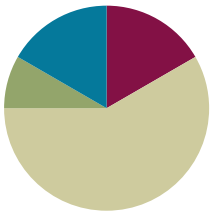
Lesson 13

Objective: Analyze and classify triangles based on side length, angle measure, or both.

Suggested Lesson Structure



Fluency Practice	(10 minutes)
Application Problem	(5 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (10 minutes)

- Divide Three Different Ways **4.NBT.6** (5 minutes)
- Physiometry **4.G.3** (3 minutes)
- Lines of Symmetry **4.G.3** (2 minutes)

Divide Three Different Ways (5 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews concepts covered in Module 3. Alternately, have students choose to solve the division problem using just one of the three methods.

T: (Write 532 ÷ 4.) Solve this problem by drawing place value disks.
S: (Solve by drawing place value disks.)
T: Solve 532 ÷ 4 using the area model.
S: (Solve using the area model.)
T: Solve 532 ÷ 4 using the standard algorithm.
S: (Solve using the standard algorithm.)

Continue with this possible sequence: 854 ÷ 3.

Physiometry (3 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms learned in Lesson 12.

T: Stand up.
T: Am I trying to make my body position look symmetrical?

T: (Raise left arm so fingers point directly to the wall. Leave the other arm hanging down.) Is my position symmetrical now?

S: No.

Continue with other symmetrical and non-symmetrical positions.

T: With your arms, model a line that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

T: Model a right angle. Are you modeling a symmetrical position?

S: No.

T: Model a line segment that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

Lines of Symmetry (2 minutes)

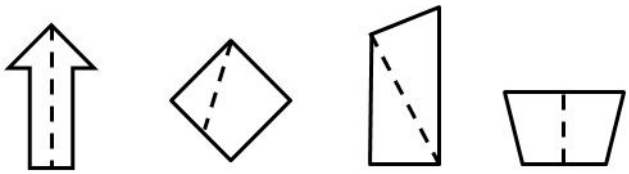
Note: This fluency exercise reviews Lesson 12.

T: (Project arrow with a line of symmetry. Point to the line of symmetry.) Is this a line of symmetry?

S: Yes.

T: (Project the diamond. Point to the non-symmetrical line.) Is this a line of symmetry?

S: No.



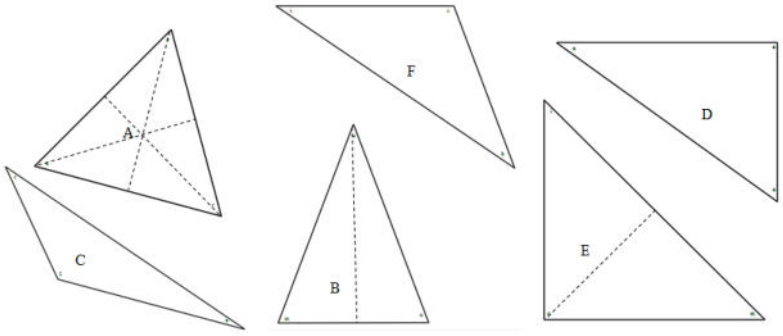
Continue process for the remaining graphics.

Application Problem (5 minutes)

Materials: (T/S) Triangles (Template)

Fold Triangles A, B, and C to show their lines of symmetry. Use a straightedge to trace each fold. Discuss with your partner the relationships of symmetric shapes to angles and side lengths.

Note: This Application Problem connects lines of symmetry in Lesson 12 to discovering the attributes of triangles in today's lesson. Prepare the triangles ahead of time by cutting them out from the triangles Template. Each student or partner group should have his or her own copy.



Concept Development (35 minutes)

Materials: (T) Triangles (Template), Practice Sheet, graph paper, ruler (S) Triangles (Template) one set per group, Practice Sheet, ruler, protractor, graph paper

Problem 1: Discover the attributes of various triangles.

T: What types of attributes can triangles have?

S: Well, they must have three sides, so they also have three angles. → But their sides can be different. Some are short, and some are long, or sometimes they are the same length. → Yeah, triangles can also have the same or different types of angles, like acute, obtuse, or right. → And some have lines of symmetry, and others don't.

T: Think about the types of angles and the lengths of the sides of triangles as we complete this activity.

Separate students into small groups of three students each. Provide each group with one of each triangle from Templates 1, 2, and 3. Instruct students to investigate the given triangle cutouts using rulers and protractors. Students should record their findings in the Attributes column of the Practice Sheet, including measures of sides and angles, as well as other general observations. It may be helpful for students to record the angle and side length measurements on the cutouts as well. Students should quickly sketch each triangle in the first column. Allow students six to eight minutes for this activity.

T: Now, take a moment with your group to compare your findings. Discuss ways in which some triangles might be classified into different groups.

Students discuss.

Problem 2: Classify triangles by side length and angle measure.

