10.1 Adding and Subtracting Polynomials

Goals • Add and subtract polynomials.
• Use polynomials to model real-life situations.

VOCABULARY

Polynomial A polynomial is an expression which is the sum of terms of the form ax^k where k is a nonnegative integer.

Standard form When the terms of a polynomial are placed in descending order, from largest degree to smallest degree, the polynomial is said to be in standard form.

Degree The degree of each term of a polynomial is the exponent of the variable.
Degree of a polynomial The degree of a polynomial is the largest degree of its terms.

Leading coefficient The leading coefficient of a polynomial is the coefficient of the first term when the polynomial is written in standard form.

Monomial A monomial is a polynomial with only one term.
Binomial A binomial is a polynomial with two terms.
Trinomial A trinomial is a polynomial with three terms.

Example 1 ***Identifying Polynomial Coefficients***

Identify the coefficients of $x + 3x^4 - 11x^3 - 9$.

Solution

First write the polynomial in standard form. Account for each degree, even if you must use a zero coefficient.

The coefficient of the x-term is 1 because $1 \cdot x = x$.

$x + 3x^4 - 11x^3 - 9 =$ $3x^4 + (-11)x^3 + 0x^2 + 1x + (-9)$

Answer The coefficients are 3, −11, 0, 1, and −9.

Example 2 *Classifying Polynomials*

Polynomial	Degree	Classified by Degree	Classified by Number of Terms
a. -3	0	constant	monomial
b. $-x + 1$	1	linear	binomial
c. $x^2 + 3$	2	quadratic	binomial
d. $5x^3 - 3x^2 + x - 8$	3	cubic	polynomial
e. $-x^4 + 2x^3 + 3$	4	quartic	trinomial

Example 3 *Adding Polynomials*

To add or subtract two polynomials, add or subtract the like terms. You can use a vertical format or a horizontal format.

Find the sum. Write the answer in standard form.

$(-3x^3 + 11x^2 - 8x + x^5 + 2) + (8x - 2x^4 + 7x^3 - 3 + 12x^2)$

Solution

Write each expression in standard form. Align like terms.

$$\begin{array}{r} x^5 - 3x^3 + 11x^2 - 8x + 2 \\ -2x^4 + 7x^3 + 12x^2 + 8x - 3 \\ \hline x^5 - 2x^4 + 4x^3 + 23x^2 - 1 \end{array}$$

Example 4 *Subtracting Polynomials*

When subtracting one polynomial from another, don't forget to distribute the subtraction sign to each term of the polynomial that's being subtracted.

Find the difference.

$(11x^4 + x^3 - x + 5) - (-x^4 - x^2 + 2x + 8)$

Solution

$$\begin{array}{r} 11x^4 + x^3 - x + 5 \\ -(-x^4 - x^2 + 2x + 8) \end{array}$$

Add the opposite.

$$\begin{array}{r} 11x^4 + x^3 - x + 5 \\ +\,x^4 + x^2 - 2x - 8 \\ \hline 12x^4 + x^3 + x^2 - 3x - 3 \end{array}$$

Checkpoint Find the sum or difference.

1. $(2x^6 - x^5 + 3x^3 - 14x^2 + 13) + (7x^5 - x^4 + 9x^3 + 13x^2 + 2)$

$2x^6 + 6x^5 - x^4 + 12x^3 - x^2 + 15$

2. $(-x^3 - 5x^2 + x - 1) - (-x^3 + 3x^2 + 10x - 9)$

$-8x^2 - 9x + 8$

Example 5 *Adding Polynomials*

Population The resident populations, in thousands, of California and Nevada from 1995 through 2001 can be modeled by the following equations, where x is the number of years since 1990.

CA: $C = 4.8106x^4 - 155.662x^3 + 1855.45x^2 - 9171.3x + 47{,}623$

NV: $N = 0.8826x^4 - 27.659x^3 + 315.64x^2 - 1465.3x + 3924$

Find a model for the resident population P of California and Nevada combined for 1995 through 2001.

Solution

You can find a model P by adding the models C and N.

$$\begin{array}{r} 4.8106x^4 - 155.662x^3 + 1855.45x^2 - 9171.3x + 47{,}623 \\ +\ 0.8826x^4 - 27.659x^3 + 315.64x^2 - 1465.3x + 3924 \\ \hline 5.6932x^4 - 183.321x^3 + 2171.09x^2 - 10{,}636.6x + 51{,}547 \end{array}$$

Answer The model for the resident population P of California and Nevada combined for 1995 through 2001 is

$P = 5.6932x^4 - 183.321x^3 + 2171.09x^2 - 10{,}636.6x + 51{,}547$.