Texas Higher Education Coordinating Board

Making Opportunity Affordable in Texas:

A Student-Centered Approach

Tuning of Mathematics

Texas Higher Education Coordinating Board

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Tuning Oversight Council for Mathematics, Business, and Information Systems
Mathematics Committee

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Definition of Tuning

"Tuning" is a faculty-led pilot project designed to define what students should know, understand, and be able to demonstrate after completing a degree in a specific field, and to provide an indication of the knowledge, skills, and abilities students should achieve prior to graduation at different levels along the educational pipeline – in other words, a body of knowledge and skills for an academic discipline in terms of outcomes and levels of achievement of its graduates.

Tuning provides an expected level of competency achievement at each step along the process of becoming a professional: expectations at the beginning of pre-professional study, at the beginning of professional study, and at the transition to practice. It involves seeking input from students, recent graduates, and employers to establish criterion-referenced learning outcomes and competencies by degree level and subject area. Through Tuning, students have a clear "picture" of what is expected and can efficiently plan their educational experience to achieve those expectations. The objective is not to standardize programs offered by different institutions, but to better establish the quality and relevance of degrees in various academic disciplines.

An overview of Lumina Foundation for Education's "Tuning USA" Initiative is available at: http://www.tuningusa.org; an overview of Tuning work to date in Texas is available at: http://www.thecb.state.tx.us/tuningtexas.

Definition of Mathematics

Mathematics is a broad-ranging, ever-expanding, vibrant body of knowledge providing tools for a quantitative description of the world around us. Mathematics is used to create and evaluate models of all types of phenomena, provide optimal solutions to questions, and generate algorithms to solve complex problems in such diverse areas as the natural sciences, social and behavioral sciences, computing, and engineering.

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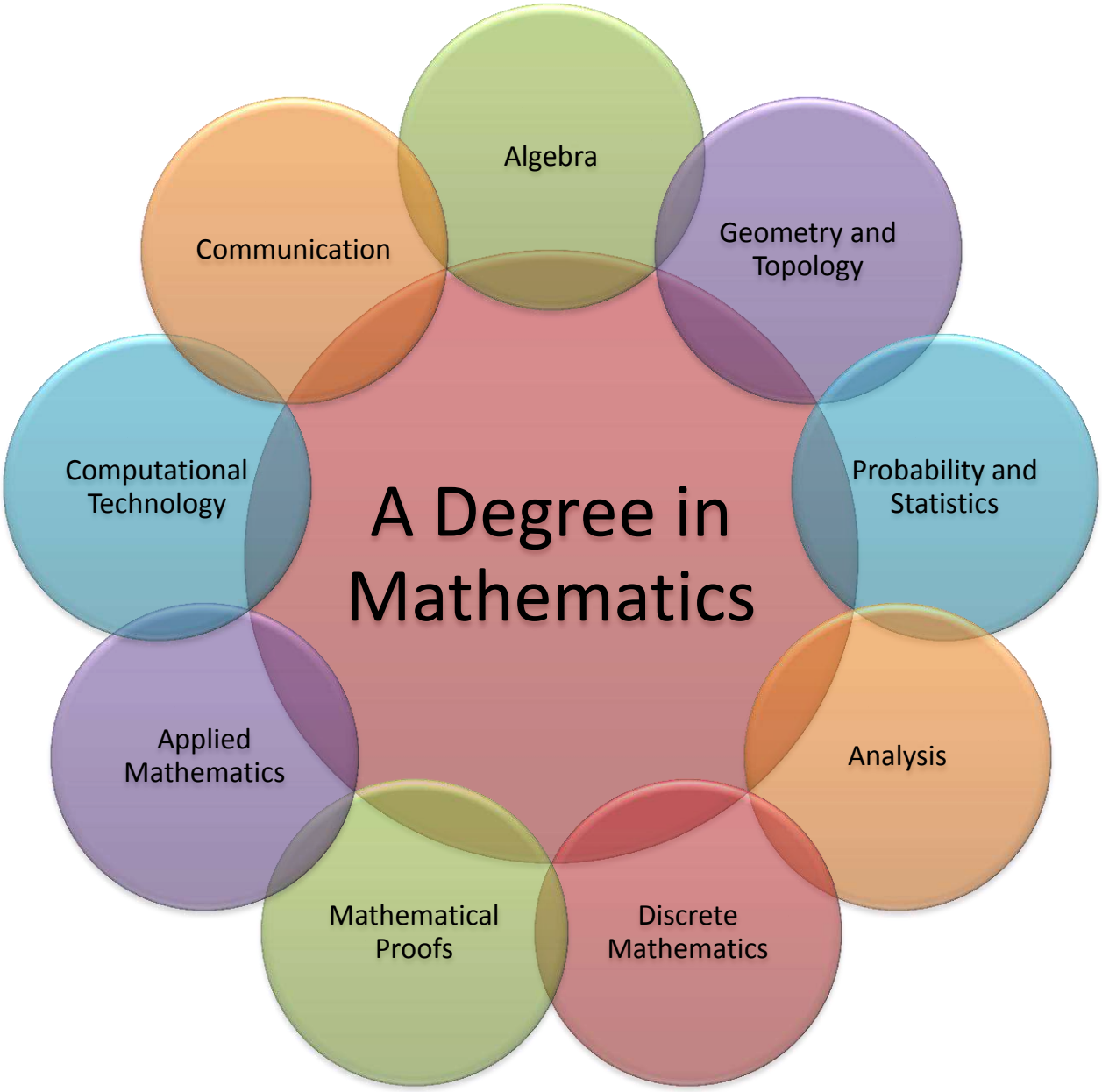
Mathematics Expertise Profile

