pdf
- Rigor Relevence Framework (Daggett, William)
  http://www.leadered.com/pdf/rigor_relevance_framework_2014.pdf
- A Framework for types of questions used in mathematics teaching (e.g. Boaler and Brodie 2004; Chapin and O'Connor 2007)

1. Gathering Information - Students recall facts, definitions, or procedures.
ex. When you write an equation, what does that equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?

2. Probing Thinking - Students explain, elaborate, or clarify their thinking including articulating the steps in solution methods or the completion of a task. ex. As you drew that number line, what decisions did you make so that you could represent 7 fourths on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans tasks? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

3. Making the Mathematics Visible - Students discuss mathematical structures and make connections among mathematical ideas and relationships. ex. What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?

4. Encouraging Reflection and Justification - Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work. ex. How might you prove that 51 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but becomes more expensive in the long run?

Important Instructional Considerations

- Questioning should encourage students to explain and reflect on their thinking.
- Questioning should allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections and support students in posing their own questions.
- Questions should attend to what students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates.

6. Build procedural and fact fluency from conceptual understanding.

Effective teaching of mathematics promotes students to be able to

- Build fluency with procedures on a foundation of conceptual understanding
- Become skillful in flexibly using procedures as they solve contextual and mathematical problems

Instructional Research Based Strategies

"Strategies that Work" Marzano:

Non-linguistic Representations: Ask students to…

- Generate mental images representing content
- Draw pictures or pictographs representing content
- Construct graphic organizers representing content
- Act out content
- Make physical models of content
- Make revisions in their mental images, pictures, pictographs, graphic organizers, and physical models

7. Support productive struggle in learning mathematics

Effective teaching of mathematics

- Consistently provides students with opportunities and supports to engage in productive struggle

Teacher Actions	Student Actions
Acknowledge the importance of both conceptual understanding and procedural fluency but also ensure that the learning of procedures is developed over time, on a strong foundation of understanding and the use of student-generated strategies in solving problems. Provide students opportunities to use their own reasoning strategies and methods for solving problems. Ask students to discuss and explain why the procedures that they are using work to solve particular problems. Connect student generated strategies and methods to more efficient procedures as appropriate. Use visual models to support students' understanding of general methods. Provide students with opportunities for continuous practice of procedures. Provide students time to practice math facts.	Know which procedure is appropriate and most productive in a given situation. Demonstrate the ability to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently. Access procedures that they can use with understanding on a broad range of problems. Demonstrate knowledge by practicing on a moderate number of carefully selected problems once they have a strong conceptual foundation and can explain the use of the strategy.

- Opportunities for delving more deeply into understanding the mathematical ideas
- Able to apply their learning to new problem situations

Teacher Actions	Student Actions
Anticipate what students might struggle with during a lesson and being prepared to support them productively through the struggle. Give students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Help students realize that confusion and errors are a natural part of learning, by facilitating discussion on mistakes, misconceptions, and struggles. Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggle at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Ask questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevere in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Help one another without telling their classmates what the answer is or how to solve the problem.

Instructional Research Based Strategies

- "Rescuing" students when they face difficulties undermines the efforts of students, lowers the cognitive demands of the task, and deprives the students of opportunities to engage fully in making sense of the mathematics. (Reinhart 2000; Stein et al. 2009)
- "If you are not struggling, you are not learning". (Carter 2008, p.136)
- Provide students with specific descriptive feedback on their progress related to their making sense of math. (Clarke 2003; Hattie and Timperley 2007)

Important Instructional Considerations

- Create a safe environment for learning.
- Consider student struggles and misconceptions.
- Classrooms should embrace productive struggle to necessitate rethinking on the part of both the teacher and student.
- Students question and critique the reasoning of their peers and reflect on their own understanding.
- Students have access to tools that will support their thinking processes.
- Teacher plans for tasks that promote reasoning and problem solving; a solution pathway is not straightforward, but requires some struggle to arrive at the solution.

8. Elicit and use evidence of student thinking

Effective teaching of mathematics

- Uses evidence of student thinking to assess progress toward understanding
- Uses evidence to adjust instruction continually in ways that support and extend learning

Teacher Actions	Student Actions
Identify what counts as evidence of student progress toward mathematics learning goals. (Formative Assessment) Elicit and gather evidence of student understanding at strategic points during instruction. Interpret student thinking to assess mathematical understanding, reasoning and methods. Make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflect on evidence of student learning to inform the planning of next instructional steps.	Reveal mathematical understanding, reasoning, and methods in written work and classroom discourse. Reflect on mistakes and misconceptions to improve their mathematical understanding. Ask questions, respond to, and give suggestions to support the learning of their classmates. Assess and monitor their own progress toward mathematics learning goals and identifying areas in which they need to improve.

