

5.3

Writing Linear Equations Given Two Points

- Goals**
- Write an equation of a line given two points on the line.
 - Use a linear equation to model a real-life problem.

WRITING AN EQUATION OF A LINE GIVEN TWO POINTS

Step 1 Find the slope. Substitute the coordinates of the two given points into the formula for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Step 2 Find the y-intercept. Substitute the slope m and the coordinates of one of the points into the slope-intercept form, $y = mx + b$. Then solve for the y-intercept b .

Step 3 Write an equation of the line. Substitute the slope m and the y-intercept b into the slope-intercept form, $y = mx + b$.

Example 1 Writing an Equation Given Two Points

Write an equation of the line that passes through the points $(2, 8)$ and $(-5, 1)$.

Find the slope of the line. Let $(x_1, y_1) = (2, 8)$ and $(x_2, y_2) = (-5, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 8}{-5 - 2} = \frac{-7}{-7} = 1$$

Find the y-intercept. Let $m = 1$, $x = 2$, and $y = 8$ and solve for b .

$$y = mx + b$$

Write slope-intercept form.

$$8 = (1)(2) + b$$

Substitute for m , x , and y .

$$8 = 2 + b$$

Simplify.

$$6 = b$$

Solve for b .

Write an equation of the line.

$$y = mx + b$$

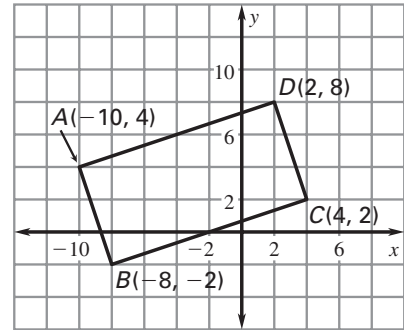
Write slope-intercept form.

$$y = 1x + 6$$

Substitute for m and b .

Example 2
Writing Equations of Perpendicular Lines

Geometry Connection Two different nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other.



- Show that \overline{AB} and \overline{BC} are perpendicular sides of $ABCD$.
- Write equations for the lines containing \overline{AB} and \overline{BC} .

Solution

- Find the slopes.

$$\text{For } \overline{AB}: m = \frac{4 - (-2)}{-10 - (-8)} = \frac{6}{-2} = -3$$

$$\text{For } \overline{BC}: m = \frac{2 - (-2)}{4 - (-8)} = \frac{4}{12} = \frac{1}{3}$$

Answer \overline{AB} and \overline{BC} are perpendicular because $\frac{1}{3}$ is the negative reciprocal of -3 .

- Find the y -intercepts of the lines containing \overline{AB} and \overline{BC} . Substitute the slopes from part (a) and the coordinates of one point into $y = mx + b$.

For \overline{AB}

$$y = mx + b$$

$$-2 = (-3)(-8) + b$$

$$-26 = b$$

For \overline{BC}

$$y = mx + b$$

$$-2 = \left(\frac{1}{3}\right)(-8) + b$$

$$\frac{2}{3} = b$$

Write an equation in slope-intercept form by substituting for m and b .

Equation of line for \overline{AB}

$$y = mx + b$$

$$y = -3x - 26$$

Equation of line for \overline{BC}

$$y = mx + b$$

$$y = \frac{1}{3}x + \frac{2}{3}$$

- ✓ **Checkpoint** Write an equation in slope-intercept form of the line that passes through the points.

1. $(-7, 3), (-4, 1)$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

2. $(5, -6), (-2, 1)$

$$y = -x - 1$$

3. $(-2, -9), (6, -3)$

$$y = \frac{3}{4}x - \frac{15}{2}$$

4. Write an equation of a line through $(5, 4)$ that is perpendicular to $y = 3x - 4$.

$$y = -\frac{1}{3}x + \frac{17}{3}$$