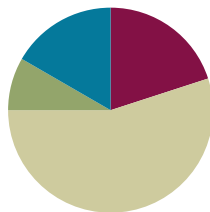


Lesson 20

Objective: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



A NOTE ON STANDARDS ALIGNMENT:

In Lessons 20 and 21, students add fractions with related denominators where one denominator is a factor of the other. In Grade 5, students find sums and differences of fractions with unrelated denominators (**5.NF.1**). Because students are able to generate equivalent fractions (**4.NF.1**) from their work in Topics A, B, and C and are very familiar with the idea that units must be the same to be added, this work makes sense and prepares them well for work with decimals in Module 6 where tenths are converted to hundredths and added to hundredths (**4.NBT.5**).

Fluency Practice (12 minutes)

- Count by Equivalent Fractions **4.NF.1** (6 minutes)
- Add Fractions **4.NF.3** (3 minutes)
- Subtract Fractions **4.NF.3** (3 minutes)

Count by Equivalent Fractions (6 minutes)

Note: This activity builds fluency with equivalent fractions. The progression builds in complexity. Work students up to the highest level of complexity in which they can confidently participate.

T: Count by ones to 10 starting at 0.

S: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

T: Count by 1 fifths to 10 fifths starting at 0 fifths. (Write as students count.)

S: $\frac{0}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, \frac{10}{5}$.

$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$	$\frac{10}{5}$
0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$	2
0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	$1\frac{1}{5}$	$1\frac{2}{5}$	$1\frac{3}{5}$	$1\frac{4}{5}$	2

T: 1 one is the same as how many fifths?

S: 5 fifths.

T: (Beneath $\frac{5}{5}$, write 1.) 2 ones is the same as how many fifths?

S: 10 fifths.

T: (Beneath $\frac{10}{5}$, write 2.) Count by fifths again from 0 to 2. This time, when you come to the whole number, say the whole number. (Write as students count.)

S: $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2.$

T: (Point to $\frac{6}{5}$.) Say 6 fifths as a mixed number.

S: $1\frac{1}{5}.$

T: Count by fifths again. This time, convert to whole numbers and mixed numbers. (Write as students count.)

S: $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1\frac{1}{5}, 1\frac{2}{5}, 1\frac{3}{5}, 1\frac{4}{5}, 2.$

T: 2 is the same as how many fifths?

S: $\frac{10}{5}.$

T: Let's count backward starting at $\frac{10}{5}$, alternating between fractions and mixed numbers. Try not to look at the board.

S: $\frac{10}{5}, 1\frac{4}{5}, \frac{8}{5}, 1\frac{2}{5}, \frac{6}{5}, 1, \frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}, 0.$

Add Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 18.

T: (Write $\frac{2}{5} + \frac{1}{5} + \frac{1}{5} = \dots$.) On your personal white board, write the complete number sentence.

S: (Write $\frac{2}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}.$)

T: (Write $\frac{5}{8} + \frac{2}{8} + \frac{1}{8} = \dots$.) Write the complete number sentence.

S: (Write $\frac{5}{8} + \frac{2}{8} + \frac{1}{8} = \frac{8}{8}.$)

T: (Write $\frac{5}{8} + \frac{2}{8} + \frac{1}{8} = \frac{8}{8}.$) Rename 8 eighths as a whole number.

S: $\frac{8}{8} = 1.$

T: (Write $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \dots$.) Write the complete number sentence.

S: (Write $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}.$)

T: How many fifths are equal to 1?

S: 5 fifths.

T: Write $\frac{6}{5}$ as a mixed number.

S: (Write $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$.)

Continue the process with $\frac{3}{4} + \frac{2}{4} + \frac{2}{4}$.

Subtract Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 17.

T: (Write $1 - \frac{1}{5} = \frac{4}{5}$.) How many fifths are in 1?

S: 5 fifths.

T: Write the subtraction sentence. Beneath it, rewrite the subtraction sentence, renaming 1 as fifths.