Instructional Research Based Strategies

Attention to eliciting and using evidence is an essential component of formative assessment. (William 2007a)

"Teachers using assessment for learning continually look for ways in which they can generate evidence of student learning, use this evidence to adapt their instruction to better meet their students' learning needs." (Leahy, 2005)

"Identifying indicators of what is important to notice in students' mathematical thinking, planning for ways to elicit that information, interpreting what the evidence means with respect to students' learning, and then deciding how to respond on the basis of students' understanding." (Jacobs, Lamb, & Philipp 2010: Sleep and Boerst 2010; van Es 2010)

Important Instructional Considerations

- What mathematical strand is being taught
- Developmental considerations
- "Teachers attend to more than just whether an answer is or is not correct." (Crespo 2000)
- Each lesson needs to include intentional and systematic plans to elicit evidence that will provide information about how student learning is evolving toward the desired goal. (Heritage 2008, p. 6)

Traditional vs Guided Math

The teacher's role is not to transmit knowledge to the students, but to create an environment in which students actively explore mathematical ideas through standards-based, engaging lessons. As students investigate and solve problems, important mathematical ideas and concepts will be learned with understanding, not just through memorization. The student's role is to be actively involved in doing mathematics, thinking and reasoning about mathematics, searching for answers and ultimately, taking responsibility for their own learning.

Traditional Classroom	Guided Math Classroom
• Students complete morning work routine – i.e. Worksheet or problems on board • Teacher reviews homework • Teacher reviews morning work • Teacher introduces lesson/standard for the day • Teacher shows students how to solve a new problem • Students solve a bunch of problems demonstrated by teacher with the teacher • Students work independently to complete problems in book • Teacher assigns homework, usually a worksheet	• Number Sense Routines: i.e. Number Talks/Calendar Math • Whole Group Instruction (mini-lesson/focus lesson) • Student Conferencing • Guided Small Groups: A small group of students working with the teacher to address specific needs • Math Workstations: Activities that students work on independently or collaboratively to practice new topics or review past topics, often taking place while the teacher is working with a guided math group • Closure/Share Time: Students think and share about their learning.

The Foundational Principles of Guided Math

- All children can learn mathematics.
- A numeracy-rich environment promotes mathematical learning.
- Learning at its best is a social process.
- Learning mathematics is a constructive process.
- An organized classroom environment supports the learning process.
- Effective math programs include Number Talks, modeling, think alouds, critical writing, and purposeful conversations.
- Effective math programs empower students with responsibility for their learning

-Adapted from Principles of Guided Reading developed by Fountas and Pinnell

K-5 Guided Math Model

Framework Objective: To provide all CCSD staff with an instructional framework that helps teachers provide quality math instruction and to help students develop a deep conceptual understanding of math, acquire computational fluency, and become skilled in thinking and acting mathematically. (Sammons 2014)

South Carolina College & Career Ready Standards

Classroom Environment of Numeracy

Purpose:
Classroom math environments set the tone for learning. The environment sends the message to the students about the importance of mathematics. The classroom environment should support mathematical thinking, hands-on learning, and peer collaboration.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• Whole-group meeting area (carpet area)	• Student desks in rows
• Workstations (student desks in mixed ability groups)	• No teacher area for small groups
• Conference/Teacher Table or Designated Area	• Schedule not visible for students
• Visible Daily Schedule for Rotations	• No word walls
• Math Word Walls	• No math manipulatives
• Math Manipulatives/Toolkits	• Generic bulletin boards and posters
• Student Math Journals and/or Folders	• No math literature books
• Math Library	• Blank walls
• Class-made Anchor Charts	• No learning objective
• Calendar/Number Talk Charts	• Student work not posted
• Number lines, ladders, charts	• Rules, Routines, and Procedures not visible
• Student Learning Targets (I can statements)	
• Student Work	
• Posted Rules, Routines, and Procedures	
• Analog clocks in classrooms	

"Look-For" the following characteristics and implementation strategies within our video exemplars for this structure:

Link to Video	
1. How to create a classroom environment of numeracy	
2. Collaboration in Elementary Math	
3. Ideal Mathematics Classroom	

Gathering Spaces

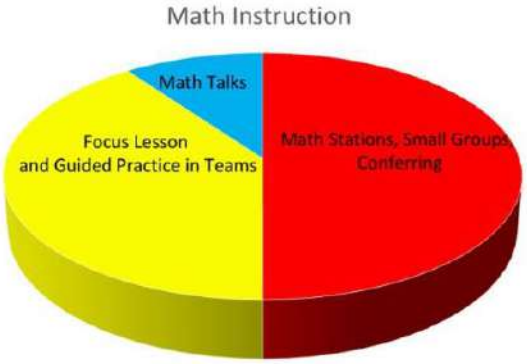
- Whole-group meeting area
  - A place to begin your math lesson, usually away from desks and distractors
- Workstations
  - Student desks assembled or clustered into groups
- Conference/Teacher Table
  - A general place for teachers to meet with small groups

