# Mathematics Curriculum Guide

## *Algebra 2*

## *2017-18*

# *Topic 7: Rational Functions*

This unit of rational functions begins with the study of direct and inverse variation. Students will learn that in a direct variation, two positive quantities either increase together or decrease together. In an inverse variation, as one quantity increases the other decreases. Students will also learn about transformations of the parent reciprocal function that includes stretches, compressions, reflections, and translations (horizontal and vertical). After that, students will learn about rational functions and their graphs including asymptotic behavior. Students will conclude the unit by learning to solve rational equations. They will apply their knowledge fraction operations to rational expressions and equations.

**Common Misconceptions and Errors:**

- **Graphing Reciprocal Functions:**
  - Functions that model inverse variations belong to a family whose parent is the reciprocal function $f(x) = \frac{1}{x}$. The branches of the parent function $y = \frac{1}{x}$ are in Quadrants I and III.
  - Stretches and compressions of the parent function remain in the same quadrants. Reflections are in Quadrants II and IV.
  - Reciprocal functions can also be translated horizontally or vertically.

# Topic 7: Rational Functions

## Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

**Timeframe:** 13 days
**Start Date:** February 13, 2018
**Assessment Dates:** March 2, 2018

## Standards:

**F-BF 1** - Write a function that describes a relationship between two quantities.

**A-SSE 1**. Interpret expressions that represent a quantity in terms of its context.

**A-SSE 2**. Use the structure of an expression to identify ways to rewrite it.

**A-REI 11**. Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x);* find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, ~~absolute value, exponential, and logarithmic functions~~.

**A-APR 3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**A-APR 6.** Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x),* where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system.

**A-APR 7.** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

**A-CED 1.** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**A-CED 2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

**F-BF 3.** Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

## Meaning-Making

### Understandings

*Students will understand that…*

- Quantities x and y are inversely proportional only if increasing x by the factor k (k > 1) **(A-CED 1,2)**
- If a rational function is in simplified form and the polynomial in the denominator is not constant, the graph of the rational function features asymptotic behavior. A rational function may have zero or one horizontal asymptote and zero or more vertical asymptotes. **(F-BF 1,3) (A-APR 3)**
- A rational expression is in simplest form when its numerator and denominator are polynomials that have no common divisors. **(A-SSE 1, 2)**
- To solve an equation containing rational expressions, first multiply each side by the least common denominator of the rational expressions**(A-APR 6, 7) (A-REI 11)**

### Essential Questions

*Students will keep considering…*

- Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?
- What kinds of asymptotes are possible for a rational function and what kinds of transformations are part of the graphs of rational functions?

## Acquisition

### Knowledge

*Students will know…*

**Vocabulary**: combined variation, complex fraction, continuous graph, discontinuous graph, inverse variation, joint variation, point of discontinuity, rational equation, rational expression, rational function, reciprocal function, extraneous solution, dependent system, equivalent systems, independent system, linear system

**Concepts:**

- Combined variations in equation form
- General form of the Reciprocal Function Family
- Vertical and horizontal asymptotes of rational functions
- Addition, subtraction, multiplication, and division rules for rational expressions and equations
- Procedures and/or methods for solving rational equations
- Graphing, substitution, and elimination methods to write and solve equivalent equations

### Skills

*Students will be skilled at and able to do the following…*

- Identify and describe inverse and direct variation functions
- Write equations to model the direct and inverse variations and graph the function
- Identify what kinds of asymptotes are possible for a rational function and what kind of transformation are part of the graphs of rational functions
- Identify whether a rational function has an asymptote and how to differentiate between vertical and horizontal asymptotes
- Identify properties of and graph rational functions, reciprocal functions, and translations of reciprocal functions.
- Factor quadratic expressions in order to determine the domain and range of rational functions
- Simplify, add, subtract, multiply and divide rational expressions.
- Solve rational equations by knowing how to solve quadratic equations.
- Define the domains of simplified rational expressions to make them equivalent to the originals.
- Evaluate the expressions on both sides of the equation for the values of the variables to make sure they are equivalent.
- Identify whether a system of equations is dependent, independent, or inconsistent.
- Graph to find a solution(s) to a system of linear equations.

