

PRE-CALCULUS: by Finney, Demana, Watts and Kennedy
 Arithmetic Sequences and Series

What you'll Learn About

- Arithmetic Sequences

Arithmetic Sequence

Common Difference
 (d)

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

2, 8, 24, 64, 160

Determine if the following sequences are arithmetic. If they are give the common difference.

A) 7, 11, 15, 19, 23, ...

$$\begin{array}{cccc} \swarrow & \swarrow & \swarrow & \swarrow \\ +4 & +4 & +4 & +4 \end{array}$$

Yes Arithmetic

Common Difference = 4

B) 2, -3, -8, -13, ...

$$-3 - 2 = -5$$

$$-8 - (-3) = -5$$

$$-13 - (-8) = -5$$

Yes Arithmetic

$$d = -5$$

$$d = a_2 - a_1$$

$$d = a_3 - a_2$$

$$d = a_4 - a_3$$

C) $1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \dots$

$$\begin{array}{cccc} \swarrow & \swarrow & \swarrow & \swarrow \\ \frac{1}{4} & \frac{5}{4} & \frac{6}{4} & \frac{7}{4} \end{array}$$

Yes Arithmetic

$$d = \frac{1}{4}$$

D) 1, 4, 9, 16, ...

$$\begin{array}{ccc} \swarrow & \swarrow & \swarrow \\ +3 & +5 & +7 \end{array}$$

Not Arithmetic

Write the first 5 terms given the rule of the sequence.

10) $a_n = 2^n n$

$$a_1 = 2^1(1) = 2$$

$$a_2 = 2^2(2) = 8$$

$$a_3 = 2^3(3) = 24$$

$$a_4 = 2^4(4) = 64$$

$$a_5 = 2^5(5) = 160$$

16. $a_n = 3 - 4(n + 6)$

$$a_1 = 3 - 4(1 + 6) = -25$$

$$a_2 = 3 - 4(2 + 6) = -29$$

$$a_3 = 3 - 4(3 + 6) = -33$$

$$a_4 = 3 - 4(4 + 6) = -37$$

$$a_5 = 3 - 4(5 + 6) = -41$$

→ Explicit

$$a_n = a_1 + d(n-1)$$

$a_1 = 1^{\text{st}}$ term

$d = \text{common Difference}$

Recursive

$$a_1 =$$

$$a_n = a_{n-1} + d$$

$$d = 10$$

Find the formula for the arithmetic series

$$(5, 190) \quad (10, 115)$$

18) $a_1 = 15 \quad d = 4$

26) $a_5 = 190 \quad a_{10} = 115$

$$a_n = 15 + 4(n-1)$$

$$\frac{115 - 190}{10 - 5} = \frac{-75}{5}$$

$$d = \boxed{-15}$$

$$a_n = a_1 - 15(n-1) \quad a_n = a_1 + d(n-1)$$

$$a_5 = a_1 - 15(5-1)$$

$$a_n = 250 - 15(n-1)$$

$$190 = a_1 - 15(5-1)$$

$$190 = a_1 - 60$$

$$250 = a_1$$

Find the first 5 terms using the recursive formula

36) $a_1 = 200 \quad a_{k+1} = a_k - 10$

A) $a_1 = 10 \quad a_{k+1} = a_k + 4$

$$a_1 = 200$$

$$200, 190, 180, 170, 160,$$

$$a_1 = 10$$

$$a_{1+1} = a_1 - 10$$

$$a_2 = 200 - 10 = 190$$

$$a_2 = 10 + 4 = 14$$

$$a_{2+1} = a_2 - 10$$

$$a_3 = 190 - 10 = 180$$

$$a_3 = 14 + 4 = 18$$

$$a_4 = 18 + 4 = 22$$

Find the missing term.

$$a_5 = 22 + 4 = 26$$

40) $a_1 = 3 \quad a_2 = 13 \quad a_9 =$

Recursive

$$a_1 = 3 \quad a_9 = 83$$

$$a_2 = 13$$

$$a_3 = 23$$

$$a_4 = 33$$

$$a_5 = 43$$

$$a_6 = 53$$

$$a_7 = 63$$

$$a_8 = 73$$

Explicit

$$a_n = a_1 + d(n-1)$$

$$a_n = 3 + 10(n-1)$$

$$a_9 = 3 + 10(9-1)$$

$$= 83$$

$$S = \frac{n}{2}(a_1 + a_n)$$

Find the sum of the first 50 terms

54) $-6, -2, 2, 6, \dots$ $\hookrightarrow n=50$

$$n=50 \quad a_1 = -6$$

$$a_{50} = -6 + 4(50-1) \\ = 190$$

$$S = \frac{50}{2}(-6 + 190) \\ = 25(184) \\ = 4600$$

56) $4.7, 3.7, 2.7, \dots$

$$d = -1$$

$$a_{50} = 4.7 - 1(50-1) \\ = 4.7 - 49 \\ = -44.3$$

$$S = \frac{50}{2}(4.7 + (-44.3)) \\ = 25(-39.6) \\ = -990$$

Ending
Sum
Starting

Find the sum of the series given in sigma notation

62. $\sum_{n=1}^{100} 2n$

$2, 4, 6, 8, 10, 12, \dots, 200$

$$n = 100$$

$$a_1 = 2$$

$$a_{100} = 200$$

$$S = \frac{100}{2}(2 + 200) \\ = 50(202) \\ = 10,100$$

66. $\sum_{n=51}^{100} n - \sum_{n=1}^{50} n$

$$S = \frac{50}{2}(51 + 100) - \left[\frac{50}{2}(1 + 50) \right] \\ = [25(151)] - [25(51)]$$