T: Tell me how you sorted your triangles by side length.

S: Triangles B, E, and F each had two sides that were the same length. → Triangles C and D had sides that all measured different lengths. → Triangle A is the only triangle that has three sides that are all the same length!

T: Let's record your findings. You just classified some triangles by the length of their sides. Let's label the first of the classification columns as *Side Length*.

MP.6

T: There are three kinds of triangles you discovered. **Equilateral triangles**, such as Triangle A, have all sides that are equal in length.

S: That's easy to remember because *equilateral* starts with the same sound as the word *equal*.

T: **Isosceles triangles** are like Triangles B, E, and F. They have at least two sides with the same length.

T: Triangles C and D are classified as **scalene triangles**. None of their side lengths are the same.

T: To show that certain sides are the same length, we draw a tick mark on each same length segment.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Remembering the names that classify triangles may present a challenge for English language learners and others. Present helpful mnemonic devices. The word *isosceles*, for example, starts with the sound *eyes*. We have *two* eyes; similarly, an isosceles triangle has at least *two* equal sides. Encourage students to come up with their own way to remember, and then to share with others.

T: (Draw a tick mark on each side of the equilateral triangle.) It's your turn. Which other triangles need tick marks?
S: Triangles B, E, and F need just two tick marks.
T: Why don't Triangles C and D need tick marks?
S: All their sides have different lengths.
T: Tell me about how you classified the triangles based on the angles you measured.
S: Triangles D and E had one right angle.
MP.6
S: Triangles C and F had one obtuse angle.
S: All of the triangles had acute angles. → Triangles A and B had only acute angles.
T: Label the second of the classification columns as *Angle Measure*. Record your findings. If a triangle has an obtuse angle, we classify it as an **obtuse triangle**. If a triangle has one right angle, we call it a **right triangle**. What are triangles called that have only acute angles?
S: **Acute triangles**!
T: What angle symbol do we know to show the classification of right triangles?
S: The small square!

Problem 3: Determine the presence of angles of specific measure in triangles.

T: Fold Triangle B on its line of symmetry. What do you notice about the two sides that line up?
S: They are the same length! That means we measured correctly. It is an isosceles triangle.
T: What about the two base angles that folded on top of each other?
S: The two angles are the same size! I wonder if that has something to do with the two sides being the same.
T: Let's check. Fold another isosceles triangle, Triangle E or F.
S: Those sides that fold together are the same, and the angles are too!
T: Use those findings to draw some conclusions about equilateral triangles. Fold Triangle A on each of its lines of symmetry.
S: No matter which symmetry line we folded, the sides were the same length, and the angles matched up. → So, if all of the angles are lining up, doesn't that mean all of the angles have the same measure? → Yeah! And that means all the sides are the same length. And we knew that when we measured with our rulers and protractors. → Equilateral triangles are a lot like isosceles triangles.
T: An isosceles triangle has at least two sides that measure the same length. Do equilateral triangles have two sides that are the same length?
S: Yes. → Yes, but actually three.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners and others may feel overwhelmed with the many new terms introduced in this lesson. Encourage students to record *isosceles, equilateral,* and *scalene* in their personal math dictionaries. Students may, for example, draw an example of each type of triangle and then define the triangles in their first language, if helpful. Create a classroom chart with examples for each type of triangle so that students may reference it during the Problem Set and further triangle work.

T: An isosceles triangle has two angles with the same measure. Do equilateral triangles have two angles with the same measure?

S: Yes. → Yes, but actually three.

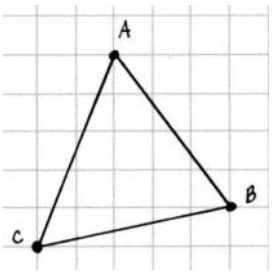
T: We can say that an equilateral triangle is a special isosceles triangle. It has everything an isosceles triangle has, but it also has a little more, such as three sides and three angles with the same measure, not just two.

T: Triangle D has a right angle. Fold the other two angles into the right angle. (Demonstrate.) It's your turn.

S: Neat, the two other angles fit perfectly into the right angle.

T: What does that tell you about the measure of both of the other angles in a right triangle?

S: The other two angles add together to make 90°.

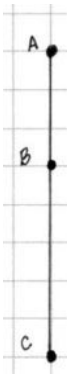


Problem 4: Define triangle.

T: What do we know about triangles that will help us to draw one?

S: Triangles have three sides and three angles. → We could draw three segments that meet together. → Those three segments will make the three angles. → When we learned about angles, we drew them by drawing two rays from one point.

T: On graph paper, plot three points, and label them *A, B*, and *C*. Connect those points with segments. What have you created?



S: A triangle!

T: (Plot three collinear points labeled *A, B*, and *C*.) What is the problem here?