# *Topic 7: Rational Functions*

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

**Transfer goals** highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

**Multiple measures** will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

| **Formative Assessment** | **Summative Assessment** |
|---|---|
| **Aligning Assessment to Stage One** | |
| • What constitutes evidence of understanding for this lesson?<br>• Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results?<br>• How will students reflect upon, self-assess, and set goals for their future learning? | • What evidence must be collected and assessed, given the desired results defined in stage one?<br>• What is evidence of understanding (as opposed to recall)?<br>• Through what task(s) will students demonstrate the desired understandings? |
| **Opportunities** | |
| • Discussions and student presentations<br>• Checking for understanding (using response boards)<br>• Ticket out the door, Cornell note summary, and error analysis<br>• *Performance Tasks* within a Unit<br>• Teacher-created assessments/quizzes | • Unit assessments<br>• Teacher-created quizzes and/or mid-unit assessments<br>• *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/)<br>• Performance tasks |

# *Topic 7: Rational Functions*

The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

<table>
<tr><th>Unit Skills</th><th>SBAC Targets (DOK)</th><th>Selected Standards</th><th>Examples</th></tr>
<tr>
<td>
<ul>
<li>Identify and describe inverse and direct variation functions</li>
<li>Write equations to model the direct and inverse variations and graph the function</li>
<li>Identify what kinds of asymptotes are possible for a rational function and what kind of transformation are part of the graphs of rational functions</li>
<li>Identify whether a rational function has an asymptote and how to differentiate between vertical and horizontal asymptotes</li>
<li>Identify properties of and graph rational functions, reciprocal functions, and translations of reciprocal functions.</li>
<li>Factor quadratic expressions in order to determine the domain and range of rational functions</li>
<li>Simplify, add, subtract, multiply and divide rational expressions.</li>
<li>Solve rational equations by knowing how to solve quadratic equations.</li>
<li>Define the domains of simplified rational expressions to make them equivalent to the originals.</li>
<li>Evaluate the expressions on both sides of the equation for the values of the variables to make sure they are equivalent.</li>
<li>Graph to find a solution(s) to a system of linear equations.</li>
</ul>
</td>
<td>
Create equations that describe numbers or relationships. (1,2)<br><br>
Represent and solve equations graphically. (1,2)<br><br>
Interpret functions that arise in applications in terms of a context. (1,2)<br><br>
Analyze functions using different representations. (1,2)<br><br>
Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)
</td>
<td>
F-BF 1 - Write a function that describes a relationship between two quantities.<br>
A-SSE 1. Interpret expressions that represent a quantity in terms of its context.<br>
A-SSE 2. Use the structure of an expression to identify ways to rewrite it.<br>
A-REI 11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, <s>absolute value, exponential, and logarithmic functions</s>.<br>
A-APR 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.<br>
A-APR 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.<br>
A-APR 7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.<br>
A-CED 1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.<br>
A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
</td>
<td>
Rational Functions<br><br>
1. Jim can paint a house in 25 hours. Alex can paint the same house in 20 hours.<br>
Write an equation that can be used to find the time in hours, t, it would take Jim and Alex to paint the house together assuming they both work at the rates they work when working alone.<br><br>
2. What value of x makes the equation $\frac{1}{\sqrt{5-x}}=3$ true?<br><br>
3. What value of t makes the equation $\frac{2}{t+3}=\frac{1}{t}$ true?<br><br>
4. Select whether each equation has no real solutions, one real solution, or infinitely many real solutions.<br>
<table>
<tr><th></th><th>No Real Solutions</th><th>One Real Solution</th><th>Infinitely Many Real Solutions</th></tr>
<tr><td>$-2x^2-\frac{3}{x}=0$</td><td></td><td></td><td></td></tr>
<tr><td>$\frac{3}{x}=\frac{3}{x+20}$</td><td></td><td></td><td></td></tr>
<tr><td>$\frac{x}{5}=\frac{2x+10}{10}-1$</td><td></td><td></td><td></td></tr>
</table>
<br>
5. Select Yes or No to indicate whether each value of b is a solution to the given equation.<br>
$3=\frac{9}{b+5}$<br>
<table>
<tr><th>Solution</th><th>Yes</th><th>No</th></tr>
<tr><td>$b=-8$</td><td></td><td></td></tr>
<tr><td>$b=-5$</td><td></td><td></td></tr>
<tr><td>$b=-2$</td><td></td><td></td></tr>
<tr><td>$b=22$</td><td></td><td></td></tr>
</table>
<br>
6. Beth is solving this equation: $\frac{1}{x}+3=\frac{3}{x}$.<br>
She says, “I can multiply both sides by x and get the linear equation $1 + 3x = 3$, whose solution is $x = \frac{2}{3}$.”<br>
Which of the following statements makes this a correct argument, or shows that it is incorrect? Select all that apply.<br>
A. You can assume $x \neq 0$ because both sides are undefined if $x = 0$.<br>
B. After multiplying both sides by x you need to subtract 1 from both sides.<br>
C. You cannot multiply both sides by x because you do not know what x is.<br>
D. The equation is not linear, so you cannot use the methods normally used for solving linear equations.
</td>
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</table>

