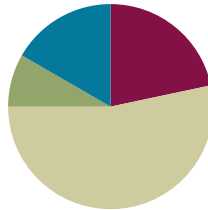# Lesson 7

Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

Suggested Lesson Structure

- ■ Fluency Practice   (13 minutes)
- ■ Application Problem   (5 minutes)
- ■ Concept Development   (32 minutes)
- ■ Student Debrief   (10 minutes)

   Total Time   **(60 minutes)**

Fluency Practice (13 minutes)

- ▪ Group Counting **3.OA.1**   (3 minutes)
- ▪ Divide Equal Groups **3.OA.2**   (5 minutes)
- ▪ Multiply with Twos **3.OA.7**   (5 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity anticipates work with those factors in Topic C.

- T: Let's count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)
- T: Let's count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

Divide Equal Groups (5 minutes)

Materials: (S) Personal white board

Note: Students directly relate repeated addition to division. They interpret the unknown in division. This activity reviews Lesson 6.

- T: (Project an array with 2 groups of 4.) Say the total as a repeated addition sentence.
- S: 4 + 4 = 8.
- T: Write a division sentence for 8 divided into 2 equal groups.
- S: (Write 8 ÷ 2 = 4.)

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T: Below that division sentence write a division sentence dividing 8 into 4 equal groups.

S: (Write 8 ÷ 4 = 2.)

Continue with this possible sequence: 5 groups of 3, 3 groups of 4, and 6 groups of 2.

Multiply with Twos (5 minutes)

Materials: (S) Personal white board, twos array (Fluency Template), blank paper

Twos Array Fluency Template

Note: Students unit count objects in an array and write multiplication sentences that match the count-by in anticipation of this lesson's objective.
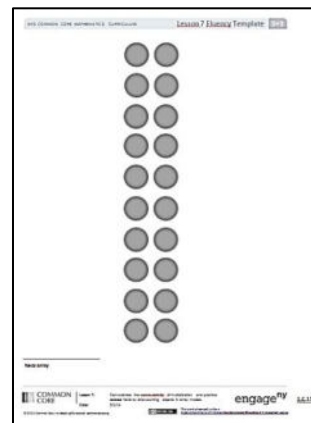

T: Slip your template into your personal white board.

T: Turn your board so that it's vertical. Use your blank paper to cover all but the first row of dots.

T: How many twos show?

S: 1 two.

T: Say the multiplication sentence to represent the array that's shown and solve.

S: 1 × 2 = 2.

T: Uncover another row.

Continue this sequence having students uncover twos for 2 × 2, 3 × 2, 10 × 2, 4 × 2, 5 × 2, 6 × 2, 7 × 2, 9 × 2, and 8 × 2.

Application Problem (5 minutes)

Anna picks 24 flowers. She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends. She keeps a bundle for herself too. How many flowers does Anna put in each bundle?

$$24 \div 8 = 3$$

Anna puts 3 flowers in each bundle.
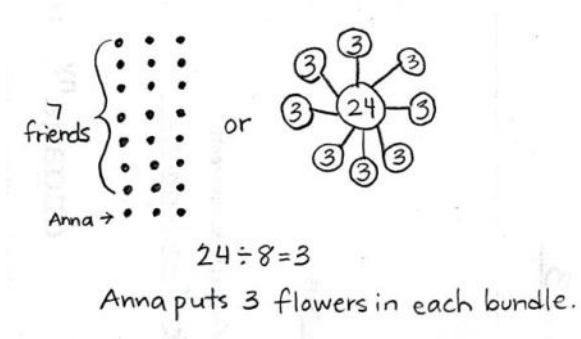
Note: This problem reviews division from Lesson 5 where the unknown represents the size of the group. The problem's complexity is in understanding that the flowers are divided equally into 8 bundles, not 7, in order to include a bundle for Anna. Students might choose to solve by drawing a division array learned in Lesson 6 or a number bond learned in Lesson 3.

Concept Development (32 minutes)

Personal White Board

Materials: (S) Personal white board

Problem 1: Rotate arrays 90 degrees.

