Chapter 4: Applications of Derivatives

4.5: Linearization

What you'll Learn About
Linearization is another term for tangent line
Differentials are part of the derivative
Mean Value Theorem

Tangent Line

a) Find the linearization of the function. b) Find L(a + .1) and f(a + .1)
c) Using concavity, determine if the Tangent Line at a is an overestimate or an underestimate. Justify your answer.

1. $f(x) = x^3 - 2x + 3 \quad a = 1$

a) (1, 2) $f'(x) = 3x^2 - 2$ $f'(1) = 1$

$L(x) = 2 + 1(x-1)$ ✓

b) $f(1.1) = 2.131$ $L(1.1) = 2.1$

c) $f''(x) = 6x$ $f''(1) = 6 > 0$ f(x) Concave up L(x) underestimate

2. $f(x) = x + \frac{1}{x} \quad a = 2$

a) (2, 2.5) $f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$f'(2) = \frac{3}{4}$

$L(x) = 2.5 + \frac{3}{4}(x-2)$

b) $L(2.1) = 2.5 + .75(.1) = 2.575$

$f(2.1) = 2.1 + \frac{1}{2.1} = 2.576$

c) $f''(x) = 2x^{-3} = \frac{2}{x^3}$

$f''(2) = \frac{2}{8} > 0$ Concave up

L(x) is an underestimate