The expertise profile lists the types of course topics included in a typical baccalaureate degree in mathematics. Note: General undergraduate degree requirements (e.g., the core curriculum) are not considered for the purpose of tuning mathematics and this report.

Any broad topic of interest may have specializations that are important for mathematics.



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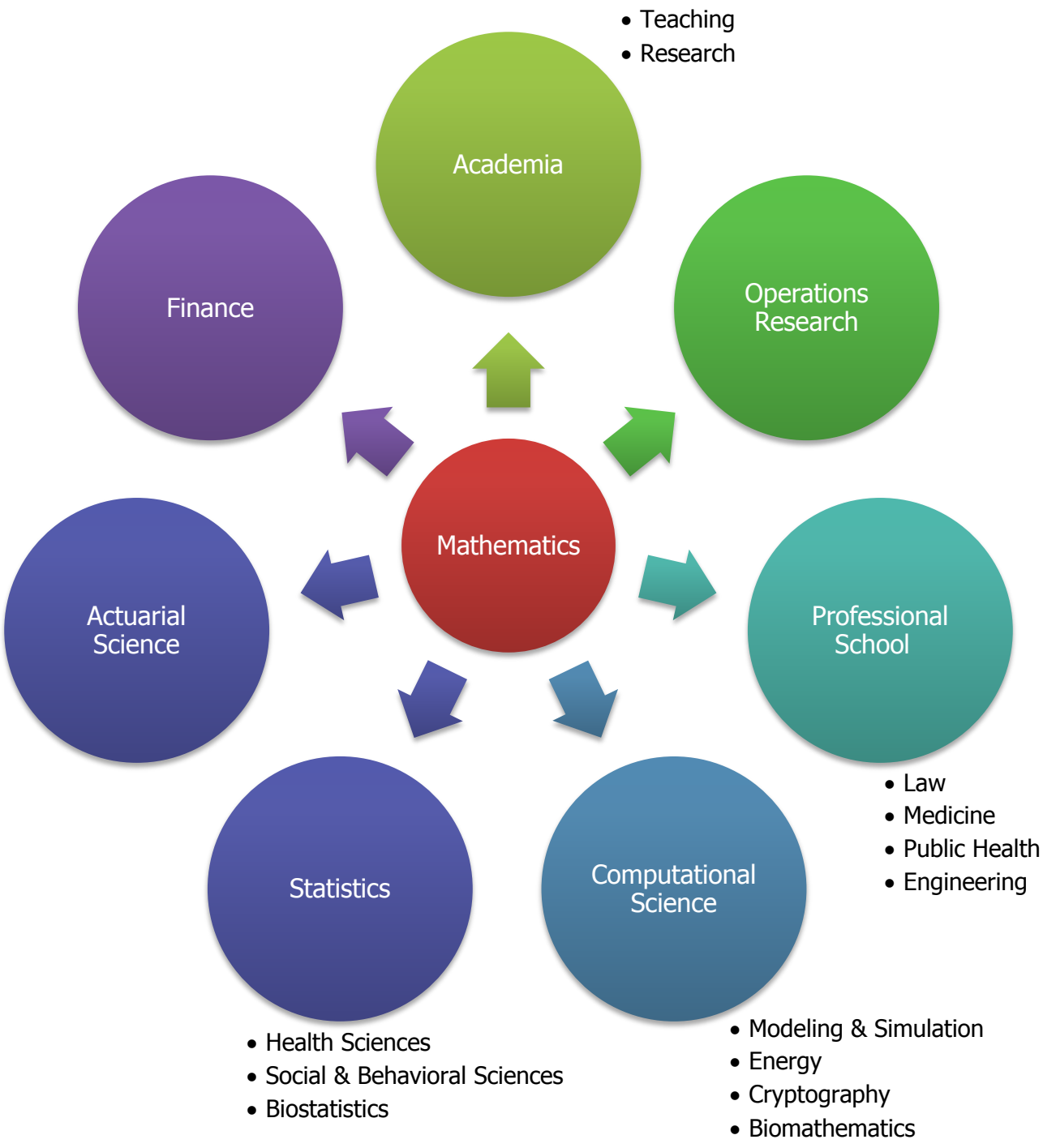
Mathematics Employment Profile

