Transforming Quadratic Functions

Common Core Math Standards

The student is expected to:

CA CC F-BF.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Also F-BF.1, F-BF.4, F-IF.4, F-IF.2

Mathematical Practices

CA CC MP.8 Patterns

Language Objective

Students work in pairs or small groups to both give and listen to oral clues about graphs of quadratic functions.

ENGAGE

Essential Question: How can you obtain the graph of $g(x) = a(x - h)^2 + k$ from the graph of $f(x) = x^2$?

Possible answer: Identify the vertex (h, k) and use the sign of a to determine whether the graph opens up or down. Generate a few points on one side of the vertex and sketch the graph using those points and symmetry.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and how the path of a ball used in sports can be modeled by a quadratic function. Then preview the Lesson Performance Task.

Name _____ Class _____ Date _____

19.2 Transforming Quadratic Functions

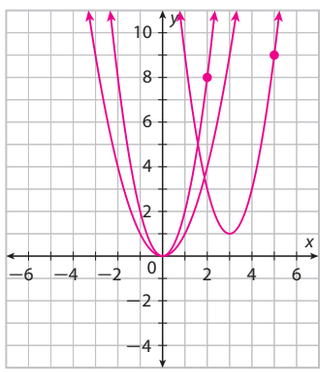
Resource Locker

Essential Question: How can you obtain the graph of $g(x) = a(x - h)^2 + k$ from the graph of $f(x) = x^2$?

Explore Understanding Quadratic Functions of the Form $g(x) = a(x - h)^2 + k$

Every quadratic function can be represented by an equation of the form $g(x) = a(x - h)^2 + k$. The values of the parameters a, h, and k determine how the graph of the function compares to the graph of the parent function, $y = x^2$. Use the method shown to graph $g(x) = 2(x - 3)^2 + 1$ by transforming the graph of $f(x) = x^2$.

A Graph $f(x) = x^2$.



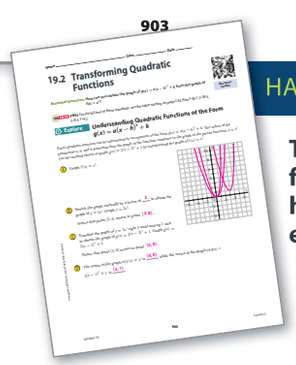
B Stretch the graph vertically by a factor of 2 to obtain the graph of $y = 2x^2$. Graph $y = 2x^2$.

Notice that point $(2, 4)$ moves to point (2, 8).

C Translate the graph of $y = 2x^2$ right 3 units and up 1 unit to obtain the graph of $g(x) = 2(x - 3)^2 + 1$. Graph $g(x) = 2(x - 3)^2 + 1$.

Notice that point $(2, 8)$ moves to point (5, 9).

D The vertex of the graph of $f(x) = x^2$ is (0, 0) while the vertex of the graph of $g(x) = 2(x - 3)^2 + 1$ is (3, 1).



HARDCOVER PAGES 709–716

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. **Discussion** Compare the minimum values of $f(x) = x^2$ and $g(x) = 2(x - 3)^2 + 1$. How is the minimum value related to the vertex?

The minimum value of $f(x) = x^2$ is 0 and the minimum value $g(x) = 2(x - 3)^2 + 1$ is 1. The minimum value is the y-coordinate of the vertex.

2. **Discussion** What is the axis of symmetry of the function $g(x) = 2(x - 3)^2 + 1$? How is the axis of symmetry related to the vertex?

The axis of symmetry of $g(x) = 2(x - 3)^2 + 1$ is $x = 3$. The axis of symmetry always passes through the vertex of the parabola. The x-coordinate of the vertex gives the equation of the axis of symmetry of the parabola.

Explain 1 Understanding Vertical Translations

A **vertical translation** of a parabola is a shift of the parabola up or down, with no change in the shape of the parabola.

Vertical Translations of a Parabola

The graph of the function $f(x) = x^2 + k$ is the graph of $f(x) = x^2$ translated vertically.

If $k > 0$, the graph $f(x) = x^2$ is translated k units up.