S: If you connect the points with segments, *A, B*, and *C* will all be on one line. *A, B*, and *C* don't make a triangle.

T: Use your triangle to help you define the word **triangle** to your partner.

S: My triangle has three segments and three angles. → My triangle was formed from three points connected by three segments. → My triangle was formed from three points that were not in a line and were connected by segments. → Two of my points can be in a line, but not all three.

T: Identify your triangle as $\triangle ABC$. (Write $\triangle ABC$.) Classify your triangle by side length and angle measure.

S: (Identify and classify.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

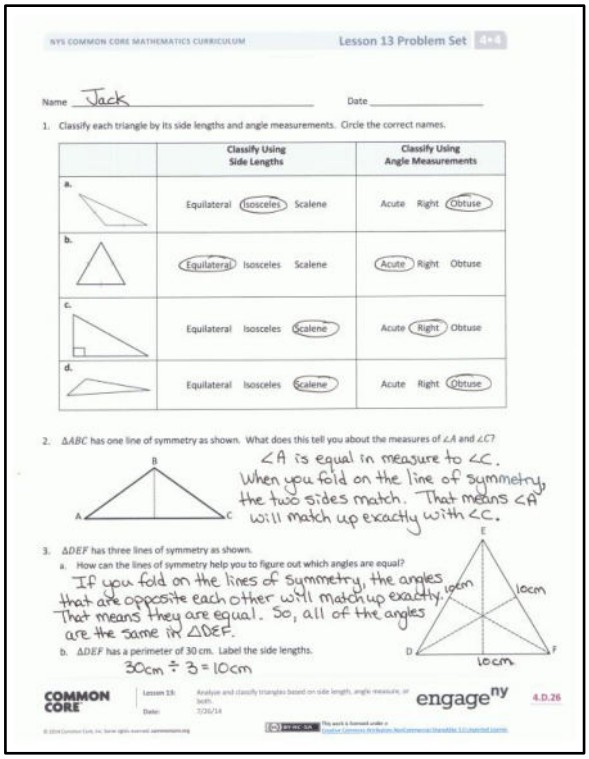
Lesson Objective: Analyze and classify triangles based on side length, angle measure, or both.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How do the tick marks and angle symbols allow classification of triangles without using tools in Problem 1?
- What strategy did you use to solve Problem 3(b)?
- Explain your answer to Problem 5(b). Recall from Lesson 6 that the word *collinear* describes three points that are on a line.
- A **triangle** can be defined as three points that are not collinear and that have line segments between them. Discuss this definition with your partner. Make sure you understand it completely.
- How many lines of symmetry can be found in **scalene triangles**? **Equilateral triangles**? **Isosceles triangles**?
- Can you determine if a triangle will have a line of symmetry just by knowing whether it is an **acute triangle** or an **obtuse triangle**? How about scalene or isosceles? Sketch an example of a scalene and isosceles triangle to verify your answer.
- Sketch some examples to prove your answer to Problem 6. How many acute angles do **right triangles** have?
- How did the Application Problem connect to today's lesson?



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 13 Problem Set 4•4

Name Jack Date

1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a.	Equilateral (Isosceles) Scalene	Acute Right (Obtuse)
b.	(Equilateral) Isosceles Scalene	(Acute) Right Obtuse
c.	Equilateral Isosceles (Scalene)	Acute (Right) Obtuse
d.	Equilateral Isosceles (Scalene)	Acute Right (Obtuse)

2. $\triangle ABC$ has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?

$\angle A$ is equal in measure to $\angle C$. When you fold on the line of symmetry, the two sides match. That means $\angle A$ will match up exactly with $\angle C$.

3. $\triangle DEF$ has three lines of symmetry as shown.
   a. How can the lines of symmetry help you to figure out which angles are equal?

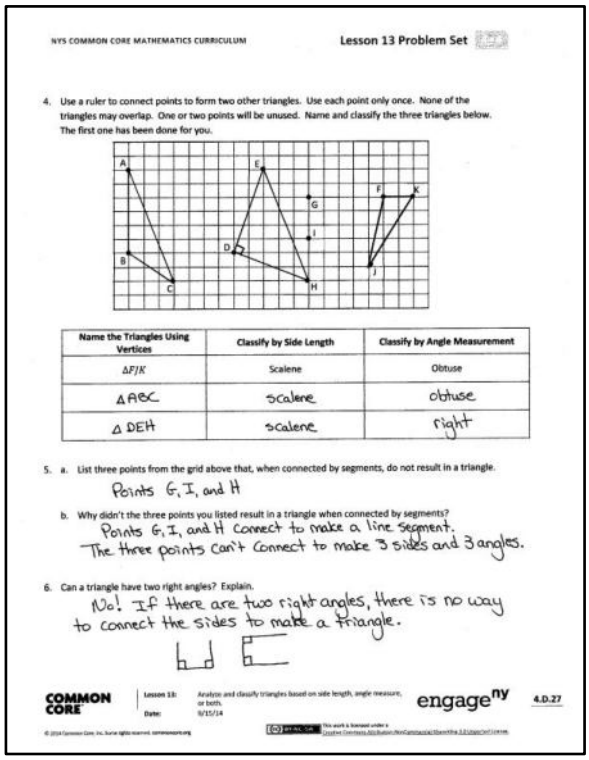
If you fold on the lines of symmetry, the angles that are opposite each other will match up exactly. That means they are equal. So, all of the angles are the same in $\triangle DEF$.