General Mathematical Environment

Evidence of a Weak Classroom Community	Evidence of a Strong Classroom Community
Students will not talk to one another; they keep their thinking to themselves.	Students talk to one another; they explain and clarify their thinking to each other.
Students see working together as cheating; they work alone.	Students see working together as learning; they collaborate, ask each other questions, and respectfully challenge ideas.
Students believe that they are either good at math, and that this really can't change – they exhibit a fixed mindset.	Students believe that they are all capable of being successful in math – they exhibit a growth mindset.
Students are reluctant to struggle; instead they wait to be rescued by the teacher or by another students.	Students constructively struggle together and hold all members of the group accountable for learning.
Students are disrespectful of their peers' ideas.	Students respect their peers' ideas and view one another as knowledgeable.
Students hide their work from each other.	Students share their work with each other.
Students feel that their opinions and ideas don't matter – it's not worth the risk!	Students feel their opinions and ideas do matter – it's worth the risk!

Daily Schedule

- CCSD Elementary Math Block time is 90 minutes
- A detailed schedule of the flow of the Math Workshop so students can learn and keep track of the flow and rotations



K5 – 1st Grade Math Workshop Schedule	
Calendar Math/Number Talks	15 minutes
Math Whole-Group Lesson	15 minutes
Exit Ticket	5 minutes
Guided Math Groups/Math Workstations	40 minutes
Teacher/Student Conferring	10 minutes
Closure/Share Time	5 minutes

2nd – 5th Grade Math Workshop Schedule	
Number Talks	10 minutes
Math Whole-Group Lesson	30 minutes
Exit Ticket	5 minutes
Guided Math Groups/Math Workstations	30 minutes
Teacher/Student Conferring	10 minutes
Closure/Share Time	5 minutes

EXAMPLE: Guided Math Rotation Schedule K5-1st (2 groups/20 min. each): Novice, Apprentice, Practitioner, Expert groups

Monday	Tuesday	Wednesday	Thursday	Friday
Novice	Practitioner	Novice	Practitioner	Novice
Apprentice	Expert	Apprentice	Expert	Apprentice

Guided Math Rotation Schedule 2nd – 5th (2 groups/15 min. each): Novice, Apprentice, Practitioner, Expert groups

Monday	Tuesday	Wednesday	Thursday	Friday
Novice	Practitioner	Novice	Practitioner	Novice
Apprentice	Expert	Apprentice	Expert	Apprentice

- **I Can Statements or Student Learning Targets** should be posted and stated at beginning and end of lessons. The learning target outlines what a student should be able to do relative to the standard. The learning targets decompose the standard into various learning chunks or ideas.

- Analog Clocks
  - Landmark your clocks
    - Kindergarten clocks marked o'clock
    - 1st grade should have o'clock and half-past
    - 2nd grade should have clocks marked by 5 minute increments, o'clock, half-past, quarter to, quarter past

- Math Word Walls
  - Math is a language and word walls help teach students the words
  - Word walls are not decorative – words should be taught and illustrated
  - Make it a habit to frequently refer to math word wall words

- Math Manipulatives/Toolkits
  - Several approaches - Central location for math manipulatives for class or students to have their own individual math toolkits

Suggested Math Manipulatives

Primary Toolkits (K-2)	Elementary Toolkits (3-5)
Materials: • Unifix cubes • Fraction bars, fraction circles • Pattern blocks • Base-ten blocks • Dice (numbered and dotted) • Dominos • Flash cards • 2 colored counters • Rulers and other measuring tools (Customary and Metric) • Coins • Rekenrek • 1-inch tiles • Attribute sorting shapes • Judy clock Templates: • Five/Ten/Double Ten frame • 100/120/200 chart • Number lines/ladders/tracks	Materials: • Unifix cubes • Fraction bars, fraction circles • Pattern blocks • Counters for fractions of sets • Base-ten blocks • Elapsed time ruler • 1-inch tiles • Decimal squares • Decimal wheels • Dice; double dice; triple dice • Dominos • Flash cards • 2 colored counters • Rulers and other measuring tools (Customary and Metric) • Protractor • Coins • Judy clock • Pattern blocks • Geoboards/Symmetry mirrors

- Make sure you have rules, procedures, and expectations for using tools

Rules for Tools!	Tools
We use tools to help us think! We keep our toolkits organized. We use them when we need them. We put them up when we are done. We use our templates to sketch out answers.	We use many different tools, Bears and counters, cubes They're cool! We know how to use them well, And clean up when we hear the bell!

