Limits - Cheat Sheet

Generally, $\lim\limits_{x \to c} f(x) = f(c)$. However, if it appears the limit does not exist (DNE) and the

value is in the indeterminate form $\dfrac{0}{0}$, try substituting into an equivalent expression (simplify/

factor/ conjugate)

$$\lim\limits_{x \to 0^+} \dfrac{1}{x} = \infty \qquad \lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty \qquad \lim\limits_{x \to \infty} \dfrac{1}{x^n} = 0 \qquad \lim\limits_{x \to \infty} x^n = \infty$$

Existence of a Limit: Let f be a function and let c and L be real number

$$\lim\limits_{x \to c} f(x) = L \quad \Leftrightarrow \quad \lim\limits_{x \to c^-} f(x) = L = \lim\limits_{x \to c^+} f(x)$$

(left-sided limit = right-sided limit)

Vertical Asymptotes:
If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach $-\infty$ or ∞.

Horizontal asymptotes:
$\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)}$ will produce a horizontal asymptote at
- $y = 0$ if g increases faster than f.
- $y = \dfrac{a}{b}$ if g and f are increasing at the relative same amount where a and b are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Continuity: A function f is continuous at $x = c$ if:

A) $f(x)$ is defined at $x = c$ \qquad B) $\lim\limits_{x \to c} f(x)$ exists \qquad C) $\lim\limits_{x \to c} f(x) = f(c)$

Three types of discontinuities: point discontinuities (a "hole" – this is removable), jump discontinuities (differing behavior on left/right), vertical asymptotes (both non-removable)

Intermediate Value Theorem: If f is continuous on the closed interval $[a, b]$ then for any number k between $f(a)$ and $f(b)$, there exists $c \in [a, b]$ with $f(c) = k$. (This means f takes on EVERY value between $f(a)$ and $f(b)$).

Limits - Cheat Sheet

Finding limits graphically

https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-continuity#ab-limits-graphically

https://www.youtube.com/watch?v=55Udw8r3Txw

https://calcworkshop.com/limits/finding-limits-graphically/

http://calculus.flippedmath.com/11-limits-graphically.html

Finding limits analytically/algebraically

https://www.youtube.com/watch?v=hewJikMkYFc

http://calculus.flippedmath.com/12-limits-analytically.html

Solving continuity of piecewise functions

http://www.sosmath.com/calculus/limcon/limcon05/limcon05.html#answer1

https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/continuitydirectory/Continuity.html

Intermediate Value Theorem

https://www.khanacademy.org/math/ap-calculus-ab/ab-existence-theorems/ab-ivt-evt/a/intermediate-value-theorem-review

https://www.mathsisfun.com/algebra/intermediate-value-theorem.html

https://www.youtube.com/watch?v=g9QRNbJLs94

https://www.youtube.com/watch?v=KBWnrApb9HE

https://www.youtube.com/watch?v=Rpug_8nTqyw