   b. $\triangle DEF$ has a perimeter of 30 cm. Label the side lengths.

30cm ÷ 3 = 10cm

10cm, 10cm, 10cm

COMMON CORE Lesson 13: Analyze and classify triangles based on side length, angle measure, or both. Date: 7/26/14 engage^ny 4.D.26



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 13 Problem Set 4•4

4. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the three triangles below. The first one has been done for you.

Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
$\triangle FJK$	Scalene	Obtuse
$\triangle AEC$	Scalene	obtuse
$\triangle DEH$	Scalene	right

5. a. List three points from the grid above that, when connected by segments, do not result in a triangle.

Points G, I, and H

   b. Why didn't the three points you listed result in a triangle when connected by segments?

Points G, I, and H connect to make a line segment. The three points can't connect to make 3 sides and 3 angles.

6. Can a triangle have two right angles? Explain.

No! If there are two right angles, there is no way to connect the sides to make a triangle.

COMMON CORE Lesson 13: Analyze and classify triangles based on side length, angle measure, or both. Date: 9/15/14 engage^ny 4.D.27

Exit Ticket (3 minutes)

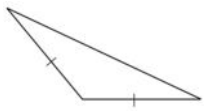
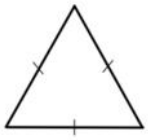
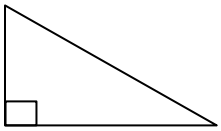
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

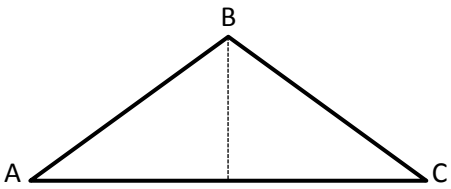
Sketch of Triangle	Attributes (Include side lengths and angle measures.)	Classification	
A			
B			
C			
D			
E			
F			

Name _____ Date _____

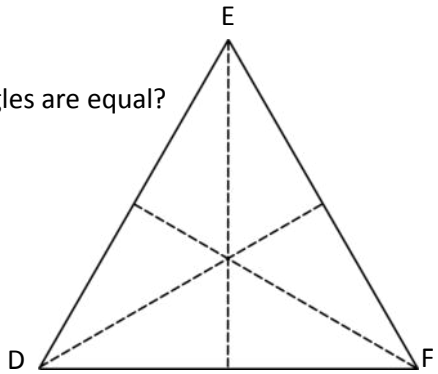
1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

2. $\triangle ABC$ has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?



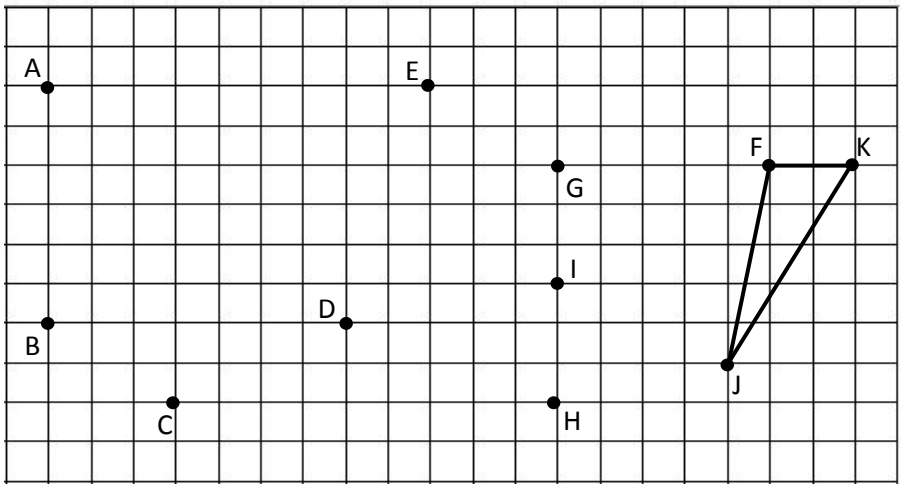
3. $\triangle DEF$ has three lines of symmetry as shown.



a. How can the lines of symmetry help you to figure out which angles are equal?

b. $\triangle DEF$ has a perimeter of 30 cm. Label the side lengths.

4. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the three triangles below. The first one has been done for you.



Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
$\triangle FJK$	Scalene	Obtuse

5. a. List three points from the grid above that, when connected by segments, do not result in a triangle.

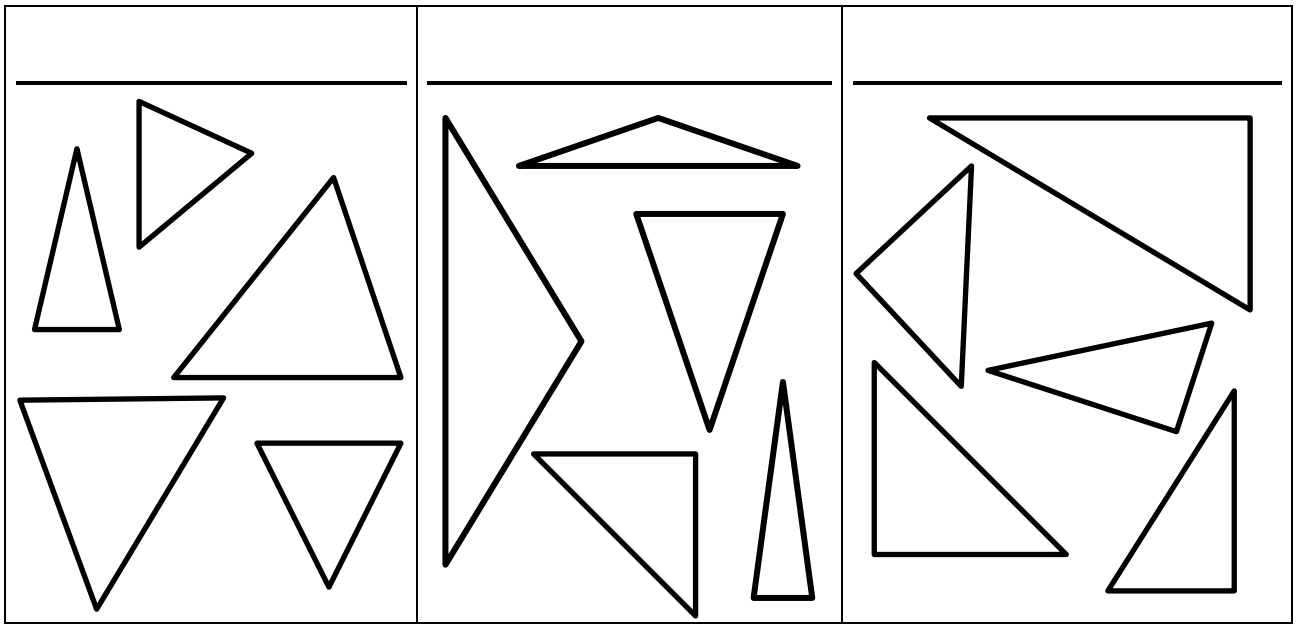
b. Why didn't the three points you listed result in a triangle when connected by segments?

6. Can a triangle have two right angles? Explain.

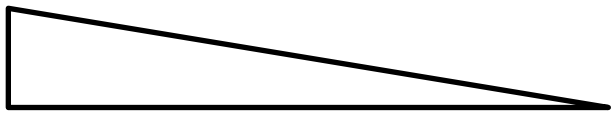
Name _____ Date _____

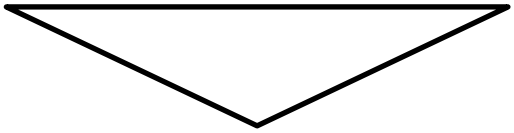
Use appropriate tools to solve the following problems.

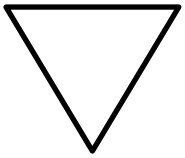
1. The triangles below have been classified by shared attributes (side length *or* angle type). Use the words *acute, right, obtuse, scalene, isosceles,* or *equilateral* to label the headings to identify the way the triangles have been sorted.



2. Draw lines to identify each triangle according to angle type *and* side length.

a. 

b. 

c. 

Acute

Obtuse

Right

Isosceles

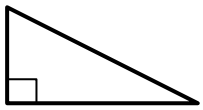
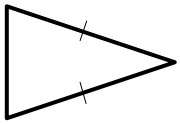
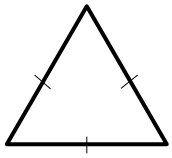
Equilateral

Scalene

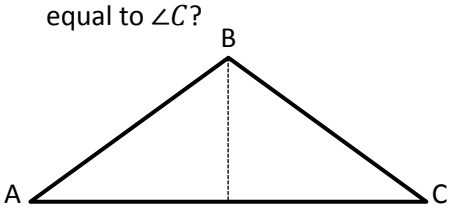
3. Identify and draw any lines of symmetry in the triangles in Problem 2.