The employment profile lists the employment pathways available for graduates of Mathematics programs.



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Mathematics Key Competencies Profile

The key competencies profile is a schematic diagram that is derived from the competency table. It lists for each learning outcome (columns) the required competency levels according to Bloom's taxonomy (rows) that should be gained at each of four educational levels:

1. secondary education competencies, marked "HS";
2. pre-mathematics competencies, marked "CC";
3. baccalaureate-level competencies, marked "BS"; and,
4. graduate-level competencies, marked "G."

The level of response for each of the Bloom's taxonomy levels is described through active verbs; examples of verbs for each level can be found at:

http://www.teach-nology.com/worksheets/time_savers/bloom/

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Mathematics Key Competency Table and Learning Outcome Descriptions

The Mathematics competency table has six learning outcome titles, one for each learning outcome description:

Competencies:

I. Broad View of Mathematical Science and Mathematical Thinking
II. Computations and Procedures
III. Data Analysis and Organizing Ideas
IV. Appreciation of Mathematics
V. Appropriate Use of Technology Tools
VI. Communication Skills

The competency table has four learning outcome categories (columns from left to right):

1. core competencies needed to enter higher education in Mathematics (HS);
2. Mathematics competencies gained during first two years of study (CC);
3. baccalaureate-level Mathematics competencies (BS); and,
4. graduate-level Mathematics competencies (G).

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Mathematics Key Competencies Diagram

G	post-graduate competencies
BS	baccalaureate level competencies
CC	pre-mathematics competencies
HS	secondary education competencies

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Mathematics Learning Outcome Descriptions

Mathematics Learning Outcome Descriptions for each of the outcome titles of the competency table explain the knowledge, skills, and attitudes that should be achieved by the graduates at each level along the educational continuum.

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Broad View of Mathematical Science and Mathematical Thinking

Much of the power of mathematics lies in the ability of the mathematician to identify patterns, to study and generalize those patterns, and to apply previously studied structures in a way that expands on those patterns. Mathematics students will learn the major results in several different mathematical fields, study the interrelations between these fields, and then apply those results to generate new solution strategies.

Broad views of mathematical sciences require comprehension and the ability to apply contrasting but complementary points of view and perspectives. These include continuous and discrete, algebraic and geometric, deterministic and stochastic, theoretical and applied perspectives and points of view.

Ideas of continuous mathematics laid a solid foundation for many mathematical theories and applications and remain crucially important as the language of science and economics. However, the development of the computational power of modern civilization and other areas of mathematics that are not based on the ideas of continuity began playing a crucial role in application and created many theoretical challenges.

Most mathematically related arguments are, in one way or another, related to algebraic computations and techniques that laid a foundation for studying various areas of mathematics and its applications.