-Newton 2014

- Student Notebooks/Journals "Thinking Notebooks":
  - Focus Lesson
  - Guided MathWord Problems
  - Workstations
  - Problem Solving
  - Vocabulary
- Math library
  - A place to store math literature books, math poems, songs, games, etc.
- Data/folder center
  - A place to store data notebooks/student data folders
  - A math data wall
- Calendar Area
  - K-1st Every Day Counts Calendar Math Bulletin Board area
- Anchor Charts
  - Most anchor charts should be made with the students each year. Students are more likely to use the anchor charts if they helped create them.
  - Make it a habit to frequently refer to anchor charts and math word wall words.
- Number lines, number charts, number ladders, or other number grids

Number Sense Routine (Daily)

Purpose:
An engaging, accessible, purposeful routine to begin your math class that promotes a community of positive mathematical discussion and thinking. To be successful in mathematics, students need to be able to solve unfamiliar problems, explain their reasoning, and make connections among different strategies. Number sense routines help students develop an awareness and understanding about what numbers are and their relationships, their magnitude, the relative effect of operating on numbers, including the use of mental math and estimation.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• Set in a whole group meeting area	• Students at their desks with dry erase boards and markers
• Teacher facilitated discussion	• Procedures and skill driven
• No more than 15 minutes (K-1st) and 10 minutes (2nd – 5th)	• Teacher lecture
• Mental math	• Lasts longer than 15 minutes
• Student to student and student to teacher discussions	• Teacher calls on students and gives no wait time
• Teacher wait time	• Worksheet activity
• Use of hand signals	• Answer getting only
• Student strategies recorded	• Student reasoning is not recorded
•	• Songs, dances, and chants

"Look-For" the following characteristics and implementation strategies within our video exemplars for this structure:

Link to Video:	
1. Jo Boaler on Number Sense	
2. Build Math Minds on Number Sense Trajectories	
3. Number Sense Through Elementary Grades	

- Number Sense routine (10 - 15) minutes per day (Non-negotiable)
  - Calendar Math/Number Talks K-1st
  - Number Talks K – 5th

- Other ideas for Number Sense Routines
  - Which One Doesn't Belong?
  - Number of the Day
  - Count Around
  - Shape Talks
  - Building Numbers

Calendar Math

Purpose: The regular use of a calendar time routine is important for the development of students' number sense, flexibility, and fluency, which will support students' performances on any mathematical task. Everyday Counts Calendar Math activities cover routine topics such as counting, time, money, positional words, patterns, tallying, and graphing. Even though there are specific units for each of these topics taught in the primary grades that emphasize key standards and big ideas at specific times of the year, these topics also need to be addressed on an ongoing basis. This can be done through the use of daily Calendar Math time experiences.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• Set in a whole group meeting area	• CM projected on promethean board
• Teacher facilitated discussion	• Students not engaging in discussion
• No more than 10 minutes	• Lasts longer than 10 minutes
• CM bulletin board attractive and inviting	• Students at desks
• CM elements reflect days in school, date, and monthly elements (see pacing guide)	• No wait time for students to participate
• Intentional planning of DOK questions and strategies	• No intentional planning
• Discussion based on appropriate numbers and elements of the month	• CM does not reflect elements of the month as set out in Everyday Counts Teacher's Guide
• Teacher wait time for DOK questions	• DOK 1 questions only
• Students actively participating, sharing their reasoning, and justifying their thinking	• Teachers calling on students with raised hands only
• Students respect each other's responses	• Songs, dances, and chants
• Students share various paths to problem solving	
• Students completing their own calendars (math workstation)	

"Look-For" the following characteristics and implementation strategies within our video exemplars for this structure:

Link to Video:	
1. Everyday Counts Calendar Math 1st grade example	
2. Calendar Math Kindergarten example	
3. Calendar Math example	

Number Talks

Purpose:

A number talk is a short routine that support students' understanding of numbers and their relationships, building a sense of number. The **purpose** of developing **number** sense in students is so that they understand the underlying concepts of the operations they perform (addition, subtraction, multiplication, division). Students who have strong **number** sense can solve problems in more than one way, and check that their answers make sense.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• Whole group meeting area	• Students at desks
• No paper or pencil	• Students have whiteboards and markers
• K-2 (use of manipulatives)	• Procedures only
• Purposeful number strings focused on strategy	• Teacher explicitly teaches strategies
• Intentional planning of questions and strategies (Teacher talk moves)	• No number strings or intentional planning
• Teacher provides wait time	• No teacher wait time
• Student accountability (Hand signals)	• Teacher calls on students with hands raised
• Recording of students' strategies	• No recording of students' strategies
• Student to student/teacher to student discussions	• Teacher to student discussion only
• Teacher facilitates Number Talks	• Lasts longer than 10 minutes
• No more than 10 minutes	
• All answers accepted, respected, and considered	

"Look-For" the following characteristics and implementation strategies within our video exemplars for this structure:

