# Math II: Unit 10, Student Lessons

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### **Lesson 1: Testing, Testing**

**Develop Understanding** 

### **Learning Focus**

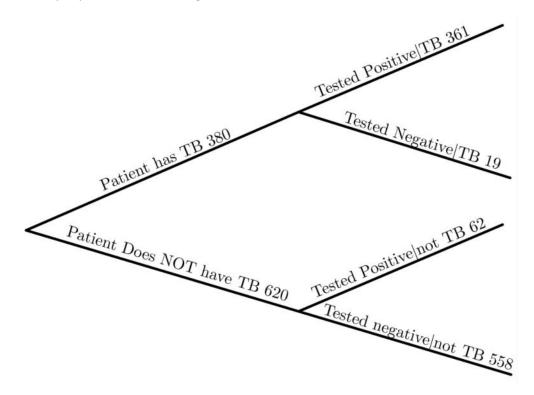
Interpret medical testing results using conditional probability.

Use a tree diagram to find probabilities.

How do I describe probabilities where one event seems related to another?

## Open Up the Math: Launch, Explore, Discuss

Tuberculosis (TB) can be tested for in a variety of ways, including a skin test. If a person has tuberculosis antibodies, then they are considered to have TB. Below is a tree diagram representing data based on 1,000 people who have been given a skin test for tuberculosis.



1. What do you notice about TB tests based on the tree diagram? What do you wonder?

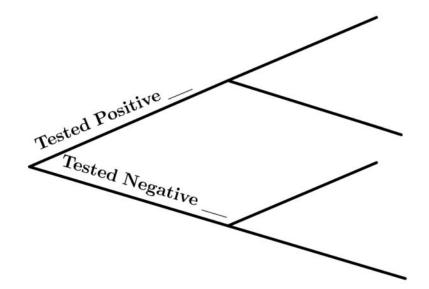
Maybe you wondered about the notation used in statements like "Tested negative|TB." The bar means "given that," so this statement would be translated as "the number of patients who tested negative, given that they have TB." We could even go one step further and write a statement about probability with this notation:  $P(\text{Tested negative}|\text{TB}) = \frac{19}{380}$ .

2. How would you translate this statement? What does it mean about TB tests?

This is an example of conditional probability, which is the measure of an event, given that another event has occurred.

3. Write several other probability and conditional probability statements based on the tree diagram.

4. Now we're going to change our perspective to see if we can gain new insights. We're going to make the primary branches of the tree diagram the test results. It's been started for you, so now you can finish it. You need to add what occurred in each branch and the number of people that it occurred to.



5. Write statements based on the diagram for the following probabilities:

a. 
$$\frac{577}{1,000} = P(\underline{\hspace{1cm}})$$

b. 
$$\frac{62}{423} = P(\underline{\hspace{1cm}})$$

c. 
$$85\% = P(____)$$

6. Find each probability and interpret the statement.

a. 
$$P(Have TB) = _{-}$$

What does this statement mean?

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b.  $P(Don't have TB) = _{-}$ 

What does this statement mean?

c. P(Tested negative|Don't have TB) =\_

What does this statement mean?

d. P(Tested positive|Don't have TB) =\_

What does this statement mean?

e. P(Have TB|Tested positive) =\_

What does this statement mean?

NAME	DATE	<b>PERIOD</b>
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- 7. Find each probability and interpret the statement.
  - a. P(Tested negative|Has TB) =\_

What does this statement mean?

b. P(Has TB|Tested negative) =\_

What does this statement mean?

c. Explain why these two statements could have different probabilities. What is the difference in what these two statements tell us about the TB test?