Geometry was the heart of mathematics for centuries and continues to be a vital part of it. Geometric thinking and visualization as well as the effective use of graphics software to enhance understanding are important aids to problem solving and scientific/engineering discovery. Discrete geometry and finite geometry recently received attention from mathematicians and specialists from different areas because of their importance to applications.

Deterministic models are among the glorious achievements of mathematics, but stochastic methods are just as valuable as deterministic. Deterministic models describe processes that can only evolve in one way under the given initial conditions. Stochastic methods are applicable to random processes, with some indeterminacy in outcomes, even if the initial conditions are given. Modern mathematics and its applications rely on fields that use data analysis and effectively draw conclusions from data deterministically and stochastically.

Theoretical ideas of mathematics are the foundation of mathematical sciences and play a crucial role in discovering opportunities to apply mathematical methods to solve quantitative problems. Recent advances in mathematical sciences and technology have implications for the balance between the theoretical and the applied in the undergraduate education.

Mathematicians largely create new mathematics through intuition, but a mathematical truth is not accepted as such until a formal proof of the result is developed. Students will first develop the ability to read a mathematical proof and recognize its logical structure, learn to appreciate the value of careful attention to definitions, and then study various patterns and techniques of sound reasoning. Once these skills are mastered, students will be expected to create proofs, building in complexity from basic proofs of known results all the way to developing new results themselves as part of post-graduate study.

BROAD VIEW OF MATHEMATICAL SCIENCE AND MATHEMATICAL THINKING			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate- Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Knowledge & Comprehension	**Application**	**Analysis**	**Synthesis & Evaluation**
Demonstrate procedural & computational understanding of mathematics. Solve problems applying algebraic, geometrical, numeric, statistical, measurement, and probabilistic reasoning. Have experience in analyzing and solving real-world problems.	Expand upon entry-level procedural and computational knowledge to comprehend and solve calculus-based problems. Solve real-world problems by developing and analyzing simple mathematical and statistical models.	Master proof techniques in various mathematical theories. Ask open-ended questions in at least one mathematical theory, and answer them through mathematical reasoning, computation, and rigorous proof. Demonstrate proficiency in applying mathematical theories to solve applied problems in various areas.	Formulate conjectures, produce new mathematical knowledge, and conduct supervised original research in one of the areas of mathematics or its applications.

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Computations and Procedures

Problem solving using mathematics frequently involves abstracting the essentials of a situation and expressing relationships between those essential elements in terms of mathematical symbols. Mathematics students must develop the ability and perseverance necessary to carry out computations in an organized and efficient manner, and follow the necessary procedures that lead to a solution of the problem. As problems become more complicated, a sophisticated system of such procedures is required to find solutions. Students will develop the ability to establish connections between the problems and required methods to solve them; successfully apply procedures and techniques, shifting them if their chosen approach fails; or modifying a known technique to fit a new situation. Successful application of computational techniques and procedures very often depends on sophisticated software and hardware, and having the ability to use it whenever appropriate.

LEARNING OUTCOME: Computations and Procedures			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate-Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Knowledge & Comprehension	**Application**	**Analysis & Synthesis**	**Evaluation**
Demonstrate computational fluency with algebra, geometry, and trigonometry and to be able to follow basic, often-repeated procedures.	Rearrange algebraic, geometric, and trigonometric techniques and concepts with a specified purpose (e.g. Calculus, Linear Algebra, etc.) in mind and make basic modifications to procedures.	Fully explain why the steps in a procedure or computation are justified and create their own procedures, as necessary.	Develop and adapt appropriate algorithms to solve problems, test hypotheses, and formulate predictions.

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Data Analysis and Organizing Ideas

As a reader or media observer, individuals must evaluate articles or reports carefully by critically thinking about the conclusions presented pertaining to a set of data because of the notorious ease in manipulating data during the collection and analysis process. The mathematics graduate should be able to determine the validity of the data source, determine misleading uses of data, recognize and interpret the basic principles used in the study design to describe and present (or not present) the data, comprehend the generalizations and limitations of the study, and explain the reliability of the statistical results.

Making informed decisions often requires collecting and analyzing data. The process begins with a plan for asking representative sample questions that clarify issues to be resolved. Some data collection is done to support or reject hypotheses. The reliability of results depends on the quality of the data, properties of the sample, and appropriate statistical analysis methods.