Link to Video:	
1. Introduction to Number Talks example	
2. First Grade Number Talks	
3. Third Grade Number Talks	

Whole Class Instruction

Purpose:

Whole Class Instruction consists of **two main parts: Mini-lesson & Focus lesson**

Mini-lesson: The first 8 minutes are crucial and can make or break a lesson. This is the mini-lesson which serves the purpose of providing a hook, or touchstone activity around a major concept or standard. Mini-lessons lay the foundation for Math Workshop. **They should excite, thrill, intrigue, and invite students to wonder "What will we learn today?"**

Focus lesson: Before beginning the focus lesson, the teacher needs to guide the students through the student learning target or "I can statement". Next, the teacher may choose to use one of two instructional models. The teacher may choose to utilize the **"I do", "We do", "You do" Gradual Release Model** (p. 52) when wanting to explicitly teach a concept standard. Or the teacher may flip the model and use the **"You do", "We do", "I do" Rapid Release Model**. (p. 53) Flipping the gradual release model by starting with a "you do" (rather than the teacher-centered "I do") approach allows children to grapple with new concepts and **deepen their understanding through problem solving and grit.** Both serve the purpose of supporting the student learning target of the day. It is teacher facilitated, student centered, and generates academic discourse.

Exit Ticket: Upon completion of the focus lesson, the next 5 minutes students will complete an exit ticket of what they learned before transitioning to Math Workstations. (p. 52-53)

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• 8 minute mini-lesson to hook students	• Standard/objective unclear
• Student Learning Target/I can statement	• DOK 1 question types
• Use of manipulatives, CRA model, or real world examples	• Longer than 10 minutes of teacher lecture
• Purposeful, rigorous and/or challenging introduction to grade level standard	• Students working alone or homogenously grouped
• No more than 15 minutes K5 – 1st and 30 minutes 2nd – 5th	• Intentional planning not evident
• Students seated and working in mixed ability groups	• Teacher demonstrates procedures of skill - i.e. Bottom, Bigger, Borrow
• Intentional planning and preparation with DOK question types	• Completing a worksheet together as a whole class
• Gradual Release or Rapid Release Instructional Model	

"Look-For" the following characteristics and implementation strategies within our video exemplars for this structure:

Link to Video:	
1. I Can Statements in Kindergarten	

"I Do", "We Do", "You Do" Gradual Release Model

Modeling (I do)

In this portion of the lesson, be explicit with students as to how to use a particular skill you are teaching or understand the concept being taught. Include think-alouds as a way to explicitly share with students what goes on in the mind of a mathematician.

Provide scaffolds for students by clearly explaining tasks and lead students step-by-step through the processes they will be using.

It is important to point out that students should still be actively engaged during the modeling process. Teachers can have students chorally call key vocabulary or next steps.

Guided practice (we do together)

As you move toward practicing with students, consider how you will call on students.

Begin with selecting some students intentionally. These may be students who have a clear understanding of the step you are practicing, or students that have a common misconception that needs to be addressed with the class.

Next move to calling on students randomly. This can be done using a cooperative structure like numbered heads, or through other methods such as using a computer program. Calling on random students holds all students to be accountable to work through the skill and demonstrate where they are in the process. Successes or errors can be addressed at this time.

Finally, call on volunteers. At times the students who always volunteer to share or practice will have something additional to add that has not come up with other students.

This practice flips what we often see in classrooms, where teachers first call on volunteers. The problem there is that often the same group of students gets called on time and again, and leave other students disengaged or lacking an opportunity to practice with direct guidance from the teacher.

Of course, the students who are helping you or sharing ideas should not be the only students engaged in the instruction. Just as in the modeling section, other students can stay engaged by chorally calling out vocabulary or steps, helping the student through "phone-a-friend" or by assisting each other.

Collaborative peer practice (you do together)

During the interactive peer-collaboration guided-practice phase, students work together in small groups to practice the skill or concept. Teachers can incorporate cooperative learning structures at this point to maximize student participation and practice of the skills or concepts.

Having students seated in pairs, triads or small groups will be helpful during this time, and structures such as having a talking stick and explicit roles within a group can keep students on task and provide an opportunity for all students to be equally involved.

The teacher, during this time, walks around monitoring student practice and assisting students as needed to clarify any misconceptions. The teacher can remind students of key points, and assesses if particular students or if the entire class needs additional instruction. Also, this is a time for teachers to pull together a small group of students that may need additional modeling or instruction by differentiating for students based on need.

Independent practice (you do independently) – EXIT TICKET

The last stage requires students to practice and apply the task independently in the form of an exit ticket. In many ways this can be said to be the purpose of education: to have students be able to apply the skills and information they have learned on their own, without the help and support of a teacher or peer.