- T: Position your board so that the long side is horizontal. Draw an array that shows 4 rows of 2.
- S: (Draw the array, as shown to the right.)
- T: Write a skip-count by twos to find the total. Then write a multiplication sentence where the first factor represents the number of rows.
- S: (Write 2, 4, 6, 8 and 4 × 2 = 8 as shown to the right.)
- T: **Rotate** your board 90 degrees so that the long side is vertical.
- S: (Rotate, as shown to the right.)
- T: What happened to the array?
- S: It has 2 rows of 4. → It has 4 groups of 2, but they're up and down instead of in rows.
- T: Now the twos are **columns**, vertical groups in an array.
- T: I'll rotate my board. You tell me if the twos are columns or rows.
- T: (Show the twos as rows.)
- S: Rows!
- T: (Rotate your board and show the twos as columns.)
- S: Columns!
- T: Skip-count the rows by four!
- S: (Point to the rows as students count.) 4, 8.
- T: Add that skip-count to your board. (Allow time.) What multiplication sentence can represent this array?
- S: 2 × 4 = 8.
- T: (Write 4 × 2 = 8 and 2 × 4 = 8 on the board with their corresponding arrays drawn as shown.) What do you notice about the multiplication sentences?
- S: The 4 and the 2 switched places.
- T: What do the 4 and 2 represent in each? Talk to your partner.
- S: In A, the 4 represents the number of rows, but in B, it represents the size of the row. → The twos are rows in A but columns in B.
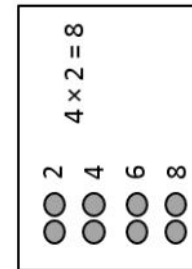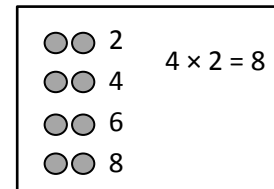- T: Did the meaning of the 8 change?
- S: No.
- T: So factors can switch places and trade meanings, but the total stays the same. We call that the **commutative property**. Talk to your partner about why the total stays the same.
- S: (Discuss.)
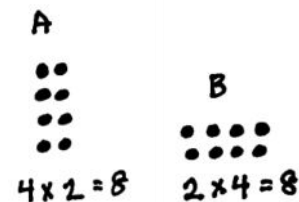
Continue with 2 × 5 and 3 × 4 arrays.

MP.7

> **NOTES ON VOCABULARY:**
>
> The word *column* was originally introduced in Grade 2, Module 6 but is treated as new vocabulary in this lesson.

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Problem 2: Interpreting rows and columns in rotated arrays.

Ask students to draw an array that shows 8 rows of 2. They should write a skip-count to find the total and a number sentence to represent the array. (See the example to the right.)

Personal White Board

⚪⚪ 2
⚪⚪ 4   8 × 2 = 16
⚪⚪ 6
⚪⚪ 8
⚪⚪ 10
⚪⚪ 12
⚪⚪ 14
⚪⚪ 16
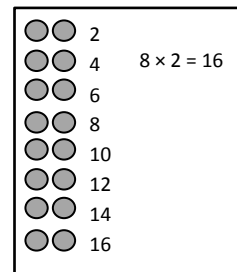
- T: What does the first factor—the 8—in your equation represent?

- S: The number of equal groups in the array. → The number of rows.

- T: Can the 8 also represent the size of the group? Talk to your partner.

- S: It can be the vertical group. → It can mean the size of the column!

- T: If we think of 8 as the size of the groups, then how many groups does the array show?

- S: 2 groups.

- T: Are those 2 groups shown by columns or by rows?

- S: By columns.

- T: Does the total change if we think of 8 as the size of the groups and 2 as the number of groups?

- S: No, the total is still 16 because you still have to multiply 8 and 2.

- T: Talk with a partner. Does 8 × 2 = 16 represent this array even if we think of 8 as the size of the groups and 2 columns as the number of groups?

- S: No, it should be written as 2 × 8 = 16. → We just learned that factors can trade meanings. → They can trade meanings, but they also switch places. → The total stays the same, so I think it works.

- T: Factors can trade meanings without always having to switch places in the equation. It's okay to write 8 × 2 = 16 and think of 8 as the size of the groups and 2 columns as the number of groups. In third grade we'll usually write multiplication sentences so that the first factor represents the number of groups. That makes them a little easier to read. But either factor can mean the size of the groups or the number of groups.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

Lesson Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How did **rotating** our boards help us see rows as **columns** and columns as rows?
- What did you learn today about changing the order of the factors?
- Can you think of different number sentences Ms. Nenadal could have written to get at the same idea in Problem 6?
- Factors can change their order without changing the total. We call that the **commutative property**. Let's test addition, subtraction, and division and see if the commutative property applies to them too.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Students need not master the term *commutative property* (**3.OA.5**). However, they will need to be familiar with the vocabulary moving forward in this module.

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____   Date _____

1. a. Draw an array that shows 6 rows of 2.

    b. Write a multiplication sentence where the first factor represents the number of rows.