Name _____ Date _____

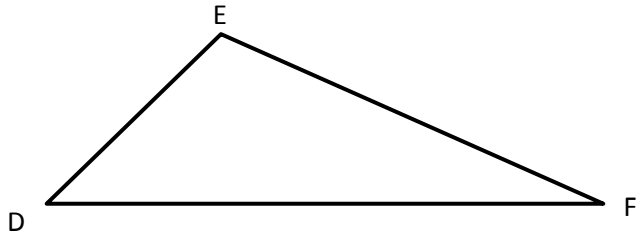
1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

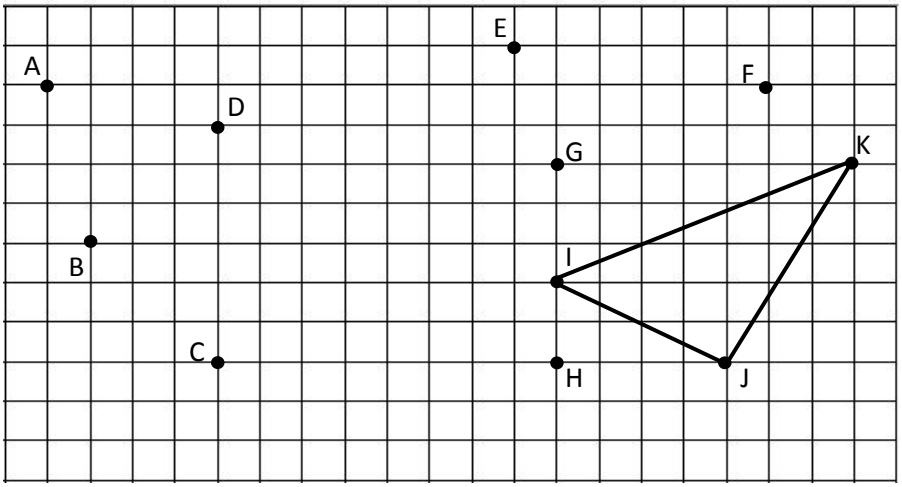
2. a. $\triangle ABC$ has one line of symmetry as shown. Is the measure of $\angle A$ greater than, less than, or equal to $\angle C$?



b. $\triangle DEF$ is scalene. What do you observe about its angles? Explain.



3. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. Two points will be unused. Name and classify the three triangles below.