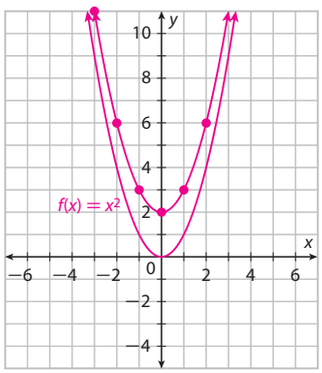
If $k < 0$, the graph $f(x) = x^2$ is translated $|k|$ units down.

Example 1 **Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.**

(A) $g(x) = x^2 + 2$

Make a table of values for the parent function $f(x) = x^2$ and for $g(x) = x^2 + 2$. Graph the functions together.

x	$f(x) = x^2$	$g(x) = x^2 + 2$
−3	9	11
−2	4	6
−1	1	3
0	0	2
1	1	3
2	4	6
3	9	11



The function $g(x) = x^2 + 2$ has a minimum value of 2.

The axis of symmetry of $g(x) = x^2 + 2$ is $x = 0$.

EXPLORE

Understanding Quadratic Functions of the Form $g(x) = a(x - h)^2 + k$

INTEGRATE TECHNOLOGY

Students have the option of completing the activity either in the book or online.

CONNECT VOCABULARY EL

This lesson discusses *translation* in terms of a transformation of a function graph. English learners may know about language *translation*. Discuss with students how the two meanings of *translate* are different.

EXPLAIN 1

Understanding Vertical Translations

QUESTIONING STRATEGIES

? How is the graph of $g(x) = x^2 + 2$ related to the graph of $g(x) = x^2 - 5$? **Both are translated graphs of the same parent function, $f(x) = x^2$, but $g(x) = x^2 + 2$ is translated 2 units up and $g(x) = x^2 - 5$ is translated 5 units down. So, the graph of $g(x) = x^2 + 2$ is 7 units higher than the graph of $g(x) = x^2 - 5$.**

? Is the vertex of the graph of $g(x) = x^2 + 2$ the same as the vertex of the graph of $g(x) = x^2 - 5$? **No; $g(x) = x^2 + 2$ has vertex $(0, 2)$, and $g(x) = x^2 - 5$ has vertex $(0, -5)$.**

PROFESSIONAL DEVELOPMENT

Math Background

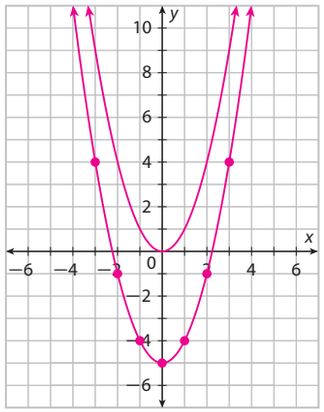
In this lesson, students graph the family of quadratic functions of the form $g(x) = a(x - h)^2 + k$ and compare those graphs to the graph of the parent function $f(x) = x^2$. Some key understandings are:

- The function $f(x) = x^2$ is the parent of the family of all quadratic functions.
- To graph a quadratic function of the form $g(x) = a(x - h)^2 + k$, identify the vertex (h, k). Then determine whether the graph opens upward or downward. Then generate points on either side of the vertex and sketch the graph of the function.

Ⓑ $g(x) = x^2 - 5$

Make a table of values for the parent function $f(x) = x^2$ and for $g(x) = x^2 - 5$. Graph the functions together.

x	$f(x) = x^2$	$g(x) = x^2 - 5$
−3	9	4
−2	4	−1
−1	1	−4
0	0	−5
1	1	−4
2	4	−1
3	9	4



The function $g(x) = x^2 - 5$ has a minimum value of -5.

The axis of symmetry of $g(x) = x^2 - 5$ is $x = 0$.

Reflect

3. How do the values in the table for $g(x) = x^2 + 2$ compare with the values in the table for the parent function $f(x) = x^2$?

For each x in the table, $g(x)$ is 2 greater than $f(x)$.

4. How do the values in the table for $g(x) = x^2 - 5$ compare with the values in the table for the parent function $f(x) = x^2$?

For each x in the table, $g(x)$ is 5 less than $f(x)$.