"You Do", "We Do", "I Do" Rapid Release Model

There are many reasons why to use this model:

- Students often learn best by doing, not by watching other people doing
- the gauge of students' readiness for a learning task isn't whether they'll complete it correctly, but whether they have the prior knowledge and/or skills to attempt it at the level of productive struggle
- it's stifling for students when teachers present one way to do something before students have a chance to discover and evaluate their own ways of doing it
- mistakes are stepping stones to meaningful learning
- new content often builds on previous content

Independent Practice (YOU DO)

Start the lesson by giving your students a risk math task and see what they can do with it. Ex. G. Fletchers 3-ACT Tasks or engaging math tasks by Dan Meyer .

Remember to:

1. Provide multiple entry points
2. Allow for varied solution paths
3. Focus on process, not necessarily the answer

These types of rich tasks allow students to play around with mathematical ideas and use the math tools they currently have in their tool chest.

Collaborative Peer Practice (WE DO TOGETHER)

After students have had an opportunity to work independently and have probably run into some roadblocks, they need time to work with their peers. During this time they are utilizing manipulatives, helping each other, connecting ideas, justifying their methods, etc. Additionally, the teacher is walking around during this time facilitating and questioning groups.

Modeling and Guided Practice (I DO)

Now your students are ready to listen to you. You have given them a need for your direct instruction, and you have a ton of data to pull together to make a rich learning experience for your students. For example, some of the data you now have are the following:

1. Students' questions and inquiries
2. Students' representations and processes
3. Student-to-student conversations
4. Students' misunderstandings and understandings
5. Students' responses to your questions

Exit Slip – Make sure students complete an exit slip at the end of your focus lesson.

Guided Math Instruction (Small group)

Purpose:
Guided math groups are small, intentional lessons with students around a particular concept or standard. Students work on same standard at their zone of proximal development. The goal of guided math is for students to become proficient mathematicians who have conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and mathematical confidence.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• No more than 5 to 6 students.	• Teachers demonstrating procedure, giving guided practice problems, and students completing problems
• Teacher table/station or designated area	• Abstract only with white boards and markers
• No more than 20 minutes per group	• Skill and drill
• Teacher facilitates discussion, observes, and assesses (anecdotals)	• Teacher doing most of the talking and work
• Use of CRA model (math tool kits/manipulatives)	• Heterogeneously grouping
• Students doing the math 85% of time	• Students completing worksheets
• Homogenous grouping based on skill/standard, differentiated instruction	• No clear purpose
• Flexible, fluid grouping over time (changes based on skill/standard)	• All groups doing the same thing (My Math Textbook)
• Based on a conceptual understanding	• No before, during, and after structure
• Quick mini-lesson introduction (before), student work period (during), debriefing or sharing (after)	• No anecdotal note taking
• Intentional planning and preparation with DOK question types	• Teacher not facilitating – busy with other groups of students or tasks
	• No preplanning

"Look-For" the following characteristics and implementation strategies within our video exemplars in TEAMS for this structure:

Link to Video in TEAMS:	*(Your video link will be from TEAMS)*
1. Fourth Grade Small Group	
2. 1st Grade Small Group	
3.	

Whole Group Instruction vs. Guided Math Instruction

What a whole class mini-lesson could include?

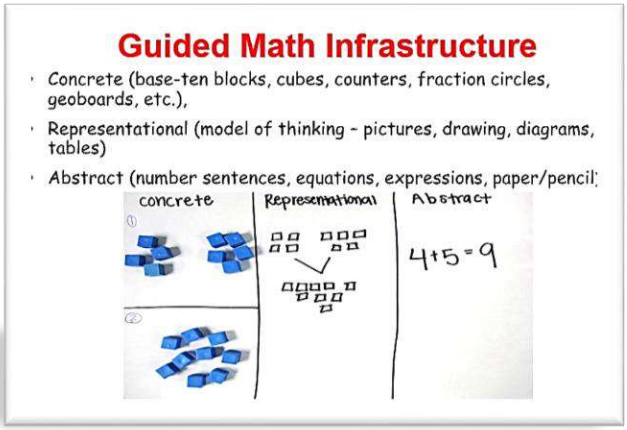
- Introducing a new unit or standard of study.
- Guided Math Read Aloud that supports the skill or standard being taught.
- Introducing new vocabulary.
- Short Video to show math in the real world.
- Anchor Chart Creation

Whole Group	Guided Math
• Student-centered • One standard • Can be differentiated • One lesson/focus for entire group • Instruction based on Scope and Sequence • Teacher acts as facilitator • Not all students have same understanding of different concepts • It's possible that only some student misconceptions are clear • Higher order questioning • Use of manipulatives • Balanced instruction • Experiential learning	• Student-centered • One standard • Differentiated • Instruction based on Scope and Sequence • Teacher acts as facilitator • Small group focused instruction using a new approach/lesson • Based on need of individual students (data-driven instruction) • Groups are flexible, change frequently • Clears up all misconceptions • Higher order questioning • Use of manipulatives • Balanced instruction • Experiential learning

