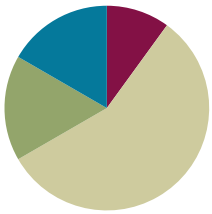
Lesson 8

Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

Suggested Lesson Structure

Fluency Practice	(6 minutes)
Application Problem	(10 minutes)
Concept Development	(34 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (6 minutes)

- Group Counting **3.OA.1** (3 minutes)
- Commutative Multiplying **3.OA.5** (3 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos, threes, and fours in this activity supports work with units of 2 and 3 in this topic and anticipates work using units of 4 in Topic E.

T: Let's count by twos to 20. Whisper the numbers, and then speak them.

T: Let's count by twos to 20 again. This time, hum the first number, and then speak it. As you hum, think of the number.

T: Let's count by twos to 20. This time, instead of humming, think every other number.

T: What did we just count by?

S: Twos.

T: Let's count by fours. (Direct students to count forward and backward to 20, periodically changing directions.)

T: Let's count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

Commutative Multiplying (3 minutes)

Materials: (S) Personal white board

Note: Practicing this concept, which was taught in Lesson 7, helps students build confidence and automaticity.

T: (Project a 3 × 2 array.) How many groups of 2 do you see?
S: 3 groups of 2.
T: Write two different multiplication sentences for the array.
S: (Write 3 × 2 = 6 and 2 × 3 = 6.)

Continue with the following possible sequence: 3 by 5 and 4 by 3.

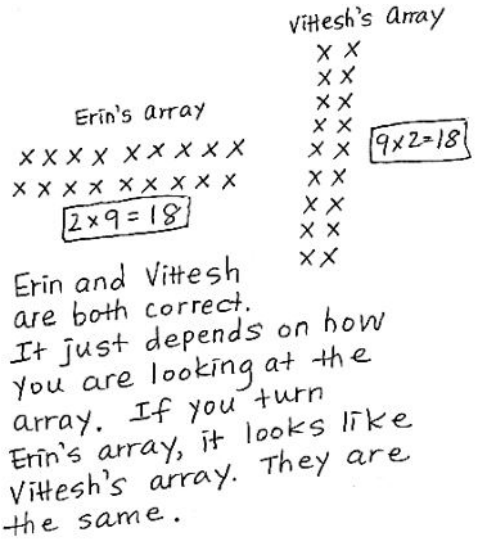
T: (Write 4 × 2 = 2 × ___.) On your board, fill in the blank.
S: (Write 4 × 2 = 2 × 4.)

Repeat the process for 9 × 5 = 5 × ___ and 3 × 6 = 6 × ___.

Application Problem (10 minutes)

Children sit in 2 rows of 9 on the carpet for math time. Erin says, "We make 2 equal groups." Vittesh says, "We make 9 equal groups." Who is correct? Explain how you know using models, numbers, and words.

Note: This problem reviews the commutativity of multiplication introduced in Lesson 7 and prepares students for Day 2 of the same concept in today's lesson.



Concept Development (34 minutes)

Materials: (S) Personal white board

Problem 1: Rotate arrays 90 degrees.

T: Turn your personal white board so that the long side is vertical. Skip-count by threes 4 times and write each number.
S: 3, 6, 9, 12.
T: Draw an array to match your count where the number of rows represents the number of groups.
T: Discuss how many rows and columns you see.
S: (Students discuss that there are 4 rows and 3 columns.)
T: Turn your board so that the long side is horizontal. How many rows and columns does it show now?

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If students are very comfortable with the way an array changes depending on how it is turned, add a bit of complexity by having them imagine turning it horizontal rather than actually doing it.

S: (Turn boards 90 degrees.) 3 rows and 4 columns.
T: Tell your partner a different skip-count that also represents the array.
S: 4, 8, 12.
T: What is the difference between the vertical and horizontal arrays?
S: In the vertical array the 4 threes were rows, and in the horizontal array they were columns. → It's the same with the 3 fours. They were columns, then rows.
MP.7 T: Did the total number of dots change?
S: No.
T: So, the total and the factors stay the same, but the factors switch places. Yesterday, we learned a special name for that. It's called…
S: Commutative! → The commutative property!
T: Use the commutative property to write two multiplication sentences for the array.
S: (Write 4 × 3 = 12 and 3 × 4 = 12.)