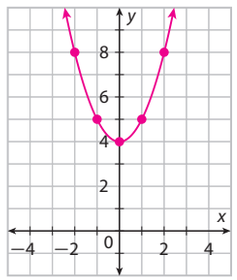
Your Turn

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

5. $g(x) = x^2 + 4$

The function $g(x) = x^2 + 4$ has a minimum value of 4.

The axis of symmetry for $g(x) = x^2 + 4$ is $x = 0$.



COLLABORATIVE LEARNING

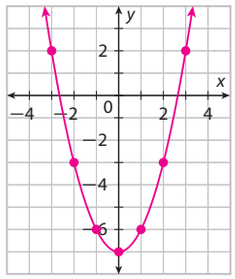
Peer-to-Peer Activity

Have students work in pairs. Have one student draw a graph of $y = x^2 + k$ for some value of k. The second student then writes the equation for the graph. Students then compare their results and determine whether the equation is correct for the given graph. Have students take turns in the two roles.

6. $g(x) = x^2 - 7$

The function $g(x) = x^2 - 7$ has a minimum value of -7.

The axis of symmetry for $g(x) = x^2 - 7$ is $x = 0$.



Explain 2 Understanding Horizontal Translations

A **horizontal translation** of a parabola is a shift of the parabola left or right, with no change in the shape of the parabola.

Horizontal Translations of a Parabola

The graph of the function $f(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

If $h > 0$, the graph $f(x) = x^2$ is translated h units right.

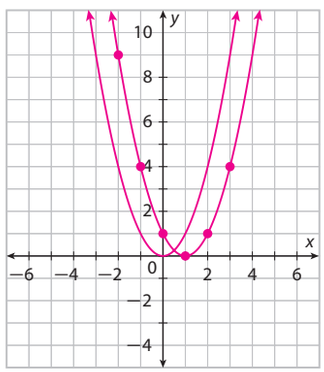
If $h < 0$, the graph $f(x) = x^2$ is translated $|h|$ units left.

Example 2 **Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.**

Ⓐ $g(x) = (x - 1)^2$

Make a table of values for the parent function $f(x) = x^2$ and for $g(x) = (x - 1)^2$. Graph the functions together.

x	$f(x) = x^2$	$g(x) = (x-1)^2$
−3	9	16
−2	4	9
−1	1	4
0	0	1
1	1	0
2	4	1
3	9	4



The function $g(x) = (x - 1)^2$ has a minimum value of 0.

The axis of symmetry of $g(x) = (x - 1)^2$ is $x = 1$.

EXPLAIN 2

Understanding Horizontal Translations

QUESTIONING STRATEGIES

? How is the graph of $g(x) = (x - 1)^2$ related to the graph of $g(x) = (x + 2)^2$? **Both are translated graphs of the same parent function, $f(x) = x^2$, but the graph of $g(x) = (x - 1)^2$ is translated 1 unit to the right and has vertex (1, 0), while the graph of $g(x) = (x + 2)^2$ is translated 2 units to the left and has vertex $(-2, 0)$. So, the graph of $g(x) = (x - 1)^2$ is 3 units to the right of the graph of $g(x) = (x + 2)^2$.**

? What is the vertex of the graph of $g(x) = (x - h)^2$? **$(h, 0)$**

DIFFERENTIATE INSTRUCTION

Visual Cues

Have students take a coordinate grid and label it "Vertex of $g(x) = (x - h)^2 + k$." Have them place these points, labels, and functions into the four quadrants.

$(2, 3)$	$h = 2, k = 3$	$g(x) = (x - 2)^2 + 3$
$(-2, 3)$	$h = -2, k = 3$	$g(x) = (x + 2)^2 + 3$
$(-2, -3)$	$h = -2, k = -3$	$g(x) = (x + 2)^2 - 3$
$(2, -3)$	$h = 2, k = -3$	$g(x) = (x - 2)^2 - 3$

Students can use this graph as a reminder of how the location of the vertex and the function are related.