NAME	DATE	PERIOD
Part of	f understanding the world around us is being able to analyze	data and explain it to others.
8.	Based on the probability statements that could be made fr you say to a friend regarding the validity of their results if t and the result came back positive?	_
9.	In this situation, explain the consequences of errors (having	g a test with incorrect results).
10.	If a health test is not $100\%$ certain, why might it be benefitoward a false positive, which means that the test indicate don't?	
Reac	dy for More?  Find these probabilities using the tree diagram:	
	a. $P(\text{Has TB and tests positive})$ b. $P(\text{Tests positive} \text{Has TB})$	

2. How are these probabilities different?

### **Takeaways**

Conditional probability of A given B,

## Vocabulary

- · conditional probability
- false negative/positive
- tree diagram

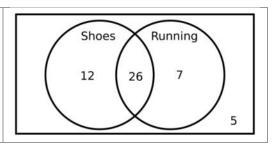
**Bold** terms are new in this lesson.

### **Lesson Summary**

In this lesson, we learned about conditional probability, the probability of an event given that another event has occurred. We used basic probability statements along with conditional probability to analyze the effectiveness of a medical test and to consider the meaning of testing errors, false positives, and false negatives.

### **Retrieval**

For problems 1–4, use the Venn diagram, which represents data collected about whether a group of people prefer wearing shoes or sandals and whether they prefer running or biking.



1. What does the overlapping section with the 26 in it represent?

NAME	DATE	PERIOD
2.	Where on the Venn diagram do you find the numbers of people that prefer wearing sand	dals?
3.	How many people prefer running?	
4.	How many total people are represented in the Venn diagram?	
5.	What is the probability of rolling the number $1$ when rolling a standard six-sided die?	
6.	What is the probability of drawing an even number from a set of cards numbered $1\ \mathrm{thro}$	ugh 15?

#### **Lesson 2: Favorite Flavors**

#### **Solidify Understanding**

### **Learning Focus**

Make Venn diagrams, tree diagrams, and two-way tables for data.

Use representations to find probabilities.

What kind of probability statements are easiest to find with each representation?

## Open Up the Math: Launch, Explore, Discuss

Amara thinks that chocolate ice cream is the greatest! She cannot even imagine someone saying that bland vanilla is better. She claims that chocolate is the favorite ice cream around the world. Her friend, Isla, thinks that vanilla is much better and more popular. To settle the argument, they created a survey asking people to choose their favorite ice cream flavor between chocolate and vanilla. After completing the survey, the following results came back:

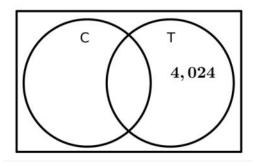
- There were 8,756 teens (ages 13–19) and 6,010 adults (age 20 or older).
- Out of all the adults, 59.73% chose vanilla over chocolate.
- 4,732 teens chose chocolate.
- 1. Upon first observations, which flavor do you think "won"? Write a sentence describing what you see at first glance that makes you think this.

2. Isla started to organize the data in the following two-way table. Complete the table using counts, not percentages:

	Chocolate	Vanilla	Total
Teens			8,756

Adults		6,010
Total		

3. Organize the same data into the Venn diagram that Amara started.



4. Now, put the same data in a tree diagram:

5.

- a. Find P(Prefers chocolate).
- b. Explain how to use the two-way table to find the probability.
- c. Explain how to use the Venn diagram.

NAME		DATE	PERIO
	d.	Explain how to use the tree diagram.	
6.	a.	Find $P(Prefers\ chocolate Teen).$	
	b.	Explain how to use the two-way table to find the probability.	
	C.	Explain how to use the Venn diagram.	
	d.	Explain how to use the tree diagram.	

- 7. Find each of these probabilities:
  - a. *P*(Adult|Prefer chocolate)
  - b. P(Teen and prefer vanilla)
- 8. Write statements to describe these probabilities:

a. 
$$\frac{2,420}{7,152} = P_{-}$$

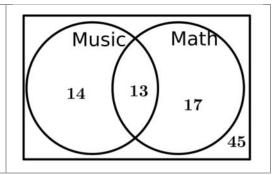
b. 
$$\frac{2,420}{14,766} = P_{-}$$

9. Which flavor do you think is actually the favorite? Using your organized data representations, write at least three probabilities that help support your claim regarding the preferred flavor of ice cream. For each probability, write a complete statement that includes probability notation.