Data quality can often be gauged by visually comparing and analyzing distributions for similarities, differences, and spreads within and between data sets using charts or graphs. In any collection process errors are likely, and many of these are clearly outliers that deviate dramatically from evident trends. Such data are eliminated, but must be explained and noted to maintain the integrity of analyses.

Statistical analyses generally include descriptions of the sample and the data, and inferences drawn based on the data collected. The skills involved in visual and oral explication are crucial to successful analyses. Knowing the limitations of statistical methods is very important, as is frequently found in confusing correlation and causation. Organization, focus, clarity, and critical thinking skills play important roles in statistical analyses. The mathematics graduate should have a good knowledge of basic statistical techniques, an in-depth understanding of the power and limitations of statistical analyses, and the ability to present statistical data in a clear and concise manner, both graphically and in writing.

LEARNING OUTCOME: DATA ANALYSIS & ORGANIZING IDEAS			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate-Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Knowledge	**Comprehension & Application**	**Analysis & Synthesis**	**Evaluation**
Identify basic arithmetic and algebraic manipulation skills to interpret fairly simple statistical problems using textual write-ups, lines, charts, graphs, and tables.	Apply mathematical logic and problem-solving skills to study data. Define and explain appropriate techniques to process data and derive logical conclusions.	Identify practical applications of statistics, including datasets with significant variation. Devise a plan to organize and analyze data by applying and interpreting advanced mathematics and statistical procedures.	Construct and assess applications of mathematical and statistical concepts. Evaluate whether applied procedures are appropriate and whether the results are accurate and clearly stated.

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Appreciation of Mathematics

Mathematics is a constantly expanding intellectual endeavor rich in history, beauty, and consequences. Preferred by science for its elegance of method, precision, unshakable proofs, and generality, it has, since time immemorial, captivated humanity's interest and imagination. In the words of Bertrand Russell, "Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show." [1] First-hand appreciation of the beauty of mathematics lies, in Keith Devlin's opinion, "beyond the techniques and the symbolic manipulation" and one has to "see the subject for what it is."[2] Greater than the sum total of its uses, mathematics is the product of both individual and collective efforts throughout human history. The nurturing of an appreciation for its history, inherent beauty, and dynamism should pervade the curriculum and guide instruction.

Students of mathematics will have glimpses of beauty of the subject at hand and be expected to demonstrate a clear perception of the role that mathematics has played, and is now playing, in society.

For those who choose a career in mathematics instruction, a profound knowledge of its history and beauty is essential. Every teacher of mathematics should know that students will not learn by the mere mastering of techniques and symbolic manipulations, but rather be given the chance to enjoy mathematics by seeing what the subject really is.

The beauty and contemporary challenges in Mathematics are often unknown or unappreciated. Research mathematicians have the responsibility of discovering new mathematical beauty and mathematical truth, and of communicating their peer-reviewed advances.

[1] Russell, Bertrand. *A History of Western Philosophy*. London: George Allen & Unwin Ltd., 1945.
[2] Devlin, Keith. "Letter to a Calculus Student." *Devlin's Angle*. Mathematical Association of America. June 2006. (http://www.maa.org/devlin/devlin_06_06.html)

LEARNING OUTCOME: APPRECIATION OF MATHEMATICS			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate-Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Knowledge	**Comprehension & Application**	**Analysis**	**Synthesis & Evlauation**
Demonstrate awareness that the current body of mathematics knowledge is the sum total of both individual and collective work throughout history.	Understand the role played by mathematics throughout history. Describe how individual and collective effort have produced mathematical knowledge, and how this knowledge has impacted humanity's understanding of nature.	Identify, examine, and classify beauty or elegance in mathematical results throughout history. Explain how individual and collective effort have produced mathematical knowledge and explain how this knowledge distinguishes itself from other bodies of knowledge.	Discover and communicate instances of mathematical beauty and mathematical truth and evaluate past and new advances.

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Appropriate Use of Technology Tools

Mathematics deals with the science of structure, order, and relation that has evolved from basic counting, measuring quantities, and describing the shapes of objects. It uses logical reasoning and quantitative calculation, and is considered the underlying language of science. Several tools have been developed to assist in calculation, visual representation, and analysis of mathematical concepts.