# *Topic 7: Rational Functions*

| Transfer Goals | |
|---|---|
| 1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.<br>2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.<br>3) Construct viable arguments and critique the reasoning of others using precise mathematical language. | |
| **Essential Questions:**<br>• Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?<br>• What kinds of asymptotes are possible for a rational function and what kinds of transformations are part of the graphs of rational functions? | **Standards:** F-BF 1, A-SSE 1, A-REI 11, A-APR 1, A-APR 3, A-APR 6, A-APR 7, A-CED 1, A-CED 2, F-BF 3<br><br>**Timeframe:** 13 days<br>**Start Date:** February 13, 2018<br>**Assessment Dates:** March 2, 2018 |

| Time | Lesson/ Activity | Focus Questions for Lessons | Understandings | Knowledge | Skills | Resources |
|---|---|---|---|---|---|---|
| 1 day | **Opening Activity:**<br>Introduction to the Common Core Performance Task p. 497 | | | | | |
| 1 day | **Lesson 8.1: Inverse Variation**<br>SMP: 1,2,3,4,6<br>(pp. 498-505)<br><br>**A-CED 2, A-CED 1** | **Focus Question(s):**<br>• How can you tell whether two sets of data show direct variation or inverse variation?<br><br>**Inquiry Question(s):**<br>Pg. 498 Solve It! | • If a product is constant, a decrease in the value of one factor must accompany an increase in the value of the other factor.<br>• In a direct variation, two positive quantities either increase together or decrease together. In an inverse variation, as one quantity increases the other decreases.<br>• Quantities x and y are inversely proportional only if increasing x by the factor k ($k \neq 0$) means shrinking y by the factor $\frac{1}{k}$. | **Vocabulary:** inverse variation, combined/joint variations<br><br>• Combined Variations in Equation Form<br>• Function models for direct and inverse variations | • Identify direct and inverse variation<br>• Recognize and use inverse variation<br>• Determine an inverse variation<br>• Apply combined/joint and other variations<br>• Write equations to model the direct and inverse variations and graph the function | **Thinking Map:** Tree Map to record characteristic of inverse, combined, inverse, and joint variation.<br><br>**CC Problems:** #3,4,5, 19-25, 42,43,46<br><br>**STEM:** #21, 23, 25 |