# **Ready for More?**

Some studies suggest that students that like math also like music. To test that idea, Luca did a survey of students in his school to find out which math or music courses they were enrolled in. He used his results to create the following Venn diagram:

Some studies suggest that students that like math also like music. To test that idea, Luca did a survey of students in his school to find out which math or music courses they were enrolled in. He used his results to create the Venn diagram shown:



Find the following probabilities:

- 1. P(Music|Math)
- 2. P(Music and Math)
- 3. P(not Math or Music)

# **Takeaways**

Tree diagram	Tree diagram
Highlighted (easier to see)	Hidden

NAME DATE PERIOD

Venn diagram

Venn diagram

# Vocabulary

Venn diagram

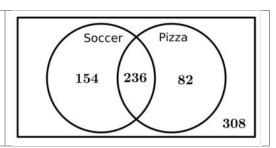
**Bold** terms are new in this lesson.

## **Lesson Summary**

In this lesson, we learned to use two-way tables and Venn diagrams, along with tree diagrams, to find conditional, compound, and basic probabilities. We compared the representations to understand what information is easy to read from a given representation and what information may not be as evident in a representation, so that we can make choices about the representations we use for a given situation.

### Retrieval

For problems 1–4, use the Venn diagram, which represents students' preferred food (pizza or hamburgers) and students' favorite sport (baseball or soccer).



- 1. How many students said that they prefer soccer?
- 2. Where do we find the number of students that prefer baseball?
- 3. How many students prefer baseball?

- 4. How many total students in the group?
- 5. What is 20% of 60?
- 6. What percent is 24 out of 192?

# **Lesson 3: Fried Freddy's**

**Solidify Understanding** 

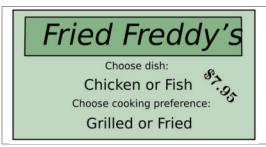
### **Learning Focus**

Use Venn diagrams to find probabilities.

What connections exist between the Venn diagram and probability notation?

# Open Up the Math: Launch, Explore, Discuss

Freddy loves fried food. His passion for the perfect fried food recipes led to him opening the restaurant Fried Freddy's. His two main dishes are fish and chicken. Knowing he also had to open up his menu to people who prefer to have their food grilled instead of fried, he created the following menu board:

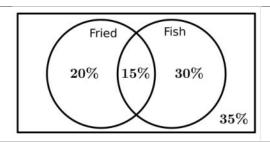


After being open for six months, Freddy realized he was having more food waste than he should because he was not predicting how much fish and chicken he should prepare in advance. His business friend, Tyrell, said he could help.

1. What information do you think Tyrell would need?

2.

Luckily, Freddy uses a computer to take orders each day so Tyrell had lots of data to pull from. After determining the average number of customers Freddy serves each day, Tyrell created the following Venn diagram to show Freddy the food preference of his customers:



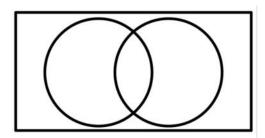
How is this Venn diagram different than the one you made in the previous lesson? What do all the percentages add up to on this diagram? Why?

To learn more about what the Venn diagram tells him about his business, Freddy computed the following probabilities:

3. What is the probability that a randomly selected customer would order fish?

Shade the part of the diagram that models this solution.

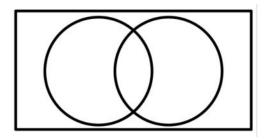
$$P(Fish) =$$



4. What is the probability that a randomly selected customer would order fried fish?

Shade the part of the diagram that models this solution.

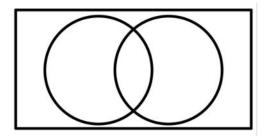
$$P(\text{Fried} \cap \text{Fish}) = P(\text{Fried and fish}) =$$



5. What is the probability that a person prefers fried chicken?

Shade the part of the diagram that models this solution.

