Name:

Tips and Tricks for Proving Trig Identities

When you don't know if something's an identity, before you try to prove algebraically:

1. Check a random angle– if the identity holds, then you should go to prove it algebraically. If it doesn't– congrats, you have a counterexample to disprove it.

Where to start:

1. Work on just one side of the equation
2. Remember your algebra! Add fractions, factor, distribute, etc.
3. Get everything in terms of sines and cosines– change $\tan\theta$ to $\frac{\sin\theta}{\cos\theta}$, $\csc\theta$ to $\frac{1}{\sin\theta}$, etc. Does that get you anywhere?
4. If there's a 1 you don't know what to do with, change it to $\sin^2\theta + \cos^2\theta$.
5. If there's a $\sin(2\theta)$, or similar, use a double angle or sum of angles identity.
6. Don't be afraid to start over!

Common missteps:

1. When trying to prove the identity, do not assume that it is true.
2. It's true that $\sin\theta = \sqrt{1-\cos^2\theta}$. However, adding in square roots usually won't help you simplify.
3. Forgetting what you're trying to show in the first place

Example: Proving

$$\csc^2\theta = 1 + \cot^2\theta.$$

Proof.

$$\begin{aligned}
\csc^2\theta &= \frac{1}{\sin^2\theta} && \text{writing } \csc\theta \text{ as } \frac{1}{\sin\theta} \\
&= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} && \text{replacing 1 with } \sin^2\theta + \cos^2\theta \\
&= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} && \text{splitting up the fraction} \\
&= 1 + \cot^2\theta.
\end{aligned}$$

□

① $\cos\theta \tan\theta \csc\theta = \sin^2\theta + \cos^2\theta$

Proof: $\cos\theta \tan\theta \csc\theta$

$= \cos\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}$ replace trig functions with sin/cos equivalents

$= \frac{\cos\theta \cdot \sin\theta}{\cos\theta \cdot \sin\theta}$ multiply fractions

cancel

$= 1$

Pythagorean identity

$= \sin^2\theta + \cos^2\theta$

$\square$

② $\frac{\cot^2\theta}{\csc\theta} \sec^2\theta \neq \tan\theta \sin\theta \csc^2\theta$ if $\theta = 19$

$$\frac{\frac{1}{(\tan(19))^2}}{\frac{1}{\sin(19)}} \cdot \frac{1}{(\cos(19))^2} \approx 6.67$$

$$\tan(19) \cdot \sin(19) \cdot \frac{1}{(\sin(19))^2} \approx 1.01$$

not an identity

(3) $\tan^2\theta \sin^2\theta = \tan^2\theta - \sin^2\theta$

Proof: $\tan^2\theta - \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$ replace tan with sin/cos

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$ get common denominator

$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$ combine fractions

$= \dfrac{\sin^2\theta\,(1-\cos^2\theta)}{\cos^2\theta}$ factor out $\sin^2\theta$ in numerator

$= \dfrac{\sin^2\theta \cdot \sin^2\theta}{\cos^2\theta}$ Pythagorean Identity

$= \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \sin^2\theta$ multiplying fractions

$= \tan^2\theta \cdot \sin^2\theta \quad \square$ replace sin/cos with tan

(4) $\dfrac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

Proof: $\dfrac{1+\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$ separate out added fractions (Distributive Law)

$= \csc\theta + \dfrac{\cos\theta}{\sin\theta}$ because $\csc = \dfrac{1}{\sin}$

$= \csc\theta + \cot\theta$ because $\dfrac{\cos}{\sin} = \cot$

$\square$

(5) $\dfrac{\sec\theta \sin\theta}{\tan\theta + \cot\theta} \neq \cos^2\theta$ if $\theta = 19$ radians

$$\frac{\frac{1}{\cos(19)} \cdot \sin(19)}{\tan(19) + \frac{1}{\tan(19)}} \approx 0.022$$

$\cos^2(19) \approx 0.978$ not an identity

(6) $\cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta = 1$

Proof: First, note that $(\cos^2\theta + \sin^2\theta)^2 = \cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta$

because:

	$\cos^2\theta$	$\sin^2\theta$
$\cos^2\theta$	$\cos^4\theta$	$\sin^2\theta\cos^2\theta$
$\sin^2\theta$	$\sin^2\theta\cos^2\theta$	$\sin^4\theta$

$= \cos^4 + 2\sin^2\theta\cos^2\theta + \sin^4\theta$

So then:

$\cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta$

$= (\cos^2\theta + \sin^2\theta)^2$ by above

$= (1)^2$ Pythagorean Identity

$= 1$ $\square$