Mathematics students need to maintain a high level of competence in technology and computer languages to be competitive in today's workforce. Scientific and engineering fields use technology in research and modeling. Computer technology helps mathematicians sort through multiple outcomes. Large-scale data gathering and analysis is more manageable with statistical software. Software is a common thread between mathematicians, scientists, and engineers. It allows the communication of ideas to be better dispersed and understood.

College mathematicians need to work and learn in a connected digital society. Digital technology will help inspire digital age students to improve higher-order thinking skills such as problem solving, critical thinking, and creativity.

LEARNING OUTCOME: APPROPRIATE USE OF TECHNOLOGY TOOLS			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate-Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Knowledge	**Comprehension & Application**	**Analysis**	**Synthesis & Evaluation**
Solve problems and visualize information in mathematics using a graphing calculator and spreadsheets.	Apply symbolic software, graphing calculators, and spreadsheets to present, visualize, model, and interpret results.	Analyze problems in mathematics using dynamic geometry software, programming languages, and symbolic mathematical and analytical software.	Construct, evaluate, and revise a complex problem to determine the relevant mathematical principles and available software and hardware to solve the problem.

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Communication Skills

Gaining knowledge in mathematics and acquiring the skills to apply it are only a beginning. It is essential to be able to communicate that knowledge and the results of those skills to others, no matter what their level of mathematics competency. Additionally, the ability to explain mathematical concepts to others is a good measure of the depth of one's own understanding of those concepts.

In the workplace, it is common for people to work in teams on complex projects. It is, therefore, necessary to function as a part of a working group. This includes communicating to the group what is being done and understanding what the others are doing in the group. Whether working alone or as part of a group, one must be able to communicate the results of the effort to those who will use them.

For those who choose a career in mathematics, success will depend on the ability to help others learn mathematics, not the ability to simply present mathematical ideas. Thus, skills in expressing mathematical ideas in a way that the audience can understand the material are paramount. A successful communicator will be able to facilitate a discussion of mathematical ideas.

Research mathematicians also work together and frequently present work in progress to colleagues and students. Completed work must be placed before the professional community in written publications and/or oral presentations at professional meetings. Clarity in the presentation is essential to having the work accepted by one's peers.

Students of mathematics will be given the opportunity to work independently and in groups on problems including both theory and application. Results of this work will include a written report and a presentation in appropriate settings using appropriate technology.

LEARNING OUTCOME: COMMUNICATION SKILLS			
Core Competencies Needed to enter Higher Education in Mathematics	**Pre-Mathematics Competencies gained during the first two years of study**	**Baccalaureate-Level Mathematics Competencies**	**Post-Graduate/ Work Experience Mathematics Competencies**
Application	**Analysis**	**Synthesis**	**Evaluation**
Present the results of individual or group work on a specific problem assigned in the student's current course in mathematics in written, graphic, or oral form.	Examine and present the results of individual or group work on a project assigned within the context of the student's course in mathematics in written, graphic, or oral form.	Summarize and present the results of individual or group work on a project defined by the individual or group outside the context of a course setting in an appropriate written, graphic, or oral form.	Evaluate presentations given by another individual or group. Present original work and new mathematical results in appropriate settings such as a class, seminar, professional meeting or publication.

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Community College Program of Study for Transfer to a Baccalaureate Mathematics Program

FRESHMAN YEAR - Recommended Scheduling Sequence*

First Semester (Fall)	SCH	Second Semester (Spring)	SCH
MATH 2413 Calculus I*	4	MATH 2414 Calculus II	4
Communication Core Curriculum Course	3	Computer Programming Course	3
Core Curriculum Course	3	Communication Core Curriculum Course	3
Core Curriculum Course	3	Core Curriculum Course	3
Core Curriculum Course	3	Core Curriculum Course	3
TOTAL	16	TOTAL	16

SOPHOMORE YEAR - Recommended Scheduling Sequence*

First Semester (Fall)	SCH	Second Semester (Spring)	SCH
MATH 2415 Calculus III**	4	MATH 2320/2420 Differential Equations**	3-4
MATH 2318 Linear Algebra**	3	Core Curriculum Course	3
Core Curriculum Course	3	Core Curriculum Course	3
Core Curriculum Course	3	Core Curriculum Course	3
Core Curriculum Course	3		
TOTAL	16	TOTAL	13

* Prerequisite courses may be required before beginning Calculus sequence.