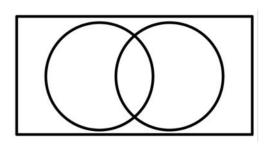
 $P(\text{Fried} \cap \text{Chicken}) = P(\text{Fried and chicken}) =$ \_\_\_



6. What is the estimated probability that a randomly selected customer would order fish and want it grilled?

Shade the part of the diagram that models this solution.

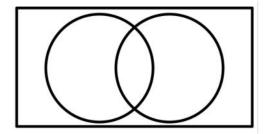
$$P(Grilled and fish) = P(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$$



7. What is the probability that a randomly selected person would choose fish or something fried?

Shade the part of the diagram that models this solution.

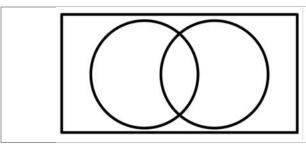
$$P(\text{Fried } \cup \text{Fish}) = P(\text{Fried or fish}) =$$



8. What is the probability that a randomly selected person would NOT choose fish or something fried?

Shade the part of the diagram that models this solution.

What other probability would describe the same space on the Venn diagram in this context?



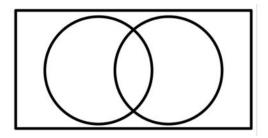
- 9. If Freddy serves 100 meals at lunch on a particular day, how many orders of fish should he prepare with his famous fried recipe?
- 10. Just as Freddy hoped, messing around with the diagrams makes him think he discovered a relationship. Here's his theory:

P(Fried or fish) = P(Fish) + P(Fried) - P(Fish and fried)

Check out Freddy's theory with numbers from his Venn diagram.

11. Unfortunately for Freddy, the statisticians of the world beat him to the theorem. (Freddy needs to keep on fryin'.) Statisticians call his idea the **Addition Rule**. (Freddy might have found a more creative name.) Label the Venn diagram below and use it to show:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



# **Ready for More?**

The two-way table below shows preferences for chocolate and vanilla ice cream among 9th and 10th grade students.

- 1. Write a statement of the Addition Rule for P(Prefers Chocolate or 10th grade):
- 2. Use the two-way table to show the statement is true:

	Chocolate	Vanilla	Total
9th Grade students	23	10	33
10th Grade students	6	8	14
Total	29	18	47

# **Takeaways**

Addition Rule for the union of two events *A* and *B*:

# **Adding Notation, Vocabulary, and Conventions**

Term	Notation	Meaning	Additional Information
The complement of A			

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Intersection of A and B		
Union of A and B		

## Vocabulary

- complement (in probability)
- intersection of sets
- union

**Bold** terms are new in this lesson.

### **Lesson Summary**

In this lesson, we used Venn diagrams to find the probability of the complement of an event, the union of two events, and the intersection of two events. We learned that the probability of the union of two events can be found using the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B).

### Retrieval

If one action affects the next, then the actions or events are dependent. If one action has no effect on the next, then the actions are considered independent. For each scenario, determine whether the actions or events are dependent or independent, and justify your choice.

- 1. Waking up when the alarm clock goes off and being on time to school.
  - A. dependent
  - B. independent
- 2. Rolling a standard six-sided die and tossing a coin.
  - A. dependent
  - B. independent

Create a proportion and use it to answer the problem.

3. If a basketball player makes 4 out of 10 shots, how many total shots would you predict they need to take in order to make 30 shots?

#### **Lesson 4: I Will Survive!**

#### **Solidify Understanding**

### **Learning Focus**

Represent probabilities with Venn diagrams.

Use conditional probability to draw conclusions.

Understand the definition of conditional probability.

What does it mean for events to be independent?

How does the Venn diagram connect to probability statements?

## Open Up the Math: Launch, Explore, Discuss

You may have heard of the *Titanic*, the biggest, fanciest cruise ship of its day. It sank in the North Atlantic after hitting an iceberg on its very first voyage. There were not enough lifeboats for all the passengers, so many people died, but some were rescued. There are many stories to be told of the *Titanic*, but we'll save them for another day. We're going to look at the data and see what relationships we can find.

