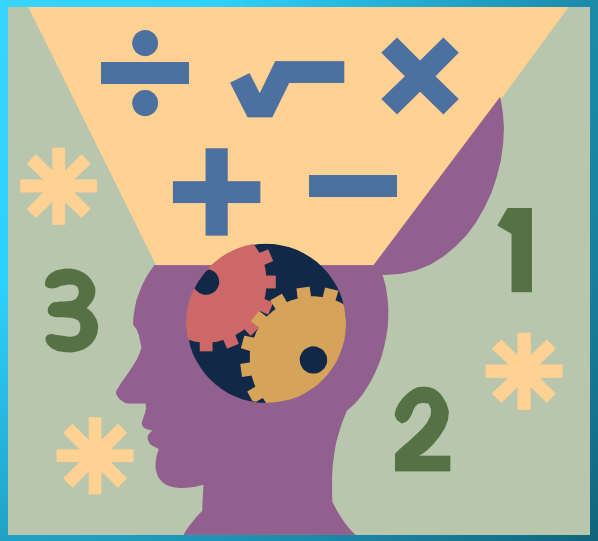
Problem Solving

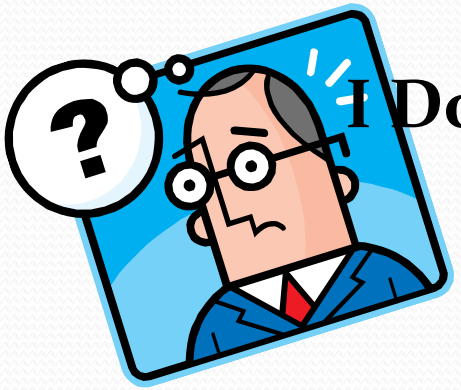


“Through problem solving, students can experience the power and utility of mathematics. Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas.”

NCTM 2000, p. 256

Seeing the BIG Picture

90 Second Sweep



I Don't Know

Group DeBriefs

Think-Puzzle-Explore:
Problem Solving

Step 1

- **What do you THINK you know?**

Step 2

- **What PUZZLES you about this topic?**

Step 3

- **How can we EXPLORE these puzzles?**

Challenges students face when solving applications in mathematics

- Brainstorm **5 or more** major challenges students face when solving rigorous problems in mathematics
- You will share with your shoulder partner when think time is up and I say go.

What are the characteristics of good problem solvers?

- Brainstorm these characteristics
- You have 2 minutes

An Effective Problem Solver

- Reads the problem carefully
- Defines the type of answer that is required
- Identifies important information and words
- Accesses background knowledge regarding a similar situation
- Eliminates extraneous information
- Uses a graphic organizer
- Sets up the problem correctly
- Uses mental math and estimation
- Checks the answer for reasonableness
- Make mental pictures (visualize)

Are you a problem solver?

