Name:

Tips and Tricks for Proving Trig Identities

When you don't know if something's an identity, before you try to prove algebraically:

1. Check a random angle– if the identity holds, then you should go to prove it algebraically. If it doesn't– congrats, you have a counterexample to disprove it.

Where to start:

1. Work on just one side of the equation
2. Remember your algebra! Add fractions, factor, distribute, etc.
3. Get everything in terms of sines and cosines– change $\tan\theta$ to $\frac{\sin\theta}{\cos\theta}$, $\csc\theta$ to $\frac{1}{\sin\theta}$, etc. Does that get you anywhere?
4. If there's a 1 you don't know what to do with, change it to $\sin^2\theta + \cos^2\theta$.
5. If there's a $\sin(2\theta)$, or similar, use a double angle or sum of angles identity.
6. Don't be afraid to start over!

Common missteps:

1. When trying to prove the identity, do not assume that it is true.
2. It's true that $\sin\theta = \sqrt{1-\cos^2\theta}$. However, adding in square roots usually won't help you simplify.
3. Forgetting what you're trying to show in the first place

Example: Proving

$$\csc^2\theta = 1 + \cot^2\theta.$$

Proof.

$$\begin{aligned}
\csc^2\theta &= \frac{1}{\sin^2\theta} && \text{writing } \csc\theta \text{ as } \frac{1}{\sin\theta} \\
&= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} && \text{replacing 1 with } \sin^2\theta + \cos^2\theta \\
&= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} && \text{splitting up the fraction} \\
&= 1 + \cot^2\theta.
\end{aligned}$$

□

Practice

Prove or disprove the following identities.

1. $\cos\theta \tan\theta \csc\theta = \sin^2\theta + \cos^2\theta$

2. $\frac{\cot^2\theta}{\csc\theta} \sec^2\theta = \tan\theta \sin\theta \csc^2\theta$

3. $\tan^2\theta \sin^2\theta = \tan^2\theta - \sin^2\theta$

4. $\frac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

5. $\frac{\sec\theta\sin\theta}{\tan\theta+\cot\theta} = \cos^2\theta$

6. $\cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta = 1$