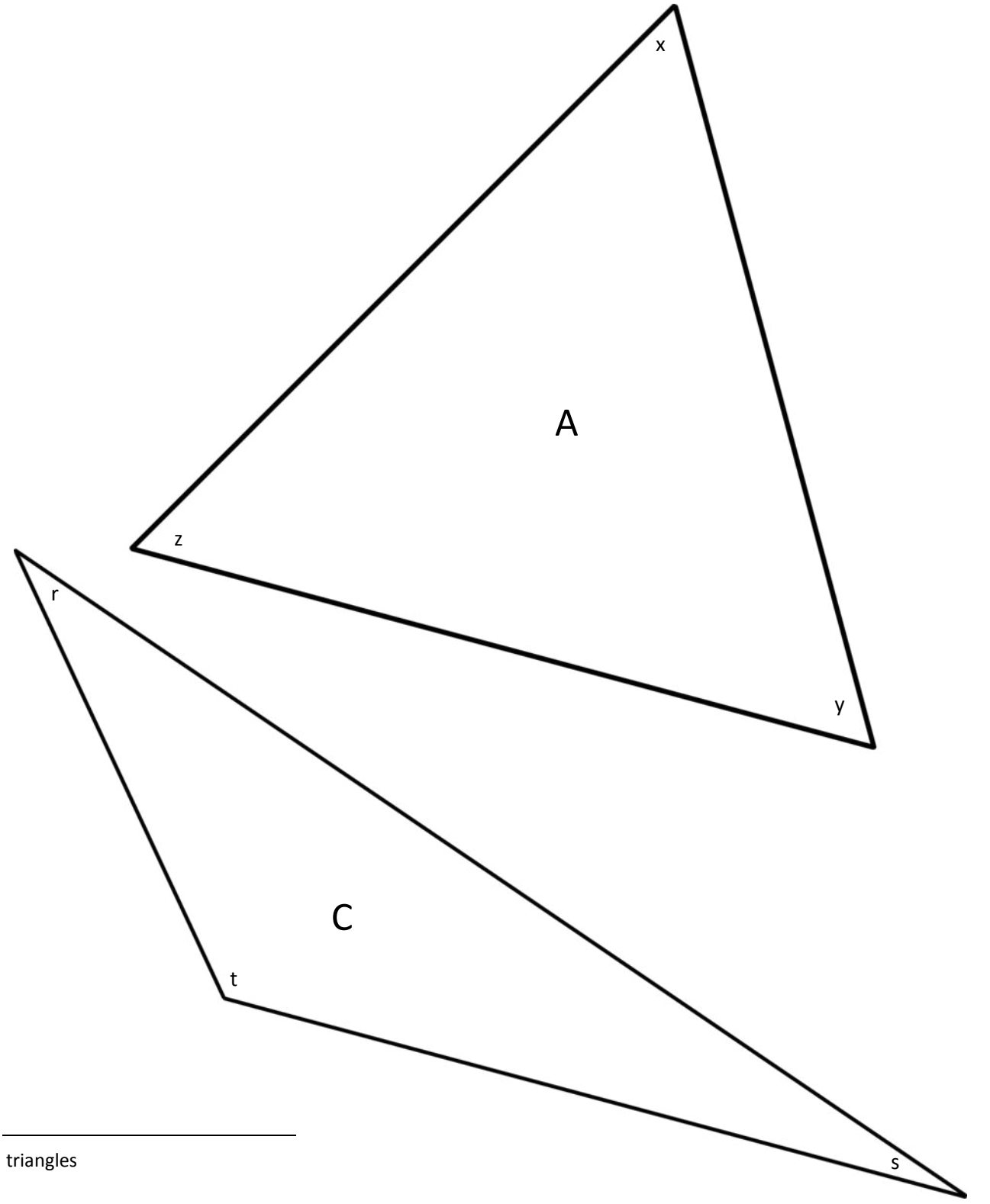


Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
$\triangle IJK$		

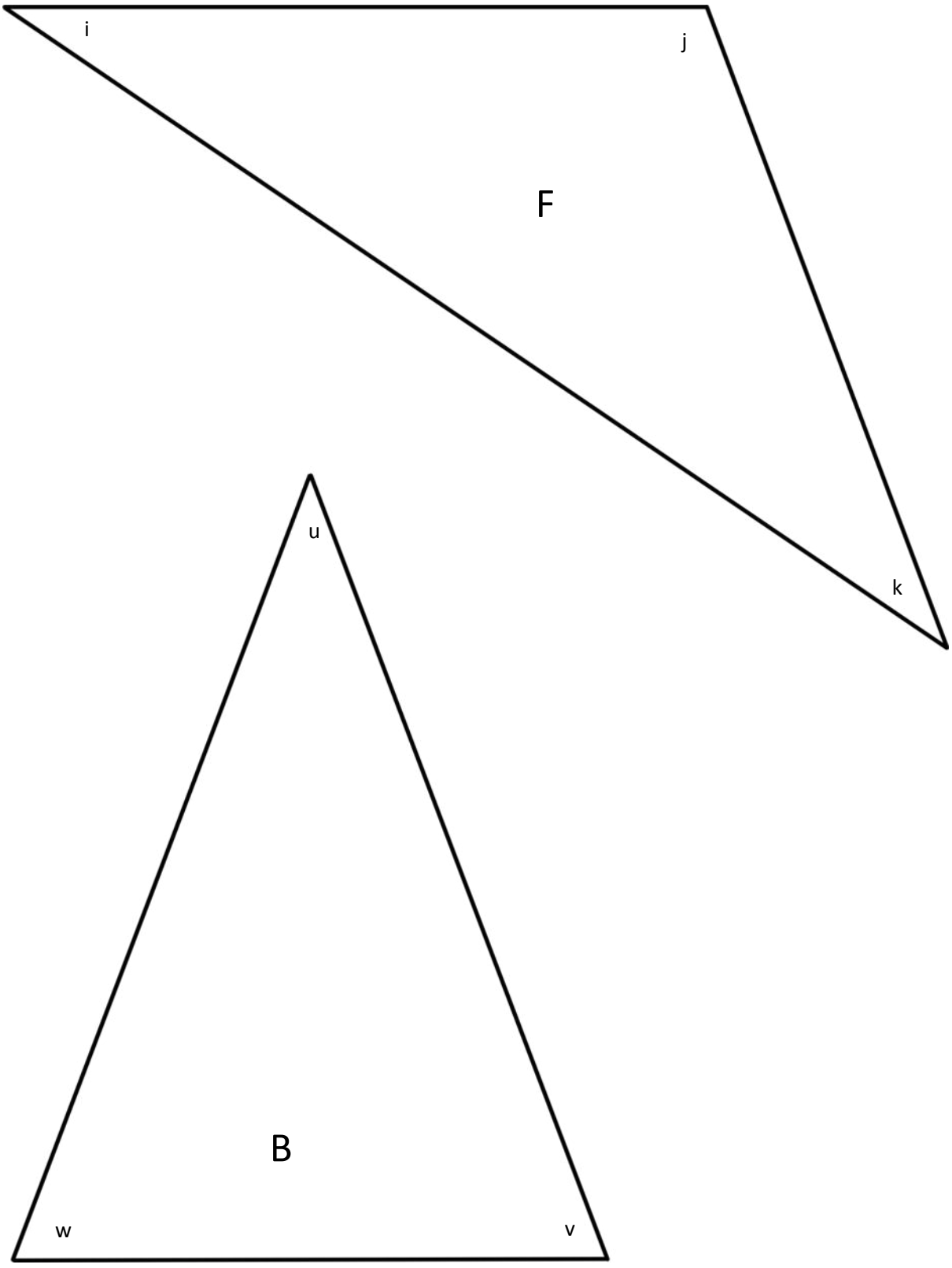
4. If the perimeter of an equilateral triangle is 15 cm, what is the length of each side?