Students practice with partners using the following examples. Partner A gives skip-counting directions. Partner B writes the count, draws an array, and writes multiplication sentences. Then, partners switch roles.

- Skip-count by twos 3 times
- Skip-count by threes 6 times

Problem 2: Interpreting rows and columns in rotated arrays.

T: Work with your partner to draw an array that shows 5 rows and 3 columns.
S: (Demonstrate one possible process.) Let's draw 5 circles going down to show the start of each row. → Then we can draw 3 circles to show the columns across the top. → Wait, we already drew 1 column when we made the rows, so we can just draw 2 more columns.
T: Write an equation to match your array where the first factor represents the number of rows. Don't solve it yet.
S: (Write 5 × 3 = ____.)
T: I'm going to change the problem slightly. Listen carefully and rotate your array to match: 3 rows and 5 columns.
S: (Turn boards 90 degrees.)
T: Write the equation for the new array. Let the first factor represent the number of rows. Don't solve it yet.
S: (Write 3 × 5 = ____.)
T: Explain the difference between these problems to your partner.
S: The array turned and the factors switched places.

NOTES ON DRAWING ROWS AND COLUMNS:

Students may not immediately recognize that they do not need to redraw the corner circle to make 3 columns. After drawing rows, they already have 1 column and, for this problem, only need to add 2 more columns. If they make a mistake, help them recognize it by encouraging them to recount their total columns.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If appropriate, provide a challenge for students by having them cover the array as they skip-count to solve.

T: When we rotated the array, we agreed the first factor would tell us the number of rows. What did that do to the order of the factors?

S: They switched!

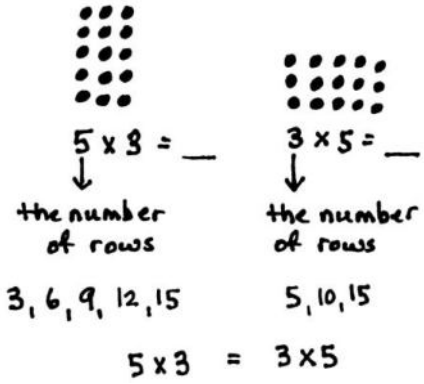
T: Did the total change?

S: No.

T: When we change the order of the factors, we are using the commutative property.

T: Solve each of your equations by skip-counting. Write each number as you say it.

S: (Write 3, 6, 9, 12, 15 and 5, 10, 15.)



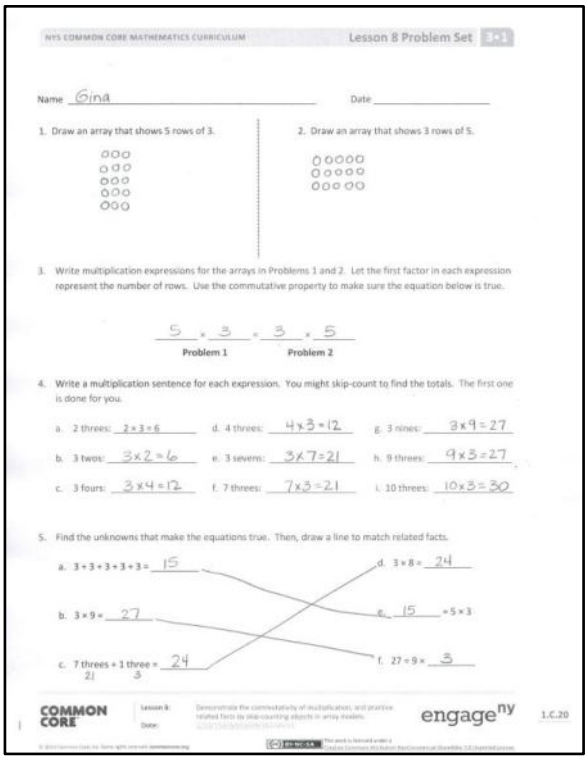
Continue with the following possible examples:

- 7 rows and 2 columns
- 3 rows and 9 columns

T: (Once students have worked through the problem, write the final example in groups language: 3 groups of 9 and 9 groups of 3.) Are these statements equal? Use your array to discuss with your partner how you know.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 8 Problem Set 3•1

Name Gina Date

1. Draw an array that shows 5 rows of 3.

2. Draw an array that shows 3 rows of 5.

3. Write multiplication expressions for the arrays in Problems 1 and 2. Let the first factor in each expression represent the number of rows. Use the commutative property to make sure the equation below is true.