Passengers on the *Titanic* purchased different classes of tickets. Passengers with first-class tickets spent more to get fancier rooms and nicer food. When the ship sank, some of the passengers were saved and some perished. The following data represents the number of passengers aboard the *Titanic* with first- and second-class tickets and whether or not they survived. Fill in the blanks for this table:

	Survived	Did Not Survive	Total
First Class	202		325
Second Class		167	285
Total			610

- 2. Use the data from the previous table to create a Venn diagram for each of the following:
  - a. First class and second class.

IAME			DATE	PERIOD
	b.	Second class and survived.		
3.	Find ea	ach probability:		
	a.	P(Survival)		
	b.	P(Not Survival)		
	C.	P(Not Survival First Class)		

NAME PERIOD DATE P(Second|Survive) P(First or Second) P(First and Second) P(Survival|First) P(Survival and First) P(First) Jack and Rose are looking at these last few probabilities and notice a relationship: 4.  $P(\mathsf{Survive}|\mathsf{First}) = \frac{P(\mathsf{Survive} \; \mathsf{and} \; \mathsf{First})}{P(\mathsf{First})}$ 

Use the probabilities you found to check their conjecture. Show your work here:

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NAME	DATE	PERIO
5.		
	- Complete a similar conjecture for: $P(Survive Second) = $	
	<ul> <li>Verify this conjecture with the appropriate probabilities and show yo</li> </ul>	our work here:
6.	As Rose and Jack are examining these probabilities, they are starting to feel a says, "I think our survival depends on the class of ticket we bought." Would y three probability statements to support your claim.	_
7.	How would you expect the $P(Survival First class)$ to compare to $P(Survival)$ depend on the class of ticket? What would you expect of the $P(Survival Second on the class of ticket?$	
Read	dy for More?	
	iffexam gave a test that had two hard problems on it. $35\%$ of students solved f students solved both problems. What is the probability that a student who s	•

problem also solved the second one?

## **Takeaways**

Mutually exclusive, disjoint events:

Joint events:

Definition of conditional probability:

Events *A* and *B* are independent if:

## Vocabulary

- disjoint
- independent event / dependent event
- joint events
- mutually exclusive
- two-way table

**Bold** terms are new in this lesson.

# **Lesson Summary**

In this lesson, we learned the definition of conditional probability and the relationship with the union of two events. We discussed two events that cannot occur together and learned that they are called mutually exclusive. Finally, we were introduced to the idea of independent events, events that may occur together, but the probability of one event does not change if the other occurs.

### **Retrieval**

Find the product or quotient.

1. 
$$\frac{1}{3} \cdot \frac{2}{5}$$

$$2. \quad \frac{3}{7} \div \frac{9}{14}$$

3.  $\frac{\frac{3}{4}}{\frac{9}{16}}$ 

4. Fill in the missing values of the two-way table and then write a conditional probability statement.

	Potato Chips	French Fries	Total
9th Grade	74		
10th Grade		48	64
Total		90	

## **Lesson 5: Declaring Independence**

**Solidify Understanding** 

### **Learning Focus**

Determine if two events are independent.

Find the probabilities of independent events.

Under what conditions is an event independent of another event?

How does knowing that events are independent help find probabilities?

## Open Up the Math: Launch, Explore, Discuss

At Fried Freddy's, Tyrell helped Freddy determine the amount and type of food Freddy should prepare each day for his restaurant. As a result, Freddy's food waste decreased dramatically. As time went by, Freddy noticed that another factor he needed to consider was the day of the week. He noticed that he was overpreparing during the week and sometimes under-preparing on the weekend. Tyrell and Freddy worked together and started collecting data to find the average number of orders he received of chicken and fish on a weekday and compared it to the average number of orders he received of each on the weekend. After two months, they had enough information to create the two-way table below:

	Fish	Chicken	Total
Weekday	65	79	144
Weekend	130	158	288
Total	195	237	432

- 1. As usual, Freddy starts with a little analysis of the data. Help Freddy by finding these probabilities. Write your answers both as fractions and percentages.
  - a. P(Fish|Weekday)
  - b. *P*(Chicken|Weekend)