http://www.youtube.com/watch?v=oXCuGvsThEw

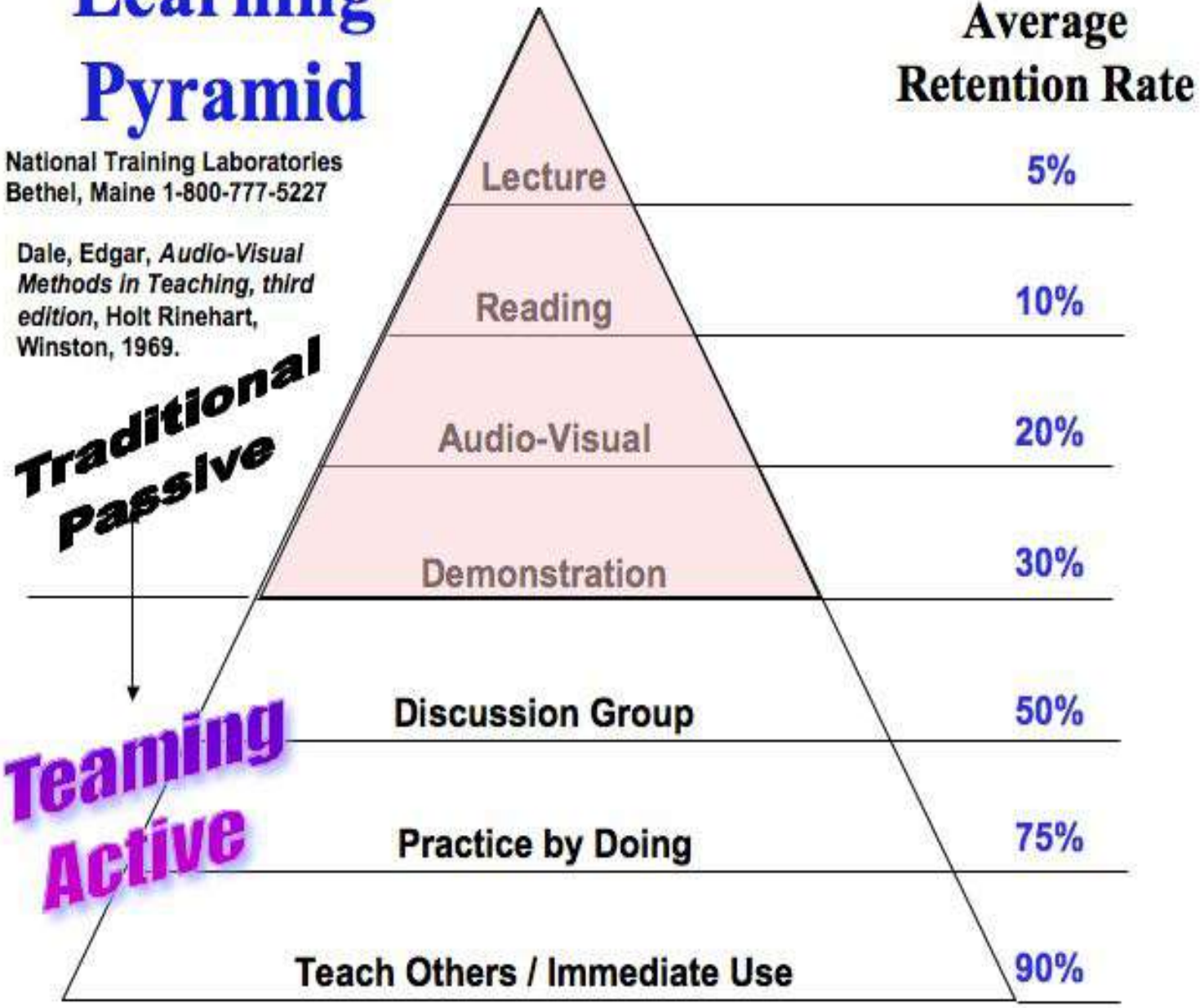
What Does Math Involve?

- Memory
- Language
- Sequencing
- Spatial ordering
- Critical thinking
- Good problem-solving strategies
- Number sense
- Reasoning
- Making connections
- Organizational skills

Learning Pyramid

National Training Laboratories
Bethel, Maine 1-800-777-5227

Dale, Edgar, *Audio-Visual Methods in Teaching, third edition*, Holt Rinehart, Winston, 1969.



Problem Solving…Revised TEKS

3.14B, 4.14B, 5.14B
Solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, evaluating the solution for reasonableness

3.14C, 4.14C, 5.14C
Select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking , or acting it out in order to solve a problem

3.1B, 4.1B, 5.1B
Use problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution

NEW process standards

Problem-Solving Model	
Current TEKS	**Revised TEKS (2012)**
Understanding the Problem	Analyzing given information
Making a plan	Formulating a plan or strategy
Carrying out the plan	Determining a solution
Evaluating the solution for reasonableness	Justifying the solution Evaluating the problem-solving process and the reasonableness of the solution

UPS

UNDERSTAND	**PLAN**
SOLVE	**CHECK**

UPS

UNDERSTAND Use a problem solving model that incorporates • **analyzing given information**	**PLAN** Use a problem solving model that incorporates • **formulating a plan or strategy**
SOLVE Use a problem solving model that incorporates • **determining a solution**	**CHECK** Use a problem solving model that incorporates • **justifying the solution** • **evaluating the problem-solving process and the reasonableness of the solution**

Problem Solving Plan Implementation

- The plan creates a process that students should use with all problem activities, not just stated/word problems.
- The plan is designed to promote critical thinking and self evaluation.
- The plan is NOT an extra thing to do in math instruction.
- The plan should be teacher directed through the use of questions that engage students in the process, not just guiding them blindly with students copying what to do.

Problem Solving Plan Implementation

- Students should be making the decisions and explaining why and how.
- Estimation, writing equations and number sense should be integrated whenever possible.
- During the **PLAN** component you have the opportunity to ask students to **create** the equation they would use, **estimate** the solution, how large/small the answer should be….