CRA Model – (Concrete- Representational-Abstract)

Effective Uses:

- Implementation of CRA method of instruction: Concrete-Representational-Abstract
- Reinforce Whole Group Lesson (Mini-Lesson)
- Practice of New Skills
- Instruction with the use of Manipulatives
- Introduction of activities that will later become independent work.
- Informal Assessment
- Re-teaching Based on Formative Assessment



Math Workshop Stations

Purpose:
Math Workstations are differentiated, standards-based, engaging activities that foster mathematical thinking. The purpose of stations are to encourage students to make sense of math by engaging in sustained practice over time that allows them to build conceptual understanding, procedural fluency, and problem solving-skills.

What SHOULD this structure look like?	What this structure SHOULD NOT look like.
• Students working at Zone of Proximal Development	• Overuse of TpT or Pinterest worksheets and games
• Homogenously or heterogeneously grouping	• Students working on same activities, not differentiated
• Students taking turns and respecting each other	• Students off-task and not talking about math
• Students actively engaged and talking about math	• Intentional planning and preparation not evident
• Intentional planning of materials, materials readily available (math toolkits or manipulatives	• Rotation schedule and rules and procedures not in place or not visible
• Students working independently, with a partner, or with a group	• No accountability piece
• Math Workshop Models i.e. M-A-T-H acronym (Math Facts, At Your Seat, Teacher Time, Hands On Math)	• Time wasted on cutting and pasting of materials
• Rotation schedule and rules and procedures displayed and visible	• Students working only in My Math textbooks or worksheets
• Timer utilized for active flow of workshop stations	• All students on laptops
• Accountability piece for workstations i.e. exit slips, reflection journals, SeeSaw, answer checking, etc.	
• CFAs can be administered as a workstation	

"Look-For" the following characteristics and implementation strategies within our video exemplars in TEAMS for this structure:

Link to Video in TEAMS:	*(Your video link will be from TEAMS)*
1.	
2.	
3.	

Math Centers vs. Math Workstations

Math Centers	Math Workstations
• Games and activities introduced the day of center and rarely used for instructional purposes. • Centers are often thematic and change weekly. • Centers are often made available to students after they complete their regular work. • All students work on the same centers, and activities are seldom differentiated.	• Tasks are derived from materials previously used during instruction, so students are familiar with them. • Tasks changed for instructional purposes, not because it is the end of the week. • Tasks provide ongoing practice to help students retain and deepen their understanding and are an important part of students' mathematical instruction. • Tasks are differentiated to meet the identified learning needs of the students.

Student Math Conferences

Purpose: One-on-one discussion between the teacher and the student can help the teacher discover what their **students** are thinking mathematically and then identify what to do to help them progress in both their understanding and skill.

- Targeted - focused on one math goal of content or practice.
- It's individualized and differentiated.
- It creates the timely opportunity to immediately impact student learning.

What <u>SHOULD</u> this structure look like?	**What this structure <u>SHOULD NOT</u> look like.**
• A conversation about math	• Group conferences
• No more than 5 minutes per student	• Longer than 5 minutes
• 4 to 5 students per day; every student each week	• Teaching or lecturing
• Predictable structure: Ask a question, Assess Understanding, Teach for improvement, Document the work	• More teacher talk than student talk
• Feedback and individualized instruction	• No structure or planning
• Can be integrated during guided math teacher station	• No documentation of conference

"Look-For" the following characteristics and implementation strategies within our video exemplars in TEAMS for this structure:

Link to Video in TEAMS:	***(Your video link will be from TEAMS)***
1. Math Conferences	
2.	
3.	

How do you have a math conference?

1. **Ask a question.** The question you ask will help you discover what the student is doing and understanding about the math. The question may lead you to a discovery of deeper understanding that needs to be fostered or a misunderstanding that needs to be addressed. You might find the student understands the procedure, but not the math concept. You may discover the student doesn't understand that what they are doing fits into a larger picture of the concept. Let the student have the opportunity to tell you what they know, need, or don't understand. This is powerful in helping all students develop self-efficacy for learning. See a list below of questions to get you started.
2. **Assess understanding.** Use the conversation to assess the student's edge of understanding. It's an opportunity to discover the student's ability to model and explain the math. Getting a right answer is necessary, but not sufficient.
3. **Teach for improvement.** The goal of a conference is to move a student's understanding forward in math. This takes explicit teaching. Just as Anderson suggested, ***cue the student*** that you are about to start teaching by saying, "There's something I want to teach you today..." Then, ***name the strategy*** you are teaching. Next, ***explain the strategy*** including *what* it is and *why* it is important to learn. ***Describe*** how to do it. ***Give examples*** and use the strategy, Now, let the student ***try the strategy***. All students learn by doing. End the conference **reminding the student to use this strategy** when faced with similar math.
4. **Document the work.** Create a simple system to record conferences. This will help you track student progress, give feedback, and direct your instruction. The information may give guidance to creating flexible groupings for small group instruction or partnering students to maximize learning.