NAME

P(Fish and Weekend) d. P(Fish)P(Weekday) 2. What do you notice about these probabilities? Would you say that the probability that a customer orders fish depends on the day of the week? Explain. Would you say that the amount of fish needed depends on the day of the week? Explain. 3. Using the test for independence from the last lesson, show that the probability of a customer ordering fish is independent of whether it is a weekday or weekend. Another relationship we worked with in the last lesson was  $P(A|B) = \frac{P(A \text{ and } B)}{B}$ . Write a statement of this relationship using:

DATE

**PERIOD** 

$$P(_{-}) = \frac{P(\text{Fish and Weekday})}{P(_{-})}$$

b. Solve this relationship for P(Fish and Weekday).

Since these probabilities are independent, P(Fish | Weekday) = P(Fish)

Make this substitution and write the equation for P(Fish and Weekday).

Verify that the equation is true using the probabilities you found earlier.

This relationship between probabilities of independent events is used in two ways. It can be used to determine if events are independent and to find probabilities when independence has been established. It is called the Multiplication Rule and is written:

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Sometimes the nature of the problem makes it apparent that events are independent.

- 5. Let's start with a classic number cube that has six sides labeled 1–6 that is rolled once, picked up, and rolled again.
  - a. Would you say that the probability of rolling a 6 in the first roll is independent of the probability of rolling a 6 in the second roll? Why?
  - b. What is the probability of rolling a 5 in the first roll?

$$P(R5) =$$

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c. What is the probability of rolling a 5 in the second roll?

$$P(R5) =$$

d. What is the probability of rolling 5 in the first roll and 5 in the second roll?

$$P(R5 \text{ and } R5) =$$

- 6. You're at a carnival and you're watching a game that has a big color wheel with eight different colors of equal area. People pay money to spin the wheel to win a giant stuffed animal if the wheel lands on yellow.
  - a. Is each spin an independent event? Explain.
  - b. As you watch, you see that 6 people in a row have landed on red. You think about jumping up and playing the game because now you are more likely to get a yellow. Is this a good idea?
  - c. You reach in your pocket and don't have any money to spin the wheel, so you just sit and watch. The wheel landed on red again and you start to wonder if this is a fair game. What argument can you make to the carnival manager about the game?

# **Ready for More?**

Johnny K has a bag of sour candies colored red, blue, green, and yellow that contains 25 of each candy. What is the probability that he pulls out a red, eats it, and then pulls out another red?

# **Takeaways**

Multiplication Rule for independent events:

Using the Multiplication Rule to find the probability of the intersection of events A, B, and C:

### **Lesson Summary**

In this lesson, we used two methods to show that events are independent. The first method relies on conditional probability, and the second method is the Multiplication Rule. We also used the Multiplication Rule to find the probability of the intersection of two independent events.

### **Retrieval**

Find the x-intercepts, y-intercept, line of symmetry, and vertex for the quadratic functions.

1. 
$$f(x) = (x - 7)^2 - 4$$

2.  $g(x) = x^2 + 8x + 7$ 

For problems 3–5, use the two-way frequency table to find the probabilities.

	Own Bike	Don't Own Bike	Total
Age 16-25	152	48	200
Age 26-35	86	114	200
Total	238	162	400

- 3. P(Own Bike) =
- 4.  $P(\text{Own Bike} \mid \text{Age 26-35}) =$
- 5. P(Age 16-25 or Don't Own a Bike) =

## **Lesson 6: Striving for Independence**

**Practice Understanding** 

### **Learning Focus**

Determine if events are independent.

Use representations to find probabilities.

How do I determine if one event is independent of another event?

