Pre-calc: Algebra 2 Unit 3 Name:

Lesson 6 Closed Minded part 2

A **polynomial** function has the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where n is a natural number and all of the as are real numbers.

These are polynomials:	These are not polynomials:
$f(x) = x^3$	$g(x) = 3^x$
$f(x) = 2x^2 + 5x - 12$	$g(x) = \dfrac{2x^2}{x^2 - 3x + 2}$
$f(x) = -x^3 + 3x^2 - 2x - 7$	$g(x) = x^3 + 3x^2 - 2x + 10x^{-1} - 7$
$f(x) = \frac{1}{2}x$	$g(x) = \dfrac{1}{2x}$
$f(x) = x^2$	$g(x) = x^{\frac{1}{2}}$

You can add, subtract, multiply, and divide polynomials. Why not think of them as a number system?

For each of the following, decide whether the claim is true or false. If it is true, explain how you know that it has to be true. If it is false, provide a counterexample to show that it is false.

1. The sum of a $\underbrace{\text{quadratic}}_{\text{degree 2}}$ polynomial function and a $\underbrace{\text{linear}}_{\text{degree 1}}$ polynomial function is a $\underbrace{\text{cubic}}_{\text{degree 3}}$ polynomial function.

false! For example, $\underbrace{x^2}_{\text{quadratic}} + \underbrace{x}_{\text{linear}} = \underbrace{x^2 + x}_{\text{quadratic}}$

2. The sum of a linear polynomial function and an exponential function is a polynomial function.

false! For example, $\underbrace{x}_{\text{linear}} + \underbrace{2^x}_{\text{exponential}} = x + 2^x$ → not a polynomial because x is in the exponent

3. A cubic polynomial function subtracted from a cubic polynomial function is a cubic polynomial function.

false! For example, $\underbrace{(x^3+1)}_{\text{cubic}} - \underbrace{(x^3 - x)}_{\text{cubic}} = \underbrace{x+1}_{\text{linear}}$

since it's not always true, this counts as false

(but! sometimes true. For example, $\underbrace{(2x^3+1)}_{\text{cubic}} - \underbrace{(x^3 - x)}_{\text{cubic}} = \underbrace{x^3 + x + 1}_{\text{cubic}}$)

4. A cubic polynomial function divided by a linear polynomial function is a quadratic polynomial function.

false! For example, $(x^3+1) \div (x+2)$

$$
\begin{array}{r}
x^2 - 2x + 4 \quad \text{rem } -7 \\
x+2 \overline{\smash{)}\, x^3 + 0x^2 + 0x + 1} \\
-(x^3 + 2x^2) \\
\hline
-2x^2 + 0x \\
-(-2x^2 - 4x) \\
\hline
4x + 1 \\
-(4x + 8) \\
\hline
-7
\end{array}
$$

so $\dfrac{x^3+1}{x+2} = x^2 - 2x + 4 + \dfrac{7}{x+2}$

this part makes it not a polynomial

5. The set of polynomial functions is closed under addition.

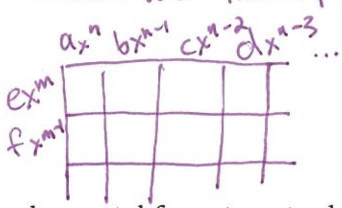
true! when adding two polynomials, you add like terms.
so the result will be a bunch of terms like $ax^n + bx^n = (a+b)x^n$.
if all the terms are like that, it's a polynomial
(note: real numbers like 5 and 0 count as polynomials of degree 0.)

6. The set of polynomial functions is closed under subtraction.

true! same as #5

7. The set of polynomial functions is closed under multiplication.

true! when we multiply polynomials, we get:



each term is something like $a \cdot e \cdot x^n \cdot x^m$
$= ae\,x^{n+m}$

so you end up with a polynomial

8. The set of polynomial functions is closed under division.

false! see #4. if you get a remainder, the result is not a polynomial.