Lesson 5 Is This the End?

Name:

Board work

Consider the following functions.

$$f(x) = 2^x$$

$$h(x) = x^5 - 4x^2 + 1$$

$$p(x) = x^3 + x^2 - 4$$

$$k(x) = x + 30$$

$$g(x) = x^2 - 20$$

$$m(x) = x^4 - 1$$

$$r(x) = x^5$$

$$n(x) = \left(\frac{1}{2}\right)^x$$

$$q(x) = x^6$$

(I recommend doing the following problems in the numbered order.)

3. List the functions from greatest to least based on their output values when x is a very large negative number (as $x \to -\infty$.)

$$n(x) = \left(\tfrac{1}{2}\right)^x$$
$$q(x) = x^6$$
$$m(x) = x^4 - 1$$
$$g(x) = x^2 - 20$$
$$f(x) = 2^x$$
$$k(x) = x + 30$$
$$p(x) = x^3 + x^2 - 4$$
$$r(x) = x^5$$
$$h(x) = x^5 - 4x^2 + 1$$

higher even-order polynomials are at the top, higher odd-order polynomials are on the bottom

1. List the functions from greatest to least based on their output values when $x = 0$.

$$k(0) = 0 + 30 = 30$$

$$f(0) = 2^0 = 1$$
$$n(0) = \left(\tfrac{1}{2}\right)^0 = 1$$
$$h(0) = 0^5 - 4 \cdot 0^2 + 1 = 1$$

$$r(0) = 0^5 = 0$$
$$q(0) = 0^6 = 0$$

$$m(0) = 0^4 - 1 = -1$$

$$p(0) = 0^3 + 0^2 - 4 = -4$$

$$g(0) = 0^2 - 20 = -20$$

the y-intercept of a polynomial is the constant term at the end

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2. List the functions from greatest to least based on their output values when x is very very large. (We say "as x approaches infinity" or "as $x \to \infty$".)

$$f(x) = 2^x$$
$$q(x) = x^6$$
$$r(x) = x^5$$
$$h(x) = x^5 - 4x^2 + 1$$
$$m(x) = x^4 - 1$$
$$p(x) = x^3 + x^2 - 4$$
$$g(x) = x^2 - 20$$
$$k(x) = x + 30$$
$$n(x) = \left(\tfrac{1}{2}\right)^x$$

higher-order polynomials are higher

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Some of the functions are different now.

$$f(x) = 2^x$$
$$h(x) = -x^5 - 4x^2 + 1$$

$$r(x) = -x^5$$

$$p(x) = -x^3 + x^2 - 4$$
$$k(x) = -x + 30$$

$$n(x) = \left(\frac{1}{2}\right)^x$$

$$g(x) = -x^2 - 20$$
$$m(x) = -x^4 - 1$$

$$q(x) = -x^6$$

6. List the functions from greatest to least based on their output values when x is a very large negative number (as $x \to -\infty$.)

$$n(x) = \left(\tfrac{1}{2}\right)^x$$
$$r(x) = -x^5$$

$$h(x) = -x^5 - 4x^2 + 1$$

$$p(x) = -x^3 + x^2 - 4$$

$$k(x) = -x + 30$$

$$f(x) = 2^x$$

$$g(x) = -x^2 - 20$$

$$m(x) = -x^4 - 1$$

$$q(x) = -x^6$$

with the negative leading term, the order is switched compared to the previous list of functions (mostly) (just for the polynomials)

4. List the functions from greatest to least based on their output values when $x = 0$.

$$k(0) = 30$$

$$f(0) = 1$$
$$n(0) = 1$$
$$h(0) = 1$$

$$r(0) = 0.$$
$$g(0) = 0$$

$$m(0) = -1$$

$$p(0) = -4$$

$$g(0) = -20$$

the y-intercepts are all the same as last time

5. List the functions from greatest to least based on their output values when x is very very large (as $x \to \infty$.)

$$f(x) = 2^x$$

$$n(x) = \left(\tfrac{1}{2}\right)^x$$

$$k(x) = -x + 30$$

$$g(x) = -x^2 - 20$$

$$p(x) = -x^3 + x^2 - 4$$

$$m(x) = -x^4 - 1$$

$$r(x) = -x^5$$

$$h(x) = -x^5 - 4x^2 + 1$$

$$q(x) = -x^6$$

with the negative leading term, the order is switched from last time (for the polynomials) (mostly)

Table work

For each function, give its function type (including what degree of polynomial,) its y-intercept, and its end behavior.

7. $f(x) = 3 + 2x$ (polynomial

function type: linear degree 1)

$f(0) = 3$

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$

8. $f(x) = x^4 - 16$

function type: polynomial degree 4

$f(0) = -16$

as $x \to -\infty$, $f(x) \to \infty$

as $x \to \infty$, $f(x) \to \infty$

9. $f(x) = 3^x$

function type: exponential

$f(0) = 1$

as $x \to -\infty$, $f(x) \to 0$

as $x \to \infty$, $f(x) \to \infty$

10. $f(x) = x^3 + 2x^2 - x + 5$

function type: degree 3 polynomial

$f(0) = 5$

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$

11. $f(x) = -2x^3 + 2x^2 - x + 5$

function type: degree 3 polynomial

$f(0) = 5$

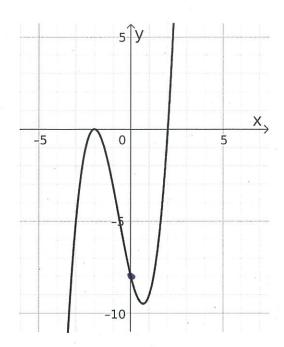as $x \to -\infty$, $f(x) \to \infty$

as $x \to \infty$, $f(x) \to -\infty$

12. $f(x) = \log_2 x$

function type: logarithm

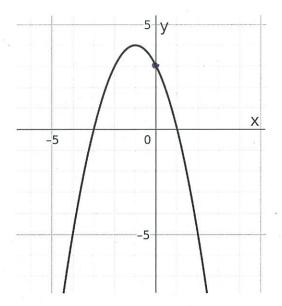$f(0) = $ undefined

as $x \to -\infty$, $f(x) \to$ undefined

as $x \to \infty$, $f(x) \to \infty$

13. $f(x)$:

function type: degree 3 polynomial

$f(0) = -8$

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$

14. $f(x)$:

function type: degree 2 polynomial (quadratic)

$f(0) = 3$

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to -\infty$

15. $f(x) = -(x - 3)(x - 3i)(x + 3i)$

function type: degree 3 polynomial

$f(0) = 27$

as $x \to -\infty$, $f(x) \to \infty$

as $x \to \infty$, $f(x) \to -\infty$

$f(0) = -(0-3)(0-3i)(0+3i)$

$= -(-3)(-3i)(3i)$

$= -(-3)(-9i^2)$

$= -(-3)(9)$

$= -(-27)$

$= 27$

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