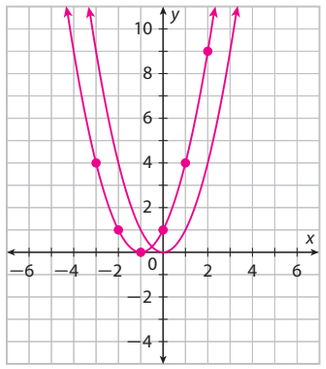
AVOID COMMON ERRORS

Students may forget that they can use a pattern to write equations from graphs. Remind students that adding k to x^2 moves the graph up for $k > 0$ or down for $k < 0$ and that *subtracting* h from x moves the graph left for $h < 0$ or right for $h > 0$. This is true for all nonzero values of k and h.

Ⓑ $g(x) = (x+1)^2$

Make a table of values and graph the functions together.

x	$f(x) = x^2$	$g(x) = (x+1)^2$
−3	9	4
−2	4	1
−1	1	0
0	0	1
1	1	4
2	4	9
3	9	16



The function $g(x) = (x+1)^2$ has a minimum value of 0.

The axis of symmetry of $g(x) = (x+1)^2$ is $x = -1$.

Reflect

7. How do the values in the table for $g(x) = (x-1)^2$ compare with the values in the table for the parent function $f(x) = x^2$?

For each x in the table, $g(x)$ is the same as $f(x-1)$.

8. How do the values in the table for $g(x) = (x+1)^2$ compare with the values in the table for the parent function $f(x) = x^2$?

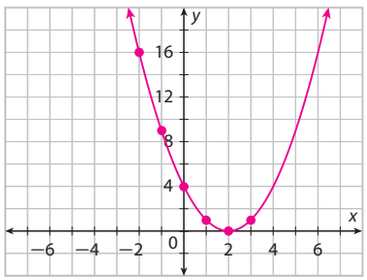
For each x in the table, $g(x)$ is the same as $f(x+1)$.

Your Turn

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

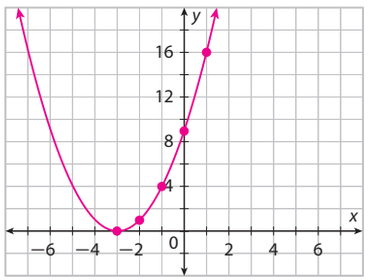
9. $g(x) = (x-2)^2$

Minimum: 0; axis of symmetry: $x = 2$



10. $g(x) = (x+3)^2$

Minimum: 0; axis of symmetry: $x = -3$



LANGUAGE SUPPORT EL

Communicate Math

Have each student sketch a graph of a parabola on a card, and write a quadratic function in any form on another card. Then have them write clues about the graph and about the function. For example, "The parabola opens upward/downward. Its axis of symmetry is _____; the vertex is at the point _____. The function's graph will open downward/upward." Provide sentence stems if needed to help students begin their clues. Collect the graph and function cards in one pile, and the clue cards in another. Have other students match graph and function cards to fit the clues.

Explain 3 Graphing $g(x) = a(x - h)^2 + k$

The **vertex form of a quadratic function** is $g(x) = a(x - h)^2 + k$, where the point (h, k) is the vertex. The *axis of symmetry* of a quadratic function in this form is the vertical line $x = h$.

To graph a quadratic function in the form $g(x) = a(x - h)^2 + k$, first identify the vertex (h, k). Next, consider the sign of a to determine whether the graph opens upward or downward. If a is positive, the graph opens upward. If a is negative, the graph opens downward. Then generate two points on each side of the vertex. Using those points, sketch the graph of the function.

Example 3 **Graph each quadratic function.**

Ⓐ $g(x) = -3(x + 1)^2 - 2$

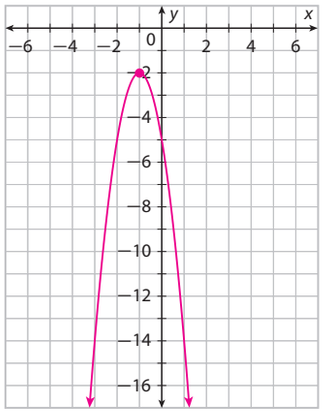
Identify the vertex.

The vertex is at $(-1, -2)$.

Make a table for the function. Find two points on each side of the vertex.

x	−3	−2	−1	0	1
$g(x)$	−14	−5	−2	−5	−14

Plot the points and draw a parabola through them.