S: (Write $1 - \frac{1}{5} = \frac{4}{5}$. Beneath it, write $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$.)

T: Say the subtraction sentence.

S: $1 - \frac{1}{5} = \frac{4}{5}$.

Continue with the following possible sequence: $1 - \frac{3}{5}$ and $1 - \frac{3}{10}$.

T: (Write $1\frac{1}{5} - \frac{4}{5} = \frac{1}{5}$.) Write the complete number sentence.

S: (Write $1\frac{1}{5} - \frac{4}{5} = \frac{1}{5}$.)

T: Can we take $\frac{4}{5}$ from $\frac{1}{5}$?

S: No.

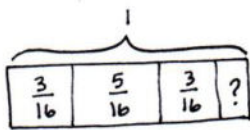
T: (Break apart $1\frac{1}{5}$, writing $\frac{5}{5}$ as one of the parts.) Take $\frac{4}{5}$ from $\frac{5}{5}$, and solve using an addition sentence.

S: (Break apart $1\frac{1}{5}$ into $\frac{1}{5}$ and $\frac{5}{5}$. Write $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$.)

Continue with the following possible sequence: $1\frac{3}{8} - \frac{7}{8}$.

Application Problem (5 minutes)

Krista drank $\frac{3}{16}$ of the water in her water bottle in the morning, $\frac{5}{16}$ in the afternoon, and $\frac{3}{16}$ in the evening. What fraction of water was left at the end of the day?



$$\frac{3}{16} + \frac{5}{16} + \frac{3}{16} = \frac{11}{16}$$

$$1 - \frac{11}{16} = \frac{16}{16} - \frac{11}{16} = \frac{5}{16}$$

$\frac{5}{16}$ of the water in the bottle was left.

Note: This Application Problem builds on Lesson 18 where students added and subtracted two or more addends, as well as Lesson 19 where students solved word problems involving fractions. This problem invites counting on to the whole number as a solution strategy, too.

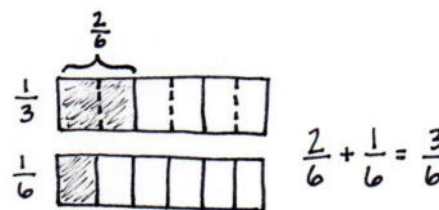
Concept Development (33 minutes)

Materials: (S) Personal white board

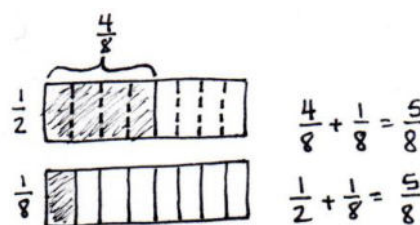
Problem 1: Add unit fractions with related denominators using tape diagrams.

- T: 1 banana + 1 orange = ____?
- S: 2 banana-oranges! No, that’s not right! We can’t add them, because the units are not the same.
- T: What do bananas and oranges have in common?
- S: They are both fruits.
- T: So, what is 1 banana + 1 orange?
- S: 2 pieces of fruit.
- T: You had to rename, to find a way to name the banana and orange as the same unit.
- T: $\frac{1}{3} + \frac{1}{6} = \underline{\hspace{1cm}}$?
- S: The units are different. → The units need to be the same. If the units are different, we cannot add the fractions together.
- T: Let’s decompose to make like units. Discuss a strategy with your partner.

S: I just know that a third is the same as 2 sixths. → We can draw a tape diagram to represent $\frac{1}{3}$ and another one to represent $\frac{1}{6}$. Then, we can decompose each third into two equal parts. $\frac{1}{3} = \frac{2}{6}$. → I can multiply in my head to rename $\frac{1}{3}$ as $\frac{2}{6}$. → I can use an area model or number line, too.



