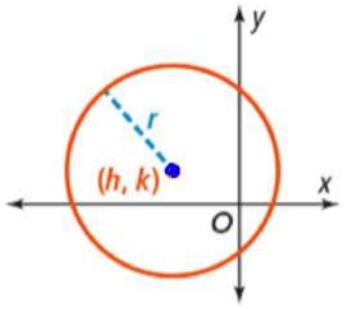
Equation of a Circle

An equation of a circle with center (h, k) and radius r is

$(x - h)^2 + (y - k)^2 = r^2$.

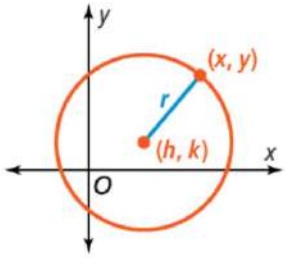
PROOF: SEE EXERCISE 13.

If...



Then... $(x - h)^2 + (y - k)^2 = r^2$

Use variables that can apply to any circle on the coordinate plane. Draw a circle with point $(h,\ k)$ as the center of the circle. Then select any point $(x,\ y)$ on the circle.



Use the Distance Formula to find the distance r between the two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

Because the radius is the same from the center to any point $(x,\ y)$ on the circle, this equation satisfies all points of the circle.

$(x-h)^2+(y-k)^2=r^2$

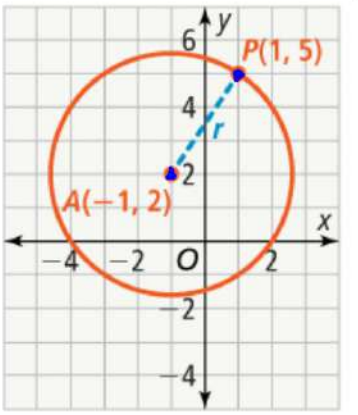
1. What are the radius and center of the circle with the equation $(x-2)^2+(y-3)^2=25$?

Center $(2,3)$

$r^2=25$

$r=5$

What is the equation for $\odot A$?



SOLUTION

$(-1, 2)$

$h \quad k$

$r = \sqrt{13}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - (-1))^2 + (5 - 2)^2}$

$\sqrt{(2)^2 + (3)^2}$

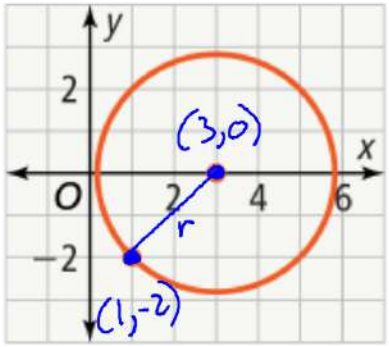
$\sqrt{4 + 9} = \sqrt{13}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 1)^2 + (y - 2)^2 = (\sqrt{13})^2$

$(x + 1)^2 + (y - 2)^2 = 13$

2. a. What is the equation for the circle?



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-1)^2 + (0-(-2))^2}$$

$$= \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4} = \sqrt{8}$$

$(h, k) \rightarrow (3, 0)$

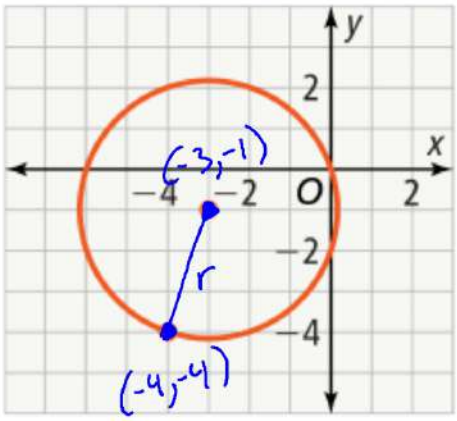
$r = \sqrt{8}$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-0)^2 = (\sqrt{8})^2$$

$$(x-3)^2 + y^2 = 8$$

2. b. What is the equation for the circle?



$r = \sqrt{(-3-(-4))^2 + (-1-(-4))^2}$

$= \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1+9} = \sqrt{10}$

$(h, k) \rightarrow (-3, -1)$

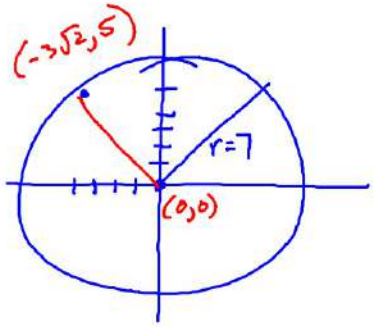
$r = \sqrt{10}$

$(x+3)^2 + (y+1)^2 = (\sqrt{10})^2$

$(x+3)^2 + (y+1)^2 = 10$

Circle Q has radius 7 and is centered at the origin. Does the point $(-3\sqrt{2}, 5)$ lie on $\odot Q$?