UPS

UNDERSTAND	**PLAN**
SOLVE	**CHECK**

The Steps of UPS	Teacher Actions	Student Actions
<u>Understand/ Analyze</u> ***Focal Point: students slow down, comprehend the main idea(question), think before they plan and solve***	•Model thoughts and actions, question students on vocabulary and key ideas using a strategy such as think a-louds · Model how to summarize question · Use questioning to focus and guide students' thinking · Facilitate the reading of words, graphics, and symbolic notation using strategies such as choral reading, small group reading, individual reading, reflection, think-pair- shares	•Read application at least 2 times · Comprehend and rephrase the question · Restate information from the problem · Demonstrate understanding of vocabulary, graphics, and/or symbols · Define the type of answer that is required · Recognize the mathematical concept

Let's Choral Read

Alex used blue, red, and green pieces of plastic to make a design. He used 84 green pieces of plastic. He used 20 more green pieces of plastic than blue pieces of plastic. He used 15 more red pieces of plastic than blue pieces of plastic. What is the number of red pieces of plastic Alex used?

Five people each bought a box of straws. In Aaron's box 6/20 of the straws were green. The table below shows the fraction of green straws in the other four boxes.

Straws

Name	Fraction of Green Straws
Lance	3/10
Cindy	10/40
Eric	3/30
Fred	7/10

Based on the table, which two people had boxes in which less than 6/20 of the straws were green?

Remove Numbers

Ben is making wooden toys for the next arts and crafts sale. Each toy costs Ben ▭ to make. If he sells the toys for ▭ each, how many would he have to sell to make a profit of exactly ▭?

Ben is making wooden toys for the next arts and crafts sale. Each toy costs Ben $1.80 to make. If he sells the toys for $3.00 each, how many would he have to sell to make a profit of exactly $36.00?

Strategies to Read the Problem for Understanding

- Choral Read
- A B partners, Paired read
- Team reading – odds stand across evens. Odds read to evens and then reverse. Listen for drop in volume for words they may not know or mispronunciation and then go over the words.
- Choral read followed by individual reading (to yourself)
- Small group
- Cover and read line by line
- Read/analyze graphics
- Summary Statement

UPS

UNDERSTAND	**PLAN**
SOLVE	**CHECK**

The Steps of UPS	Teacher Actions	Student Actions
<u>Plan</u> ***Focal Point: students allow the question to drive the plan; the students are making the decisions***	•Use probing questions such as: Have we seen a problem like this before? What tools do we need? (Formula/conversion chart, ruler, protractor, technology, etc.) · Describe types of strategies; show/discuss alternate plans · Help students get a mental picture of what the question is asking.	•Choose or adapt a method/ strategy · Identify necessary information (use highlighting, circling or underlining) · Explain steps (orally or verbally) using sequential order · Use notes and resources to develop plan · Setup expression or equation · Use estimation/mental math to establish benchmark · Reflect on question

Problem Solving Detective

- Select a partner
- One of you will be the detective and the other the witness
- Only the witness sees the problem
- The detective must solve the problem

Mrs. Evans wants to buy enough dog food to feed her dog for 90 days. Her dog eats 4 ounces of dog food twice a day. How many pounds of dog food should she buy?

Application 4:

Ben is making wooden toys for the next arts and crafts sale. Each toy costs Ben $1.80 to make. If he sells the toys for $3.00 each, how many would he have to sell to make a profit of exactly $36.00?

Understand

Understand	**Plan**
Display/project the problem with the numbers removed as follows: Ben is making wooden toys for the next arts and crafts sale. Each toy costs Ben [redacted] to make. If he sells the toys for [redacted] each, how many would he have to sell to make a profit of exactly [redacted]? Direct students to read entire application aloud as a class or with a small group Listen to the reading. Model reading: clarify any mistakes once students have read it if you hear mistakes in the reading. Questioning on the following: • Comprehension of the task- ask students to restate task/question • Meaning of the word "profit" • Call on several students to provide what they think is the needed information	Questioning on the following: • Is this a multiple step problem? • Think about your plan. What will be the steps you will take to solve in sequential order? • What operation(s) will you use? • Read again if needed. • Revisit the question summary, will your plan provide an answer to the question? • Write an equation that you would use to solve the problem. •Is there a picture than can help you with this problem?
Solve	**Check**
Display/project the problem with the numbers included.	Questioning on the following:

UPS

UNDERSTAND/ANALYZE	**PLAN**
SOLVE	**CHECK**

The Steps of UPS	Teacher Actions	Student Actions
Solve ***Focal Point: students are applying concepts and procedures***	•Check/re-teach for understanding · Examine different methods students used · Assess the application of the plan · Allow students to explain their thinking process	•Work problem according to strategy chosen and process provided · Solve problem out loud · Explain thinking process

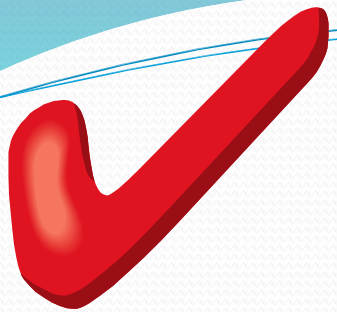
Solve

- Reread if needed.
- Students apply the plan and create an equation
- Compare the equation to the answer choices if it is multiple choice
- Which answer choice would you select?