- T: Add $\frac{2}{6} + \frac{1}{6}$. How many sixths are there altogether?
- S: $\frac{3}{6}$. → $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$. → And $\frac{3}{6}$ is also $\frac{1}{2}$.
- T: (Display $\frac{1}{2} + \frac{1}{8}$.) Draw tape diagrams to represent $\frac{1}{2}$ and $\frac{1}{8}$. Which fraction are we going to decompose?
- S: We can decompose the halves into eighths. → You can’t decompose eighths into halves, because halves are bigger than eighths. → We don’t have enough eighths to compose one half, so we have to convert halves to eighths.



- T: How many eighths are in $\frac{1}{2}$?
- S: 4 eighths.
- T: Add.
- S: $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$.

Problem 2: Add fractions with related denominators using tape diagrams.

T: (Display $\frac{2}{3} + \frac{3}{12}$.) Draw tape diagrams to show $\frac{2}{3}$ and $\frac{3}{12}$. Is one of the denominators a factor of the other?

S: Yes!

T: Which unit is larger—thirds or twelfths?

S: Thirds.

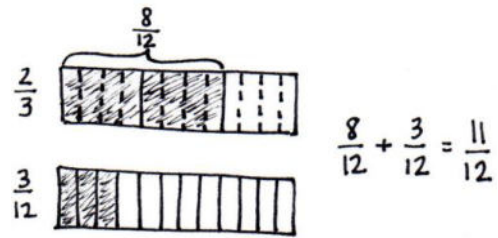
T: So, which unit do we have to decompose?

MP.4 S: Thirds.

T: Go ahead and do that.

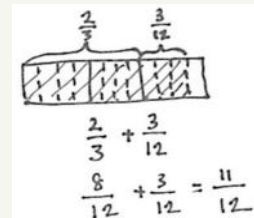
S: Thirds into twelfths. I can draw dotted vertical lines to show each third decomposed into 4 equal parts since there are 4 times as many twelfths in 1 as there are thirds. → There are $\frac{8}{12}$ shaded.

S: $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.



A NOTE ON MULTIPLE MEANS OF REPRESENTATION:

Students can also model both fractions on the same tape diagram. Have them model the larger units first and then partition with dotted lines to show the decompositions.



Problem 3: Add fractions with related denominators using a number line.

T: Write $\frac{1}{6} + \frac{3}{12}$. Let's estimate the sum as we draw a number line to model the addition. I'll mark zero. Do I need my number line to go past 1?

S: No. You are adding two small fractions, so it shouldn't go past 1.

T: Yes. Both fractions are less than 1 half. When we add them, the sum will be less than 1.

T: Draw a number line with endpoints 0 and 1. Partition the number line into sixths. Next, partition the number line further into twelfths. Each sixth will be decomposed into how many parts?

S: 2. → There are twice as many twelfths as there are sixths.

T: Use dashed lines to partition each sixth into twelfths.

T: Show the addition of $\frac{1}{6}$ and $\frac{3}{12}$. Start at 0, and hop to $\frac{1}{6}$. Draw another arrow to show the addition of $\frac{3}{12}$. What is the sum?

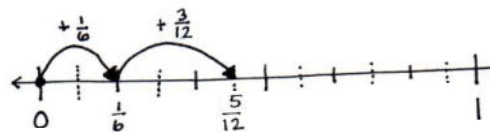
S: $\frac{5}{12}$.

T: Say the addition sentence with like denominators.

S: $\frac{2}{6} + \frac{3}{12} = \frac{5}{12}$.

T: Write $\frac{3}{4} + \frac{5}{8}$. Estimate the sum. Will it be greater than or less than 1?

S: Greater than 1.