Questions to Get the Math Conference Started

- Tell me about the problem you are solving.
- What are you working on?
- How can you model that?
- What strategy are you using to solve?
- What do you know about this problem?
- What other math does this make you think of?
- What do you do when you get stuck?
- How will this work help you answer the essential question?
- What patterns do you notice?
- What is another strategy that will allow you to solve?
- Is there a more efficient strategy? (a quicker way)
- Have you found a possible rule?
- What is the problem asking?
- Does this make sense, and how do you know?
- What is confusing?
- How can I help you?
- Why did you choose this strategy?
- What kind of errors have you made?
- What can you do to correct your errors?

Closure or Share Time

Purpose: A quick summary of the student learning target (I can statement) and couple of students sharing what they learned.	
What <u>SHOULD</u> this structure look like?	**What this structure <u>SHOULD NOT</u> look like.**
• No more than 5 minutes	• Extended reteaching of lesson
• No reteaching	• Teacher telling them what they learned
• Student led discussions	• No closure or share time evident
	• Teacher just assigns and discusses homework

"Look-For" the following characteristics and implementation strategies within our video exemplars in TEAMS for this structure:	
Link to Video in TEAMS:	***(Your video link will be from TEAMS)***
1.	
2.	
3.	

Ongoing Assessments

Purpose: Assessments are the key to a productive Guided Math Workshop Model. Teachers should utilize a variety of assessments ranging from on-the-spot, in-the-moment, spontaneous assessments, to formal, pre-planned assessments. Additionally, feedback facilitates learning, therefore, teachers should provide feedback throughout the learning process, not just at the end of teaching a topic.

What <u>SHOULD</u> this structure look like?	**What this structure <u>SHOULD NOT</u> look like.**
• Daily anecdotals of students in small groups	• Teachers not taking notes
• Notes from student conferences	• Teacher not asking DOK questions
• Informal assessments "i.e. thumbs up, thumbs down	• Math 4 Today or Study Island quizzes
• Student self-assessments or exit tickets	
• Math workstation artifacts	
• Quizzes and Tests	
• Pre and Post Common Formative Assessments	
• Quarterly Benchmarks	
• DOK questioning	

"Look-For" the following characteristics and implementation strategies within our video exemplars in TEAMS for this structure:

Link to Video in TEAMS:	***(Your video link will be from TEAMS)***
1. Guiding Instruction Through CFAs	
2.	
3.	

K-5th Homework Policy

Homework is an opportunity for students to practice skills and concepts. If homework is sent home, it should have maximum of four standards-based problems. It should include a combination of real-world problem solving, computation, and fluency practice. Additionally, homework should only be given Monday – Friday.

Homework Considerations

- It should be a task that students can complete independently.
- Academic grades should not be jeopardized because of incomplete homework.
- Homework should be purposeful rather than "just something to do." (Vatterott, 2009)
- The task is as important as the time required for homework. In other words, quality is greater than quantity. (Vatterott, 2009)
- Homework access should be equitable, therefore, computer assignments should not be given as tasks to complete.

Maximizing Instructional Time

Instructional time is lost when too much time is dedicated to homework review and/or correction. Time to discuss problems can happen in different ways. Mathematics engagement and momentum can also be lost when the class opening is dedicated to homework. Discussing homework may be a small portion of small group discussion.

Suggested Homework Review

- Post answers to questions and tell the students two answers are incorrect. Ask which ones. Share their thinking with a partner.
- Working with partners, students share answers for particular questions and explain how they found solutions. (Or problems of their choice.)
- Students self-assess by reviewing homework answers displayed on the board or with a completed copy.

Other options include:

- Collect the homework and correct it outside of class.
- Have a co-teacher, paraeducator, or volunteer record completed homework.
- Ask students to vote on a problem they would like to discuss.
- Go over a small subset of the problems. This shouldn't take place during the first five minutes of class.

Reflecting on Homework

Teachers or students can use homework for feedback. Students may reflect on problems or homework as a whole in the general categories below (primary options in parenthesis).

- Got it/understood ("smiley" face)
- Sort of got it/not sure ("straight" face)
- Didn't get it/totally lost ("frowny face") (Vatterott, 2009)

Other ideas for student self-reflection include:

- What questions/undertaintities do you still have about ____?
- What was most effective in ____?
- What was least effective in ____?
- How difficult was ____ for you?

When reviewing homework during independent time, students might record

- item number
- reason missed
- correction

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