Ⓑ $g(x) = 2(x - 1)^2 - 7$

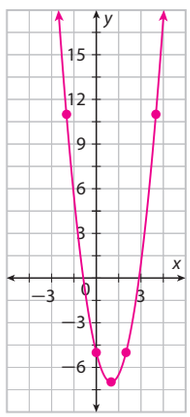
Identify the vertex.

The vertex is at (1, −7).

Make a table for the function. Find two points on each side of the vertex.

x	−2	0	1	2	4
$g(x)$	11	−5	−7	−5	11

Plot the points and draw a parabola through them.



Reflect

11. How do you tell from the equation whether the vertex is a maximum value or a minimum value?

If the value of a is positive, the vertex is a minimum value. If the value of a is negative, the vertex is a maximum value.

EXPLAIN 3

Graphing $g(x) = a(x - h)^2 + k$

QUESTIONING STRATEGIES

What can you tell about the graph of a function from its equation in the form $g(x) = a(x - h)^2 + k$? **the location of the vertex and whether the graph opens upward or downward**

What are the domain and range for a quadratic function whose graph opens downward? **The domain is all real numbers, and the range is the set of all values less than or equal to the maximum value.**

AVOID COMMON ERRORS

Students may try to graph a quadratic function of the form $g(x) = a(x - h)^2 + k$ by using a value other than $x = h$. Remind them that they need to first identify and plot the vertex. Then they should identify and plot other points and use the plotted points to draw a parabola.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Tell students that a transformed quadratic function models the height of an object dropped from a given height, based upon the time since it was dropped. Sketch a quadratic function that models the situation, and draw students' attention to the vertex $(0, k)$ being the maximum point of the graph. Ask about the sign of a in the function $g(x) = ax^2 + k$, and note that the values to the left of the y-axis are not considered.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Give students a function in the form $g(x) = a(x-h)^2 + k$. Have students use the whiteboard to identify and plot the vertex and then identify and plot other points on the graph before drawing the graph of the function.

SUMMARIZE

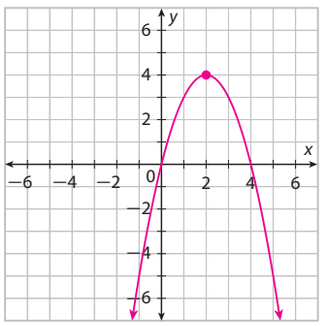
? How do you graph a quadratic function of the form $g(x) = a(x-h)^2 + k$? **First, identify and plot the vertex. Then, identify and plot other points on the graph. Finally, draw the graph.**

Your Turn

Graph each quadratic function.

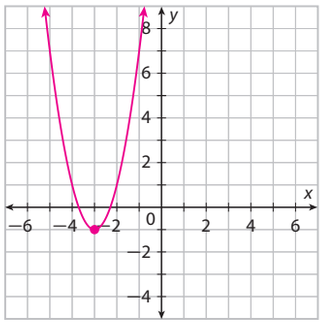
12. $g(x) = -(x-2)^2 + 4$

x	−1	0	2	4	5
$g(x)$	−5	0	4	0	−5



13. $g(x) = 2(x+3)^2 - 1$

x	−5	−4	−3	−2	−1
$g(x)$	7	1	−1	1	7



Elaborate

14. How does the value of k in $g(x) = x^2 + k$ affect the translation of $f(x) = x^2$?
If $k > 0$, the graph $f(x) = x^2$ is translated k units up.
If $k < 0$, the graph $f(x) = x^2$ is translated k units down.

15. How does the value of h in $g(x) = (x-h)^2$ affect the translation of $f(x) = x^2$?
If $h > 0$, the graph $f(x) = x^2$ is translated h units right.
If $h < 0$, the graph $f(x) = x^2$ is translated h units left.

16. In $g(x) = a(x-h)^2 + k$, what are the coordinates of the vertex?
(h, k)

17. **Essential Question Check-In** How can you use the values of a, h, and k, to obtain the graph of $g(x) = a(x-h)^2 + k$ from the graph $f(x) = x^2$?
The graph of $f(x) = x^2$ is stretched or compressed by a factor of $|a|$, and reflected across the x-axis if a is negative; it is translated h units horizontally and k units vertically.