| Time | Lesson/ Activity | Focus Questions for Lessons | Understandings | Knowledge | Skills | Additional Resources |
|---|---|---|---|---|---|---|
| 1 day | **Lesson 8.2: The Reciprocal Function Family**<br>SMP: 1,2,3,4,5<br>(pp. 507-514)<br><br>**F-BF 3**, **A-CED 2**, **A-APR 1**<br><br>*Prep for Performance Task (Apply What You Have Learned) p. 514 (Lesson 8.2)* | **Focus Question(s):**<br>• How are functions of the form $y = \frac{a}{x-h} + k$ related to the parent function $y = \frac{1}{x}$?<br><br>**Inquiry Question(s):**<br>Pg. 512 #30 | • Transformations of the parent reciprocal function include stretches, compressions (or shrinks), reflections, and horizontal and vertical translations.<br>• A rational function may have zero or one horizontal or oblique asymptote and zero or more vertical asymptotes.<br>• Quantities x and y are inversely proportional only if increasing x by the factor k (k≠0) means shrinking y by the factor $\frac{1}{k}$. | **Vocabulary:** reciprocal function, branch<br><br>• General form of the reciprocal function family<br>• The Reciprocal Function Family (parent function, stretch, shrink, reflection, translation, combined) | • Identify the x-and y-intercepts and the asymptotes of a graph and state the domain and range of the function.<br>• Write an equation for the translation of a graph that has given asymptotes<br>• Identify the effect of *a* on the graph for $= \frac{a}{x}$.<br>• Sketch the graph of a function. | **Thinking Map:** Double-bubble Map to compare and contrast a function to its reciprocal function.<br><br>**CC Problems:** #5,6,7,29,30,31,37, 38,42,43<br><br>**STEM:** #29 |
| 1 day | **Lesson 8.3: Rational Functions and Their Graphs**<br>SMP: 1,3,4<br>(pp. 515-523)<br><br>**A-APR 3**, **F-BF 1** | **Focus Question(s):**<br>• What causes discontinuities in a graph and how can you find them?<br>• How does the graph of a function behave as it approaches removable and non-removable discontinuities?<br><br>**Inquiry Question(s):**<br>Pg. 522 #41 | • A rational function is a ratio of polynomial functions.<br>• If a function has a polynomial in its denominator, its graph has a gap in each zero of the polynomial. The gap could be a one-point hole in the graph, or it could be the location of a vertical asymptote for the graph.<br>• A rational function may have zero or one horizontal or oblique asymptote and zero or more vertical asymptotes.<br>• A reasonable graph for a rational function can be sketched by finding all intercepts and asymptotes. Sometimes a few extra points should be plotted to get a good sense of the shape of the graph. | **Vocabulary:** rational function, continuous graph, discontinuous graph, point of discontinuity, removable discontinuity, non-removable discontinuity<br><br>• Point of discontinuity<br>• Vertical Asymptotes of Rational Functions<br>• Horizontal Asymptotes of Rational Functions | • Identify the domain, points of discontinuity, and x- and y- intercepts for rational functions.<br>• Describe any vertical or horizontal asymptotes and any holes of a graph of a rational function.<br>• Sketch the graph of a rational function. | **Thinking Map:** Flow Maps to show how to find the vertical and horizontal asymptotes of a rational function.<br><br>**CC Problems:** #12, 35, 39, 40, 41, 46, 47, 48, 49<br><br>**STEM:** #35 |

