6.4 Solving Absolute-Value Equations and Inequalities

Goals • Solve absolute-value equations and inequalities.

Example 1 *Solving an Absolute-Value Equation*

Solve $|x + 3| = 11$.

Because $|x + 3| = 11$, the expression $x + 3$ can be equal to 11 or -11.

$x + 3$ is positive

$|x + 3| = 11$

$x + 3 = +11$

$x = 8$

$x + 3$ is negative

$|x + 3| = 11$

$x + 3 = -11$

$x = -14$

Answer The equation has two solutions: 8 and -14.

Check Substitute both values into the original equation.

$|8 + 3| = |11| = 11$ $\qquad$ $|-14 + 3| = |-11| = 11$

Example 2 *Solving an Absolute-Value Equation*

Solve $|3x - 7| + 5 = 9$.

Isolate the absolute-value expression on one side of the equation.

$3x - 7$ is positive

$|3x - 7| + 5 = 9$

$|3x - 7| = 4$

$3x - 7 = +4$

$3x = 11$

$x = \frac{11}{3} = 3\frac{2}{3}$

$3x - 7$ is negative

$|3x - 7| + 5 = 9$

$|3x - 7| = 4$

$3x - 7 = -4$

$3x = 3$

$x = 1$

Answer The equation has two solutions: $3\frac{2}{3}$ and 1. Check these solutions in the original equation.

SOLVING ABSOLUTE-VALUE EQUATIONS AND INEQUALITIES

Each absolute-value equation or inequality is rewritten as two equations or two inequalities joined by *and* or *or*.

- $|ax + b| < c$ **means** $ax + b$ ___<___ c *and* $ax + b$ ___>___ $-c$.
- $|ax + b| \leq c$ **means** $ax + b$ ___≤___ c *and* $ax + b$ ___≥___ $-c$.
- $|ax + b| = c$ **means** $ax + b$ ___=___ c *or* $ax + b$ ___=___ $-c$.
- $|ax + b| > c$ **means** $ax + b$ ___>___ c *or* $ax + b$ ___<___ $-c$.
- $|ax + b| \geq c$ **means** $ax + b$ ___≥___ c *or* $ax + b$ ___≤___ $-c$.

Notice that when an absolute value is *less than* a number, the inequalities are connected by *and*. When an absolute value is *greater than* a number, the inequalities are connected by *or*.

Example 3 *Solving an Absolute-Value Inequality*

Solve $|5x - 7| \leq 23$.

Solution

$5x - 7$ is positive

$|5x - 7| \leq 23$

$5x - 7 \leq$ ___+23___

$5x \leq$ ___30___

$x \leq$ ___6___

$5x - 7$ is negative

$|5x - 7| \leq 23$

$5x - 7 \geq$ ___−23___ ← **Reverse inequality symbol.**

$5x \geq$ ___−16___

$x \geq$ ___$-\frac{16}{5}$___ = ___$-3\frac{1}{5}$___

Answer The solution is all real numbers ___greater than or equal to $-3\frac{1}{5}$ and less than or equal to 6___, which can be written as ___$-3\frac{1}{5} \leq x \leq 6$___.

Example 4 *Solving an Absolute-Value Inequality*

Solve $|x - 11| + 2 > 8$.

Solution

$x - 11$ is positive	$x - 11$ is negative	
$\|x - 11\| + 2 > 8$	$\|x - 11\| + 2 > 8$	
$\|x - 11\| > \underline{6}$	$\|x - 11\| > \underline{6}$	
$x - 11 > \underline{+6}$	$x - 11 < \underline{-6}$	← Reverse inequality symbol.
$x > \underline{17}$	$x < \underline{5}$	

Answer The solution is all real numbers greater than 17 or less than 5, which can be written as the compound inequality $x < 5$ or $x > 17$.

✔ ***Checkpoint*** **Solve the equation.**

1. $\|x - 3\| = 7$ 10 and −4	**2.** $\|4x + 10\| = 34$ 6 and −11

Solve the inequality.

3. $\|3x\| \geq 18$ $x \leq -6$ or $x \geq 6$	**4.** $\|x - 7\| < 16$ $-9 < x < 23$