5. Can a triangle have more than one obtuse angle? Explain.

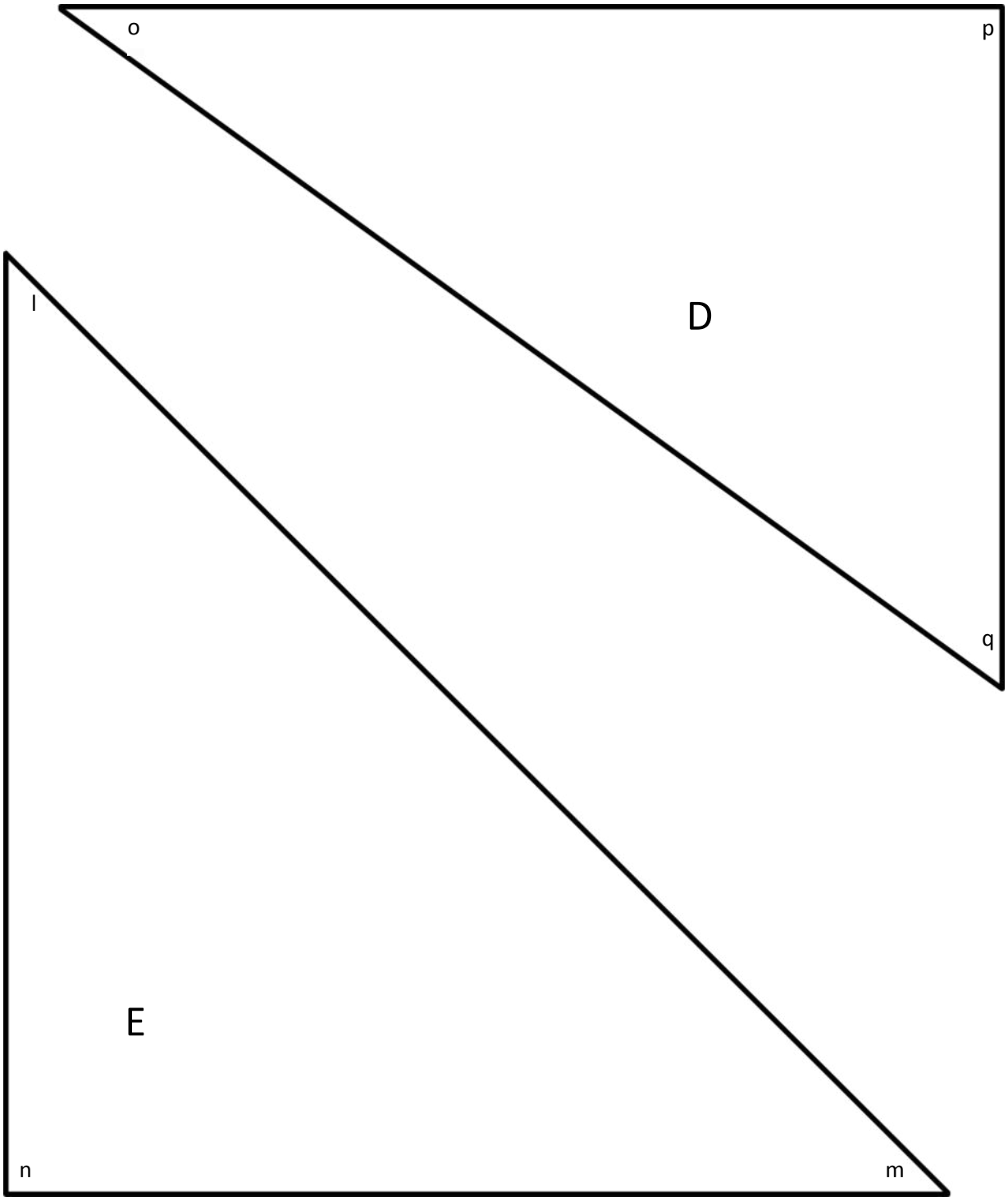
6. Can a triangle have one obtuse angle and one right angle? Explain.



triangles



triangles



triangles