UPS

UNDERSTAND/ANALYZE	**PLAN**
SOLVE	**CHECK**

The Steps of UPS	Teacher Actions	Student Actions
 ***Focal Point: students are making connections, developing critical thinking skills, evaluating the problem solving process, and becoming good problem solvers***	•Provide tools/strategies to double check work · Use clarification, comparative and transfer questions to help students make connections · Ask for validity/other ways to justify answer · Help students to reason it out; use questioning strategies to guide thinking to determine reasonableness · Model transitions from less efficient forms of representations to grade level expectations (EX: repeated addition/multiplication, column form/horizontal form, table/proportion) · Use the language of mathematics and require students to do so as well · Provide the environment/time conducive to reflection, evaluation, and error analysis	•Use oral and/or written justification which includes examples, non- examples, pictures, symbols, written and/or verbal reasoning · Demonstrate strategies to double check answer(s) and confirm reasonableness · Compare/contrast other strategies used to solve the problem · Write a complete sentence that answers the question · “Translate” less efficient mathematical representations to more efficient forms · Compare solution to estimate · Revise and edit



- Comparison of answers if multiple justifications have been found.
- Is this a situation where there could be more than one justification?
- Evaluate the problem solving process and check for reasonableness.
- Write answer in a complete sentence.

UNDERSTAND/ANALYZE	PLAN
• Read the problem (2 times) • Summary statement • Restate information from the problem • Demonstrate understanding of vocabulary, graphics, and/or symbols • Define the type of answer that is required • Recognize the mathematical concept	• Select and use tools, resources, strategies • Tap into prior knowledge • Choose or adapt a method/ strategy • Identify necessary information (use highlighting, circling or underlining) • Explain steps (orally or verbally) using sequential order • Use notes and resources to develop plan • Setup expression or equation • Use estimation/mental math to establish benchmark • Reflect on question
SOLVE •Reread if needed. •Students apply the plan and create an equation •Compare the equation to the answer choices. •Which answer choice would you select?	**CHECK** ✔ • Go back through the plan • Estimate and use number sense to justify the solution

Dual Coding

Let's take a look at how questions have been dual coded on the recent tests.

- What do you notice?
- How are they clustered?
- Why do you think they are clustered in that way?
- Are they all assessed?
- Are any assessed more than others?
- Do any of them have the same SE attached?

A Deeper Look…

Each group has two problems from one process standard. Working with your group analyze the problems and consider the following:

- What commonalities do you find with the problems?
- What makes them dual coded?
- Which SE do you think they fit?

Be prepared to share with the group.

Your Turn…

Consider the displayed problem.

- How could you make this problem dual coded?
- What could you add, tweak, or ask the student to do?

Reflecting on Problem Solving

Problem/Task:		
My original answer	**My new solution with work shown**	**The correct solution**

Reflection: Why I missed the original:

- Didn't understand
- Thought it was right
- Skipped a step
- Made a mistake in step_____
- Studied but forgot _____
- No clue
- Ran out of time and guessed
- Careless mistake made in_____
- Other:_____

Why I know I now have the right answer:

What I need to do (or not do) in the future:

What is Problem Solving?

According to Michael E. Martinez

- There is no formula for problem solving
- How people solve problems varies
- Mistakes are inevitable
- Problem solvers need to be aware of the total process
- Flexibility is essential
- Error and uncertainty should be expected
- Uncertainty should be embraced at least temporarily

Reflection

Problem Solving Idea(s)	How can I make this work in my classroom?

Improving Problem-Solving Skills

- Solve problems out loud
- Explain your thinking process
- Allow students to explain their thinking process
- Use the language of math and require students to do so as well
- Model strategy selection
- Make time for discussion of strategies
- Build time for communication
- Ask open-ended questions
- Create lessons that actively engage learners

Jennifer Cromley, *Learning to Think, Learning to Learn*

Let's revisit…

http://www.youtube.com/watch?v=oXCuGvsThEw

We need your input.