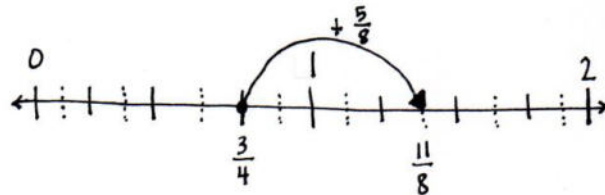
$$\frac{1}{6} < \frac{1}{2} \text{ because } \frac{1}{6} < \frac{3}{6}$$

$$\frac{3}{12} < \frac{1}{2} \text{ because } \frac{3}{12} < \frac{6}{12}$$

$$\frac{1}{6} + \frac{3}{12} = \frac{5}{12}$$

$$\frac{2}{6} + \frac{3}{12} = \frac{5}{12}$$

- T: So, our number line has to go past 1. Does it need to go past 2?
- S: No. Each fraction is less than 1.
- T: Draw a number line. Partition the number line using the larger unit first. Which is the larger unit?
- S: Fourths.
- T: What's the next step?
- S: Make the eighths by putting dashed lines to show each fourth decomposed into 2 eighths. → Just split each fourth into 2 parts.
- T: Draw arrows to show the addition. Explain to your partner what you did.
- S: I started at 0 and moved to $\frac{3}{4}$. That's equal to $\frac{6}{8}$. Then, I drew an arrow to show the addition of $\frac{5}{8}$ more at $\frac{11}{8}$. → I just started at $\frac{3}{4}$ and added $\frac{5}{8}$.
- T: Say the number sentence with like denominators.
- S: $\frac{6}{8} + \frac{5}{8} = \frac{11}{8}$.



Problem 4: Add fractions with related denominators without using a model.

- T: Today, we learned to add fractions by finding common denominators. We found equivalent fractions using models. Add $\frac{2}{5} + \frac{3}{10}$. Which unit is easiest to decompose?
- S: Fifths can be decomposed into tenths.
- T: How can we do that without a model? Talk to your partner.
- S: We can multiply both the numerator and denominator of $\frac{2}{5}$.
 → $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$.
- T: Now, add. Write a complete number sentence.
- S: (Write $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$ or $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$.)

$$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

Repeat with $\frac{3}{12} + \frac{4}{3}$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1(a–f), how was drawing tape diagrams helpful?
- In Problem 1(c), did you use sixths as the common denominator? Explain how thirds could be used as the common denominator.
- For Problem 2(a–f), how was drawing a number line helpful?
- For Problem 2(a–f), what strategies did you use to estimate if the sum would be between 0 and 1 or 1 and 2?
- Why is it important to have common denominators when adding fractions? Relate common denominators to adding with mixed units of measurement from Module 2. For example, add 3 meters to 247 centimeters.
- Explain to your partner how to determine the sum of two fractions without drawing a model. What strategies did you use?
- How did the Application Problem connect to today’s lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students’ understanding of the concepts that were presented in today’s lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 20 Problem Set 4•5

Name: Jack Date: _____

1. Use a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then, write the complete number sentence. Part (a) is partially completed.

a. $\frac{2}{8} + \frac{1}{8}$
 $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$

b. $\frac{1}{4} + \frac{1}{12}$
 $\frac{3}{12} + \frac{1}{12} = \frac{4}{12}$

c. $\frac{2}{6} + \frac{2}{6}$
 $\frac{2}{6} + \frac{2}{6} = \frac{4}{6}$

d. $\frac{1}{3} + \frac{2}{3}$
 $\frac{4}{3} + \frac{3}{3} = \frac{7}{3}$

e. $\frac{3}{10} + \frac{6}{10}$
 $\frac{3}{10} + \frac{6}{10} = \frac{9}{10}$

f. $\frac{6}{9} + \frac{2}{9}$
 $\frac{6}{9} + \frac{2}{9} = \frac{8}{9}$

COMMON CORE Lesson 20: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. 1/5/14 engage^{ny} 5.D.62

Lesson 20 Problem Set 4•5

2. Estimate to determine if the sum is between 0 and 1 or 1 and 2. Draw a number line to model the addition. Then write a complete number sentence. Part (a) has been completed for you.