5 × 3 = 3 × 5
Problem 1 Problem 2

4. Write a multiplication sentence for each expression. You might skip-count to find the totals. The first one is done for you.

a. 2 threes: 2×3=6
b. 3 twos: 3×2=6
c. 3 fours: 3×4=12
d. 4 threes: 4×3=12
e. 3 sevens: 3×7=21
f. 7 threes: 7×3=21
g. 3 nines: 3×9=27
h. 9 threes: 9×3=27
i. 10 threes: 10×3=30

5. Find the unknowns that make the equations true. Then, draw a line to match related facts.

a. 3+3+3+3+3= 15
b. 3×9= 27
c. 7 threes + 1 three = 24 (21 + 3)
d. 3×8= 24
e. 15 =5×3
f. 27=9× 3

COMMON CORE Lesson 8: Date: engage^ny 1.C.20

Student Debrief (10 minutes)

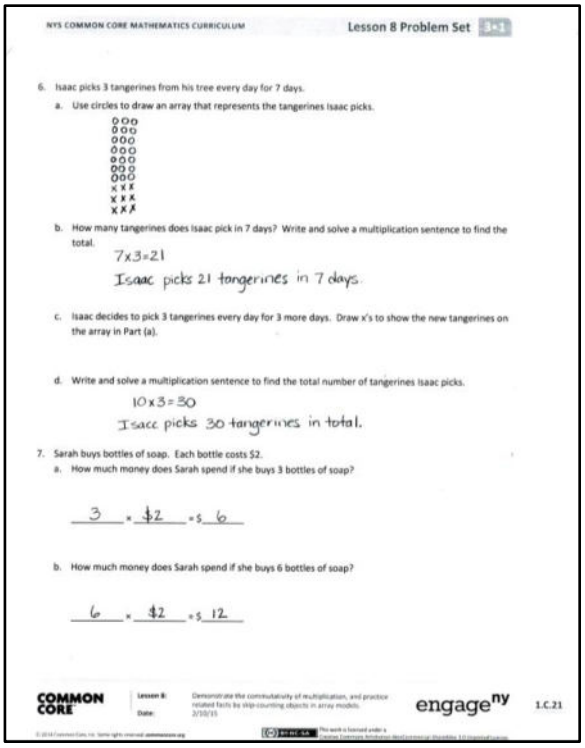
Lesson Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Share your answers to Problem 7 with a partner. Do your multiplication sentences look the same, or are they different? Why?
- Discuss the meaning of the commutative property and how it relates to equal groups, columns, rows, and arrays.
- Discuss the usefulness of skip-counting to solve multiplication problems.
- Build fluency by having students skip-count to find answers to the following expressions without the help of an array. They can keep track of their count using fingers.
    - 3 sixes, 6 threes
    - 3 eights, 8 threes
    - 5 threes, 3 fives



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 8 Problem Set 3•1

6. Isaac picks 3 tangerines from his tree every day for 7 days.
   a. Use circles to draw an array that represents the tangerines Isaac picks.
   b. How many tangerines does Isaac pick in 7 days? Write and solve a multiplication sentence to find the total.
      7x3=21
      Isaac picks 21 tangerines in 7 days.
   c. Isaac decides to pick 3 tangerines every day for 3 more days. Draw x's to show the new tangerines on the array in Part (a).
   d. Write and solve a multiplication sentence to find the total number of tangerines Isaac picks.
      10x3=30
      Isaac picks 30 tangerines in total.
7. Sarah buys bottles of soap. Each bottle costs $2.
   a. How much money does Sarah spend if she buys 3 bottles of soap?
      3 x $2 = $6
   b. How much money does Sarah spend if she buys 6 bottles of soap?
      6 x $2 = $12

COMMON CORE Lesson 8: Date: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models. engage^ny 1.C.21

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

1. Draw an array that shows 5 rows of 3.

2. Draw an array that shows 3 rows of 5.

3. Write multiplication expressions for the arrays in Problems 1 and 2. Let the first factor in each expression represent the number of rows. Use the commutative property to make sure the equation below is true.