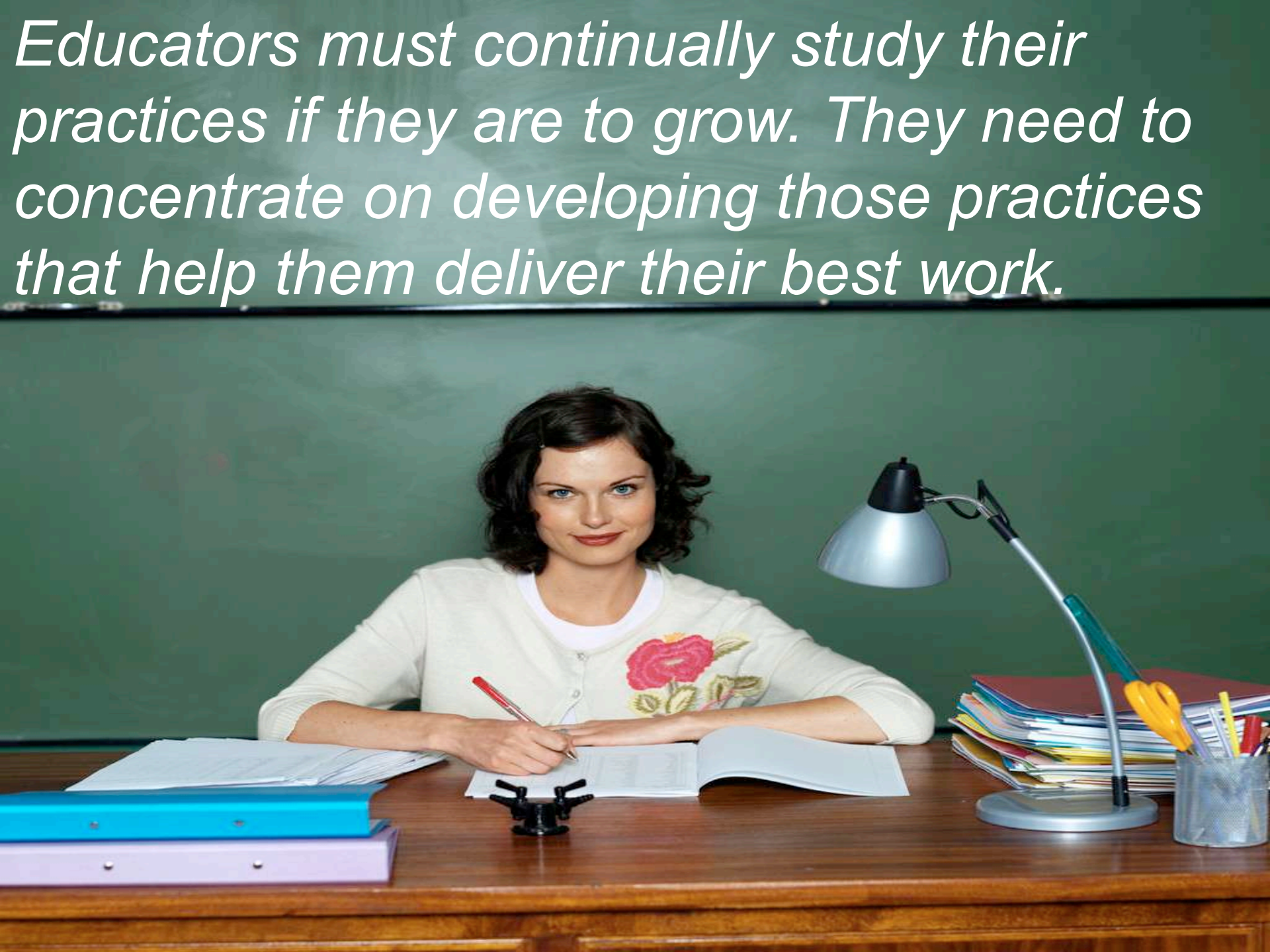
Professional Learning Surveys:

We have created some surveys so that we can elicit feedback from all of the participants about the trainings they attend. Visit the website below and complete the appropriate survey(s).

http://schools.birdvilleschools.net/surveys

The surveys will be open until April 18 and reminders will go out through Mark Thomas next week, but we would like for you to complete your survey today if possible.

Educators must continually study their practices if they are to grow. They need to concentrate on developing those practices that help them deliver their best work.



"We are what we repeatedly do. Excellence, then, is not an act but a habit." ---Aristotle

A mind once stretched by a new idea never regains its original dimensions.