** Students may need to complete additional advanced level mathematics courses to meet degree requirements.

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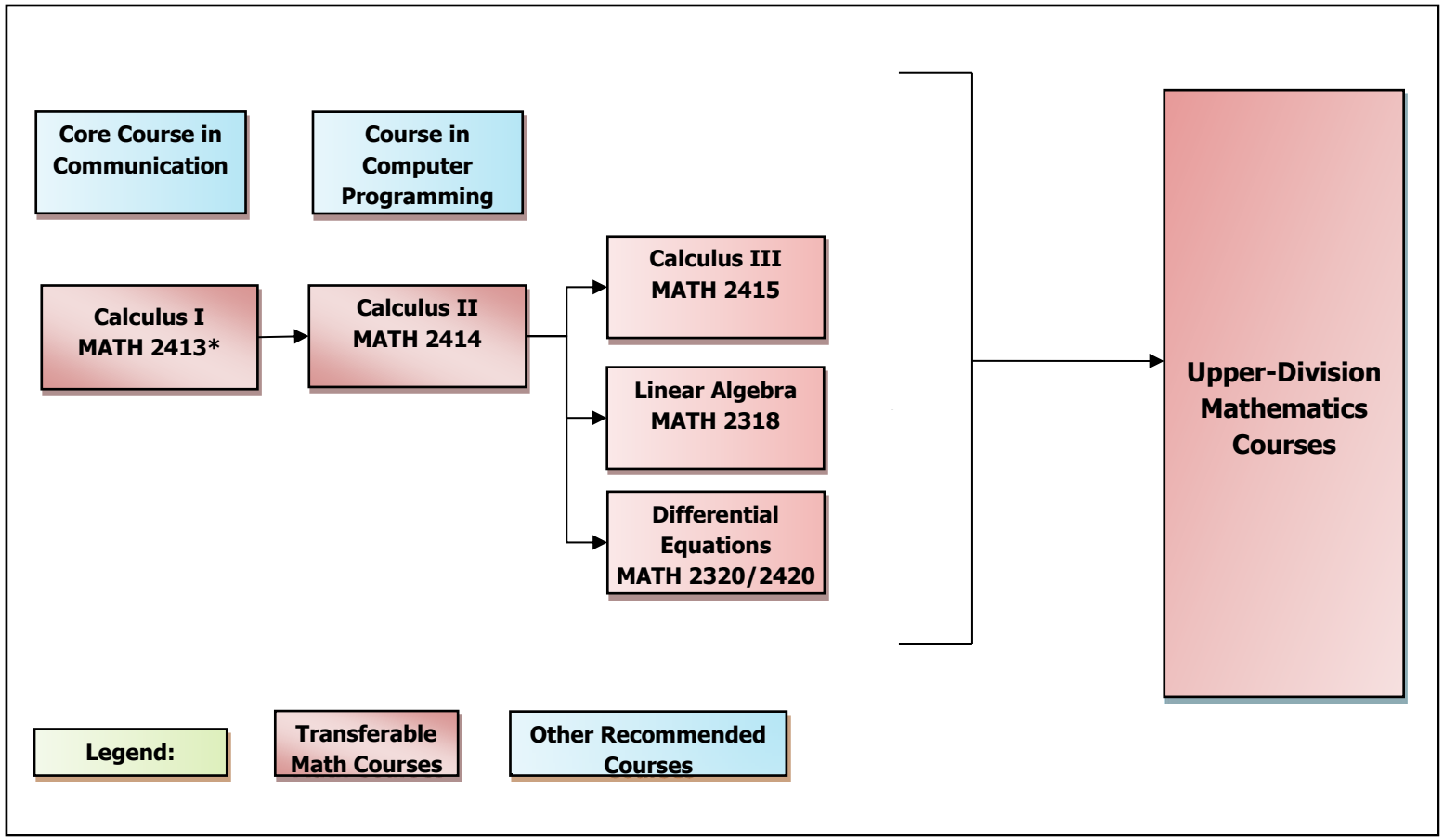
Mathematics Prerequisite Flowchart



**A student may need prerequisite courses depending upon his/her level of readiness when beginning the program of study.*

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