_____ × _____ = _____ × _____

Problem 1 **Problem 2**

4. Write a multiplication sentence for each expression. You might skip-count to find the totals. The first one is done for you.

a. 2 threes: 2 × 3 = 6

b. 3 twos: _____

c. 3 fours: _____

d. 4 threes: _____

e. 3 sevens: _____

f. 7 threes: _____

g. 3 nines: _____

h. 9 threes: _____

i. 10 threes: _____

5. Find the unknowns that make the equations true. Then, draw a line to match related facts.

a. 3 + 3 + 3 + 3 + 3 = _____

b. 3 × 9 = _____

c. 7 threes + 1 three = _____

d. 3 × 8 = _____

e. _____ = 5 × 3

f. 27 = 9 × _____

6. Isaac picks 3 tangerines from his tree every day for 7 days.

   a. Use circles to draw an array that represents the tangerines Isaac picks.

   b. How many tangerines does Isaac pick in 7 days? Write and solve a multiplication sentence to find the total.

   c. Isaac decides to pick 3 tangerines every day for 3 more days. Draw x's to show the new tangerines on the array in Part (a).

   d. Write and solve a multiplication sentence to find the total number of tangerines Isaac picks.

7. Sarah buys bottles of soap. Each bottle costs $2.

   a. How much money does Sarah spend if she buys 3 bottles of soap?

   _____ × _____ = $_____

   b. How much money does Sarah spend if she buys 6 bottles of soap?

   _____ × _____ = $_____

Name _____ Date _____

Mary Beth organizes stickers on a page in her sticker book. She arranges them in 3 rows and 4 columns.

a. Draw an array to show Mary Beth's stickers.

b. Use your array to write a multiplication sentence to find Mary Beth's total number of stickers.

c. Label your array to show how you skip-count to solve your multiplication sentence.

d. Use what you know about the commutative property to write a different multiplication sentence for your array.

Name _____ Date _____

1. Draw an array that shows 6 rows of 3.

2. Draw an array that shows 3 rows of 6.

3. Write multiplication expressions for the arrays in Problems 1 and 2. Let the first factor in each expression represent the number of rows. Use the commutative property to make sure the equation below is true.

_____ × _____ = _____ × _____

Problem 1 **Problem 2**

4. Write a multiplication sentence for each expression. You might skip-count to find the totals. The first one is done for you.

a. 5 threes: 5 × 3 = 15

b. 3 fives: _____

c. 6 threes: _____

d. 3 sixes: _____

e. 7 threes: _____

f. 3 sevens: _____

g. 8 threes: _____

h. 3 nines: _____

i. 10 threes: _____

5. Find the unknowns that make the equations true. Then, draw a line to match related facts.

a. 3 + 3 + 3 + 3 + 3 + 3 = _____

b. 3 × 5 = _____

c. 8 threes + 1 three = _____

d. 3 × 9 = _____

e. _____ = 6 × 3

f. 15 = 5 × _____

6. Fernando puts 3 pictures on each page of his photo album. He puts pictures on 8 pages.

   a. Use circles to draw an array that represents the total number of pictures in Fernando's photo album.

   b. Use your array to write and solve a multiplication sentence to find Fernando's total number of pictures.

   c. Fernando adds 2 more pages to his book. He puts 3 pictures on each new page. Draw x's to show the new pictures on the array in Part (a).

   d. Write and solve a multiplication sentence to find the new total number of pictures in Fernando's album.

7. Ivania recycles. She gets 3 cents for every can she recycles.

   a. How much money does Ivania make if she recycles 4 cans?

   _____ × _____ = _____ cents

   b. How much money does Ivania make if she recycles 7 cans?

   _____ × _____ = _____ cents