$(-4.24, 5)$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(0 - (-3\sqrt{2}))^2 + (0-5)^2}$$

$$\sqrt{(3\sqrt{2})^2 + (-5)^2}$$

$$\sqrt{18 + 25}$$

$$\sqrt{43}$$

$(-3\sqrt{2}, 5)$ Not on Circle

$$(3\sqrt{2})^2 = 3^2 \cdot (\sqrt{2})^2$$

9.2

18

3. Determine whether each point lies on the given circle.

a. $(-3, \sqrt{11})$; circle with center at the origin and radius $2\sqrt{5}$

$(0,0)$ $x_1\ y_1$ $\quad$ $x_2\ y_2$

$$d = \sqrt{(0-(-3))^2 + (0-\sqrt{11})^2}$$

$$= \sqrt{(3)^2 + (-\sqrt{11})^2}$$

$$= \sqrt{9+11}$$

$$= \sqrt{20} = \sqrt{4}\cdot\sqrt{5} = 2\sqrt{5}$$

$(-3, \sqrt{11})$ is on Circle

$(6, 3)$; circle with center at $(2, 4)$ and radius $3\sqrt{3}$

$x_2\ y_2$ $\quad$ $x_1\ y_1$

$$d = \sqrt{(2-6)^2 + (4-3)^2}$$

$$= \sqrt{(-4)^2 + (1)^2}$$

$$= \sqrt{16+1} = \sqrt{17}$$

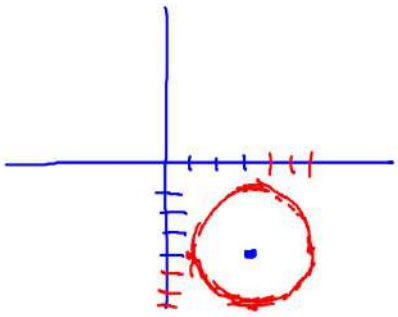
$(6,3)$ is Not on Circle

$(x-h)^2+(y-k)^2=r^2$

What is the graph of $(x-3)^2+(y+4)^2=9$?

Center $(3,-4)$

radius $= 3$

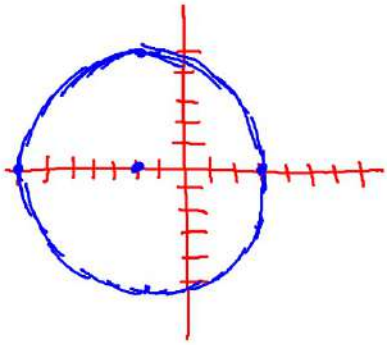


4. What is the graph of the circle?

a. $(x + 2)^2 + y^2 = 25$

Center $(-2, 0)$

radius $= 5$

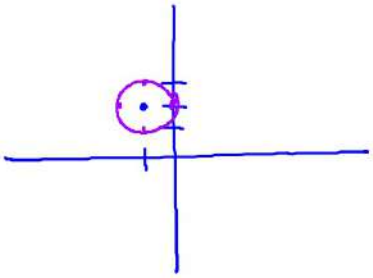


4. What is the graph of the circle?

b. $(x+1)^2 + (y-2)^2 = 1$

Center $(-1, 2)$

Radius $= 1$



5. What are the center and radius of the circle with equation $(x-4)^2+(y-9)^2=1$?

Center = (4, 9)

Radius = 1

6. What is the equation for the circle with center (6, 2) and radius 8?

h k r

$$(x-6)^2+(y-2)^2=8^2$$

$$(x-6)^2+(y-2)^2=64$$

8. What is the equation for the circle with center $(-9,\ 5)$ and radius 4?

$(x+9)^2+(y-5)^2=16$

Write equation of Circle with Center at $(-3,-1)$ and a diameter of 20.

$(h,k) \Rightarrow (-3,-1)$

Radius $= 10$

$$(x+3)^2+(y+1)^2=10^2$$

$$(x+3)^2+(y+1)^2=100$$