LESSON 4.3

NAME _____ DATE _____

Practice with Examples

For use with pages 214–221

GOAL **Determine the area of a triangle, evaluate determinants of 2 × 2 and 3 × 3 matrices, and use Cramer's Rule to solve systems of linear equations**

VOCABULARY

The **determinant** of a square matrix A is denoted by $\det A$ or $|A|$.

Cramer's Rule is a method of solving a system of linear equations using the determinants of the coefficient matrix of the linear system.

The entries in the **coefficient matrix** are the coefficients of the variables in the same order.

The area of a triangle with vertices (x_1, y_2), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

Determinant of a 2 × 2 Matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 2×2 matrix is the difference of the products of the entries on the diagonals.

Determinant of a 3 × 3 Matrix

1. Repeat the first two columns to the right of the determinant.
2. Subtract the sum of the products of the entries on the diagonals going *up* from left to right from the sum of the products of the entries on the diagonals going *down* from left to right.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

Chapter 4

Copyright © McDougal Littell Inc.
All rights reserved.

LESSON 4.3 CONTINUED

NAME _____ DATE _____

Practice with Examples

For use with pages 214–221

EXAMPLE 1 The Area of a Triangle

Find the area of the triangle with the vertices $A(-3, -1)$, $B(2, 0)$, $C(2, -4)$.

SOLUTION

$$\text{Area} = \pm\tfrac{1}{2}\begin{vmatrix} -3 & -1 & 1 & -3 & -1 \\ 2 & 0 & 1 & 2 & 0 \\ 2 & -4 & 1 & 2 & -4 \end{vmatrix}$$

Write the determinant, repeating the first two columns at the end.

$$= \pm\tfrac{1}{2}([(-3) + (-2) + (-8)] - [2 + 12 + (-2)])$$

Find the products of the diagonals.

$$= \pm\tfrac{1}{2}[-13 - 12]$$

Simplify.

$$= \tfrac{25}{2}$$

Multiply by $-\frac{1}{2}$ to get a positive value.

Exercises for Example 1

Find the area of the triangle with the given vertices.

1. $A(2, 3)$, $B(0, 5)$, $C(-1, -2)$

2. $A(0, 4)$, $B(3, 5)$, $C(-1, 4)$

3. $A(-1, -2)$, $B(2, 1)$, $C(0, 3)$

4. $A(1, 2)$, $B(2, 6)$, $C(3, 2)$

Copyright © McDougal Littell Inc.
All rights reserved.

NAME _____ DATE _____

Practice with Examples

For use with pages 214–221

EXAMPLE 2 Using Cramer's Rule for a 2 × 2 System

Use Cramer's Rule to solve this system: $3x - 2y = 22$, $x + 4y = -2$

SOLUTION

$\begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} = 12 - (-2) = 14$ Evaluate the determinant of the coefficient matrix.

$x = \dfrac{\begin{vmatrix} 22 & -2 \\ -2 & 4 \end{vmatrix}}{14} = \dfrac{88 - 4}{14} = 6$ Since the determinant is not 0, apply Cramer's Rule.

$y = \dfrac{\begin{vmatrix} 3 & 22 \\ 1 & -2 \end{vmatrix}}{14} = \dfrac{-6 - 22}{14} = -2$

The solution is $(6, -2)$.

✓ Check this solution in the original equations.

$3x - 2y = 22$ $\quad$ $x + 4y = -2$

$3(\mathbf{6}) - 2(-\mathbf{2}) \stackrel{?}{=} 22$ $\quad$ $\mathbf{6} + 4(-\mathbf{2}) \stackrel{?}{=} -2$

$22 = 22$ ✓ $\quad$ $-2 = -2$ ✓

Exercises for Example 2

Use Cramer's Rule to solve the linear system.

5. $2x + y = 1$, $-x + y = 7$

6. $3x + 4y = 2$, $2x + y = 3$

7. $x + y = 5$, $2x - y = 4$

8. $6x - 3y = 39$, $5x + 9y = -25$

9. $3x - 2y = 8$, $4x - 3y = 10$

10. $5x - 2y = -9$, $-7x + 3y = 14$

Copyright © McDougal Littell Inc.
All rights reserved.