a. $\frac{1}{2} + \frac{1}{4}$ $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
 $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

b. $\frac{1}{2} + \frac{1}{10}$ $\frac{5}{10} + \frac{1}{10} = \frac{6}{10}$
 $\frac{1}{2} + \frac{1}{10} = \frac{6}{10}$

c. $\frac{6}{10} + \frac{5}{10}$ $\frac{6}{10} + \frac{5}{10} = \frac{11}{10}$

d. $\frac{1}{2} + \frac{1}{3}$ $\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$
 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

e. $\frac{1}{2} + \frac{1}{4}$ $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
 $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

f. $\frac{1}{10} + \frac{5}{10}$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10} = \frac{3}{5}$

3. Solve the following addition problem without drawing a model. Show your work.

$\frac{2}{3} + \frac{4}{6}$
 $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ $\frac{4}{6} + \frac{4}{6} = \frac{8}{6}$

COMMON CORE Lesson 20: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. 1/5/14 engage^{ny} 5.D.63

Name _____

Date _____

1. Use a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then, write the complete number sentence. Part (a) is partially completed.

a. $\frac{1}{4} + \frac{1}{8}$

b. $\frac{1}{4} + \frac{1}{12}$



$$\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$$

c. $\frac{2}{6} + \frac{1}{3}$

d. $\frac{1}{2} + \frac{3}{8}$

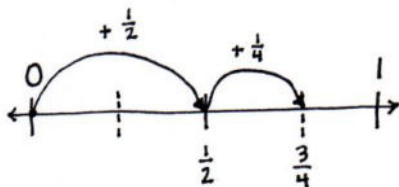
e. $\frac{3}{10} + \frac{3}{5}$

f. $\frac{2}{3} + \frac{2}{9}$

2. Estimate to determine if the sum is between 0 and 1 or 1 and 2. Draw a number line to model the addition. Then, write a complete number sentence. Part (a) has been completed for you.

a. $\frac{1}{2} + \frac{1}{4}$ $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

b. $\frac{1}{2} + \frac{4}{10}$



c. $\frac{6}{10} + \frac{1}{2}$

d. $\frac{2}{3} + \frac{3}{6}$

e. $\frac{3}{4} + \frac{6}{8}$

f. $\frac{4}{10} + \frac{6}{5}$

3. Solve the following addition problem without drawing a model. Show your work.

$$\frac{2}{3} + \frac{4}{6}$$

Name _____

Date _____

1. Draw a number line to model the addition. Solve, and then write a complete number sentence.

$$\frac{5}{8} + \frac{2}{4}$$

2. Solve without drawing a model.

$$\frac{3}{4} + \frac{1}{2}$$

Name _____

Date _____

1. Use a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then, write the complete number sentence.

a. $\frac{1}{3} + \frac{1}{6}$

b. $\frac{1}{2} + \frac{1}{4}$

c. $\frac{3}{4} + \frac{1}{8}$

d. $\frac{1}{4} + \frac{5}{12}$

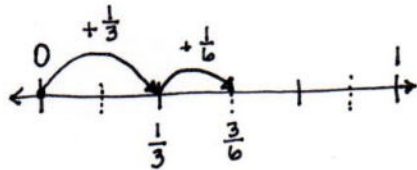
e. $\frac{3}{8} + \frac{1}{2}$

f. $\frac{3}{5} + \frac{3}{10}$

2. Estimate to determine if the sum is between 0 and 1 or 1 and 2. Draw a number line to model the addition. Then, write a complete number sentence. The first one has been completed for you.

a. $\frac{1}{3} + \frac{1}{6}$ $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

b. $\frac{3}{5} + \frac{7}{10}$



c. $\frac{5}{12} + \frac{1}{4}$

d. $\frac{3}{4} + \frac{5}{8}$

e. $\frac{7}{8} + \frac{3}{4}$

f. $\frac{1}{6} + \frac{5}{3}$

3. Solve the following addition problem without drawing a model. Show your work.

$$\frac{5}{6} + \frac{1}{3}$$