        _____ × _____ = _____

2. a. Draw an array that shows 2 rows of 6.

    b. Write a multiplication sentence where the first factor represents the number of rows.

        _____ × _____ = _____

3. a. Turn your paper to look at the arrays in Problems 1 and 2 in different ways. What is the same and what is different about them?

    b. Why are the factors in your multiplication sentences in a different order?

4. Write a multiplication sentence for each expression. You might skip-count to find the totals.

    a. 6 twos: _6 × 2 = 12___   d. 2 sevens: _____   **Extension:**
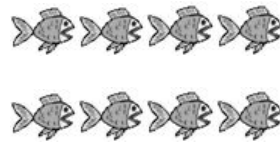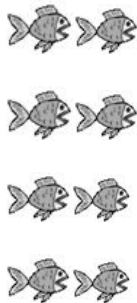
    b. 2 sixes: _____   e. 9 twos: _____   g. 11 twos: _____

   c. 7 twos: _____   f. 2 nines: _____   h. 2 twelves: _____

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5. Write and solve multiplication sentences where the second factor represents the size of the row.

_____   _____

6. Ms. Nenadal writes 2 × 7 = 7 × 2 on the board. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each equation true.

5 × 2 = 2 × _____   **_____ × 8 = 8 × 2**   **2 × 10 = _____ × 2**   **2 × _____ = 9 × 2**
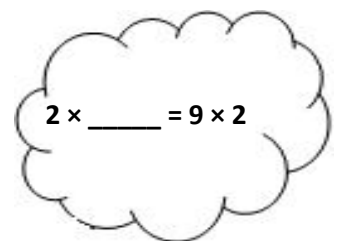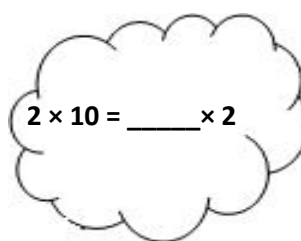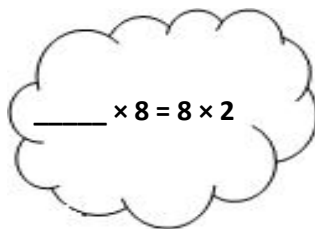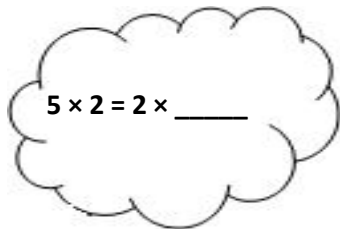
8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
   a. Draw an array to show how many erasers Jada has altogether.

   b. Write and solve a multiplication sentence to describe the array.

   c. Use the commutative property to write and solve a different multiplication sentence for the array.

Name _____   Date _____

$$2 \times 5 = 5 \times 2$$

Do you agree or disagree with the statement in the box? Draw arrays and use skip-counting to explain your thinking.

Name _____   Date _____

1. a. Draw an array that shows 7 rows of 2.

    b. Write a multiplication sentence where the
        first factor represents the number of rows.

        _____ × _____ = _____

2. a. Draw an array that shows 2 rows of 7.

    b. Write a multiplication sentence where the
        first factor represents the number of rows.

        _____ × _____ = _____

3. a. Turn your paper to look at the arrays in Problems 1 and 2 in different ways. What is the same and what
        is different about them?

    b. Why are the factors in your multiplication sentences in a different order?

4. Write a multiplication sentence to match the number of groups. Skip-count to find the totals. The first
    one is done for you.

    a. 2 twos: _2 × 2 = 4_____   d. 2 fours: _____   g. 2 fives: _____
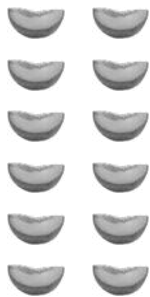
    b. 3 twos: _____   e. 4 twos: _____   h. 6 twos: _____

    c. 2 threes: _____   f. 5 twos: _____   i. 2 sixes: _____

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                                 facts by skip-counting objects in array models.

5. Write and solve multiplication sentences where the second factor represents the size of the row.

_____   _____

6. Angel writes 2 × 8 = 8 × 2 in his notebook. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each equation true.

2 × 6 = 6 × _____   **_____ × 2 = 2 × 7**   **9 × 2 = _____ × 9**   **2 × _____ = 10 × 2**

8. Tamia buys 2 bags of candy. Each bag has 7 pieces of candy in it.

   a. Draw an array to show how many pieces of candy Tamia has altogether.

   b. Write and solve a multiplication sentence to describe the array.

   c. Use the commutative property to write and solve a different multiplication sentence for the array.

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twos array

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