Precalc: Algebra 2 Unit 3 Name:

Lesson 5 I May Be Irrational, But You're Imaginary

i and multiples of i are called **imaginary numbers**. Imaginary numbers can be added to real numbers to form **complex numbers**. Complex numbers are written as $a + bi$, where a and b are both real numbers.

1. Do these addition and subtraction problems. The answers may be complex numbers.

a. $(13 + 2i) + (-12 - 5i)$

$1 - 3i$

b. $(-7 + 12i) + (3 - 6i)$

$-4 + 6i$

c. $(8 - i) - (7 - 3i)$

$1 + 2i$

d. $(-9 + 4i) - (9 - 4i)$

$-18 + 8i$

2. Do these multiplication problems. Remember how to multiply binomials?

a. $(-4 + 2i)(3 - 5i)$

	-4	2i
3	-12	6i
-5i	20i	-10·-1

$-2 + 26i$

b. $(10 - 3i)(4 - 6i)$

	10	-3i
4	40	-12i
-6i	-60i	18·-1

$22 - 72i$

c. $(1 + 3i)(1 - 3i)$

	1	3i
1	1	3i
-3i	-3i	-9·-1

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d. $(3 + 5i)(3 - 5i)$

	3	5i
3	9	15i
-5i	-15i	-25·-1

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3. Now that you know about complex numbers, solve the following quadratic equations.

a. $x^2 + 4x + 5 = 0$

$a = 1 \quad b = 4 \quad c = 5$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2} = -2 \pm i$$

b. $x^2 - 3x + 4 = 0$

$a = 1 \quad b = -3 \quad c = 4$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$= \frac{3 \pm \sqrt{9 - 16}}{2}$$

$$= \frac{3 \pm \sqrt{-7}}{2}$$

$$= \frac{3}{2} \pm \frac{\sqrt{7}}{2} i$$

4. Using your answers to #4, write the following quadratic functions in factored form. Then multiply the factored forms out to confirm that they have the correct standard form.

a. $f(x) = x^2 + 4x + 5$

$f(x) = (x - (-2+i))(x - (-2-i))$

or

$f(x) = (x+2-i)(x+2+i)$

multiplying:

	x	2	$-i$
x	x^2	$2x$	$-ix$
2	$2x$	4	$-2i$
i	ix	$2i$	$+1$

$= x^2 + 4x + 5$ ✓

b. $g(x) = x^2 - 3x + 4$

$g(x) = \left(x - \left(\frac{3}{2} + \frac{\sqrt{7}}{2}i\right)\right)\left(x - \left(\frac{3}{2} - \frac{\sqrt{7}}{2}i\right)\right)$

or

$g(x) = \left(x - \frac{3}{2} - \frac{\sqrt{7}}{2}i\right)\left(x - \frac{3}{2} + \frac{\sqrt{7}}{2}i\right)$

multiplying:

	x	$-\frac{3}{2}$	$-\frac{\sqrt{7}}{2}i$
x	x^2	$-\frac{3}{2}x$	$-\frac{\sqrt{7}}{2}ix$
$-\frac{3}{2}$	$-\frac{3}{2}x$	$\frac{9}{4}$	$\frac{3\sqrt{7}}{4}i$
$\frac{\sqrt{7}}{2}i$	$\frac{\sqrt{7}}{2}ix$	$-\frac{3\sqrt{7}}{4}i$	$-\frac{7}{4}\cdot -1$

$x^2 - \frac{3}{2}x - \frac{3}{2}x + \frac{9}{4} + \frac{7}{4} = x^2 - 3x + \frac{16}{4}$

$= x^2 - 3x + 4$ ✓

Every complex number has a **complex conjugate.** The conjugate of $a + bi$ is $a - bi$.

5. Write the conjugate for each complex number:

a. $4 + 6i$ $4 - 6i$

b. $3 + 2i$ $3 - 2i$

c. $4.5 + 7.2i$ $4.5 - 7.2i$

d. $-3 - 9i$ $-3 + 9i$

e. $4 - 8i$ $4 + 8i$

f. $4 + 8i$ $4 - 8i$

6. Multiply each number by its conjugate.

a. $4 + 6i$

	4	$6i$
4	16	$24i$
$-6i$	$-24i$	$-36\cdot -1$

$= 52$

b. $3 + 2i$

	3	$2i$
3	9	$6i$
$-2i$	$-6i$	$-4\cdot -1$

$= 13$

c. $4.5 + 7.2i$

	4.5	$7.2i$
4.5	20.25	$32.4i$
$-7.2i$	$-32.4i$	$-51.84\cdot -1$

$= 72.09$

d. $-3 - 9i$

	-3	$-9i$
-3	9	$27i$
$9i$	$-27i$	$-81\cdot -1$

$= 90$

e. $4 - 8i$

	4	$-8i$
4	16	$-32i$
$8i$	$32i$	$-64\cdot -1$

$= 80$

f. $4 + 8i$

	4	$8i$
4	16	$32i$
$-8i$	$-32i$	$-64\cdot -1$

$= 80$