| Time | Lesson/ Activity | Focus Questions for Lessons | Understandings | Knowledge | Skills | Additional Resources |
|---|---|---|---|---|---|---|
| 1 day | **Lesson 8.4: Rational Expressions**<br>SMP: 1,2,3,4<br>(pp. 527-533)<br><br>**A-SSE 2, A-SSE 1** | **Focus Question(s):**<br>• Why is it important to examine the factors of the original problem to determine variable restrictions?<br><br>**Inquiry Question(s):**<br>Pg. 530 #7 | • Much of what is true about multiplying and dividing fractions can be used to multiply and divide rational expressions.<br>• A rational expression is in simplest form when its numerator and denominator are polynomials that have no common divisors.<br>• A rational function may have zero or one horizontal or oblique asymptote and zero or more vertical asymptotes.<br>• Functions such as $f(x) = \frac{x+a}{x^2-a^2}$ and $g(x) = \frac{1}{x-a}, x \neq \pm a$, are equivalent. | **Vocabulary:** rational expression, simplest form<br><br>• Procedures for simplifying a rational expression<br>• Procedures for multiplying and dividing rational expressions | • Simplify rational expressions and state any restrictions on the variable.<br>• Multiply and divide rational expressions and state any restrictions on the variable(s).<br>• Use rational expressions to solve real world problems. | **Thinking Map:** Tree/Flow Map to show the differences and processes for simplifying, multiplying, and dividing rational expressions.<br><br>**CC Problems:** #5,6,7, 26, 30, 31, 36, 37, 38-41, 45<br><br>**STEM:** #26, 37 |
| 2 days | **Lesson 8.5: Adding and Subtracting Rational Expressions**<br>SMP: 1,3,4<br>(pp. 534-541)<br><br>**A-APR 7**<br><br>*Prep for Performance Task (Apply What You Have Learned) p. 541 (Lesson 8.5)* | **Focus Question(s):**<br>• Why should you find the *least* common denominator when adding or subtracting rational expressions?<br><br>**Inquiry Question(s):**<br>Pg. 540 #31 | • Much of what is true about operating with fractions can be used to operate with rational expressions. Rational expressions can be added or subtracted by first finding a common denominator—preferably the least common multiple (LCM) of the denominators.<br>• The LCM of denominators is the product of their prime factors, each raised to the greatest power that occurs in any of the expressions. | **Vocabulary:** LCM, LCD, complex fraction<br><br>• Procedures for finding the LCM and LCD<br>• Procedures for adding and subtracting rational expressions<br>• Procedures for simplifying complex fractions | • Find the LCD for rational expressions.<br>• Find the sum or difference for rational expressions.<br>• Simplify a complex fraction. | **Thinking Map:** Tree/Flow Map to show the differences and processes for adding, subtracting, and simplifying rational expressions and complex fractions.<br><br>**CC Problems:** #5,6, 30, 37, 38, 39, 40, 45, 46, 47<br><br>**STEM:** #38, 45, 47 |

| Time | Lesson/ Activity | Focus Questions for Lessons | Understandings | Knowledge | Skills | Additional Resources |
|---|---|---|---|---|---|---|
| 2 days | **Lesson 8.6: Solving Rational Equations**<br>SMP: 1,2,3,4,5<br>(pp. 542-548)<br><br>**A-APR 7**, **A-APR 6**, **A-CED 1**, **A-REI 11**<br><br>*Prep for Performance Task (Apply What You Have Learned)*<br>*p. 548 (Lesson 8.6)* | **Focus Question(s):**<br>• Which methods can be used to solve a rational equation?<br><br>**Inquiry Question(s):**<br>Pg. 542 Solve It! | • Solving an equation containing rational expressions begins by multiplying each side by the least common denominator of the rational expression. Doing this, however, can introduce extraneous solutions. | **Vocabulary:** rational equation, extraneous solutions<br><br>• Procedures and/or methods for solving rational equations | • Solve a rational equation<br>• Solve a rational equation for a given variable<br>• Use rational equations to solve problems | **Thinking Map:** Flow Map to sequence solving procedures<br><br>**Common Core Problems:** #5,6,46,47,48,49,58,62, 63,82,83<br><br>**STEM:** #42, 56 |
| 1 day | **Chapter 8 Performance Task**<br>Textbook p. 552 *Pull it together*<br>Have students work collaboratively to reflect on *Completing the Performance Task* and *On your Own* | | | | | |
| 2 day | **Review Topic 7 Concepts & Skills**<br>Use Textbook Resources and/or Teacher Created Items | | | | | |
| 1 day | **Topic 7 Assessment**<br>(Created and provided by PUSD) | | | | | |

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