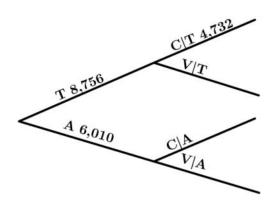
## Open Up the Math: Launch, Explore, Discuss

You've learned a lot about probability in this unit using different representations, including two-way tables, Venn diagrams, and tree diagrams. You've also learned some relationships between conditional probabilities, the probability of the union of two events, and the probability of the intersection of two events. Just in case that wasn't enough, you learned to determine if events are independent. In this lesson, you'll be asked to put all that knowledge and your best reasoning to work to solve some new probability problems with old contexts. Let's get started!

- 1. Out of the 2,000 students who attend a certain high school, 1,800 students own cell phones, 800 own a tablet, and 700 have both.
  - a. Create a Venn diagram model for this situation. Use proper probability notation as you answer the following problems.

b. What is the probability that a randomly selected student owns a cell phone?

IAME		DATE	PERIOD
	C.	What is the probability that a randomly selected students owns both a cell phon tablet?	e and a
	d.	If a randomly selected student owns a cell phone, what is the probability that this student also owns a tablet?	İS
	e.	How are problems c and d different?	
	f.	Are the outcomes "owns a cell phone" and "owns a tablet" independent? Explain	'n.
2.		llowing is a partially completed tree diagram from Lesson 2 where we compared ences for chocolate or vanilla ice cream.	
	a.	Circle the parts of the diagram that would be used to determine if choosing choosindependent of being a teen or an adult.	olate is
	b.	Complete the diagram so that choosing chocolate is independent of being teen of	or adult.



3. Many people are surprised to find that the *Titanic* picked up passengers in three ports before the disastrous attempt to sail the Atlantic. Rose and Jack found this data about survival of the passengers from two of the different ports and displayed it in the table below. Source: openup.org/USpsDV.

	Survived	Did Not Survive	Total
Southampton, UK	304	610	914
Cherbourg, France	150	120	270
Total	454	730	1,184

- a. Determine if survival is independent of boarding from Southampton for this data. Show why or why not.
- b. If survival is not independent of the port, determine how many people boarding from Southampton would need to survive in order to make it independent.
- 4. Determine whether the second event would be dependent or independent of the first event. Explain.
  - a. Rolling a six-sided die, then drawing a card from a deck of 52 cards.

AME		DATE PERI	OD
	b.	Drawing a card from a deck of $52$ cards and not replacing it, then drawing another car from the same deck.	d
	c.	Rolling a six-sided die, then rolling it again.	
	d.	Pulling a marble out of a bag, replacing it, then pulling a marble out of the same bag.	
5.	probab	obability that a student takes an art class at Imagination High School is $0.5$ . The bility that a student takes a music class is $0.3$ . The probability that they take an art or class is $0.65$ .	
	a.	Find the probability that a student takes both an art and a music class.	
	b.	Are the events "takes an art class" and "takes a music class" independent? Explain.	
	c.	Are the events "takes an art class" and "takes a music class" mutually exclusive? Expla	in.

# **Ready for More?**

When Drug A and Drug B are taken together, they operate independently. Drug A is effective 85% of the time. Drug B is effective 90% of the time. If both drugs are taken together, what is the probability that neither drug is effective?

# **Takeaways**

Helpful hints for using probability rules to determine independence:

## **Lesson Summary**

In this lesson, we combined the rules and relationships we have learned for probability to determine if events are independent and to find conditions that make events independent. We learned that when conditional probabilities such as P(A|B) are available, it is efficient to see if the conditional probability is equal to P(A) to determine if A and B are independent. If probabilities of unions and intersections are available, it may be more efficient to use the Multiplication Rule to test for independence.

### Retrieval

Solve each quadratic equation.

1. 
$$n^2 - 9n - 36 = 0$$

2. 
$$x^2 + 12x + 48 = 0$$

For problems 3–5, use the following information:

- Two events, event *A* and event *B*, are not mutually exclusive.
- $P(A) = \frac{3}{10}$ ,  $P(\text{not } B) = \frac{7}{10}$ , and  $P(A \text{ and } B) = \frac{1}{10}$ .
- 3. Find P(B).
- 4. Find P(A or B).
- 5. Find P(A|B).