Name:

Lesson 4.5: Odds and Events

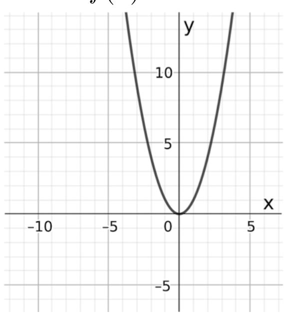
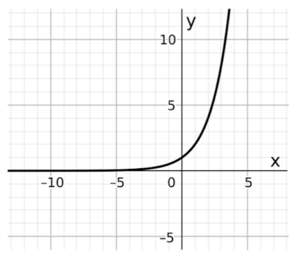
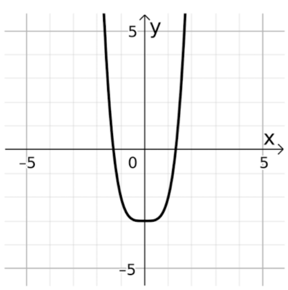
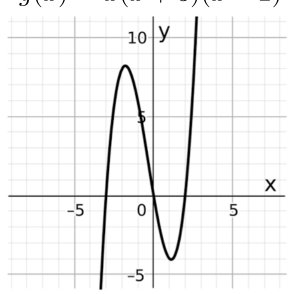
Prologue: A fun note on function notation:

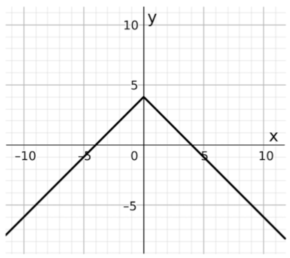
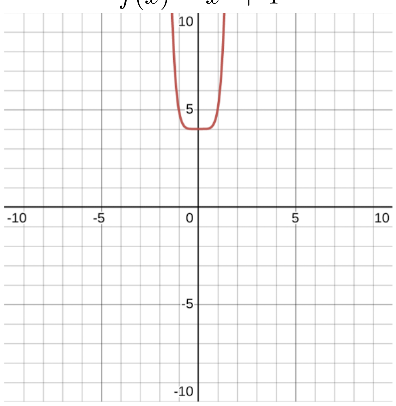
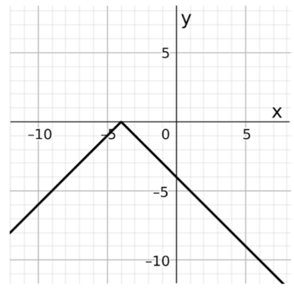
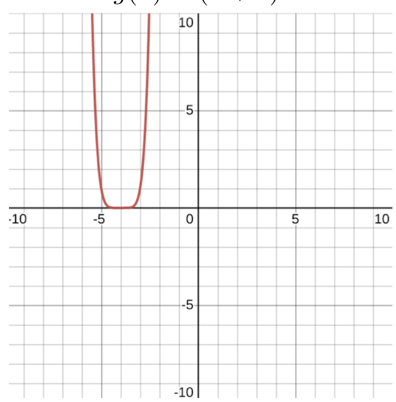
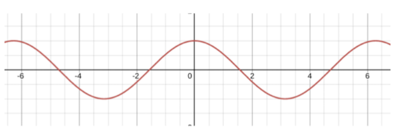
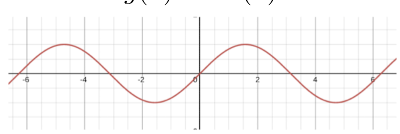
Recall: The expression $f(\star)$ means the *value of the function f when you plug in $\star$.*

Problem. For $f(x) = x^2 + 3$, find $f(-2)$, $f(17)$, $f(x+1)$, and $f(-x)$.

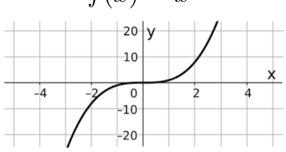
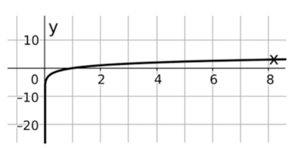
Act 1: Exploring and Investigating

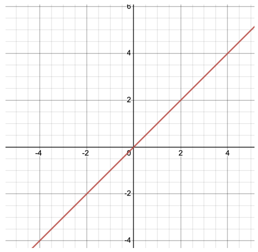
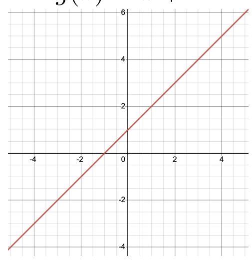
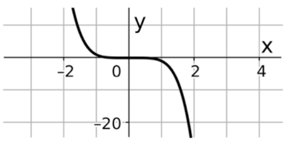
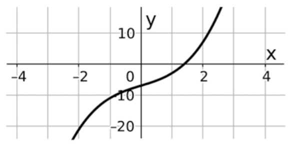
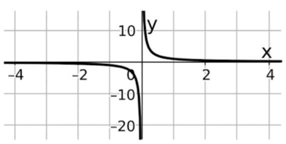
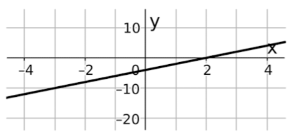
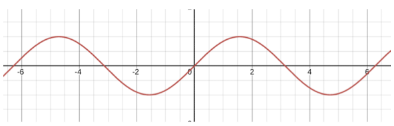
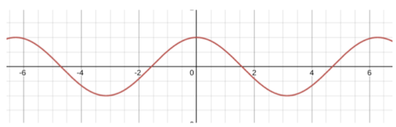
Based on the following examples, come up with a definition of an **even function**. Write your definitions on the whiteboard.

these functions are even	these functions are not even
$f(x) = x^2$ 	$g(x) = 2^x$ 
$f(x) = x^4 - 3$ 	$g(x) = x(x+3)(x-2)$ 

<table>
<tr><th>these functions are even</th><th>these functions are not even</th></tr>
<tr><td>$f(x) = -|x| + 4$

$f(x) = x^6 + 4$
</td><td>$g(x) = -|x + 4|$

$g(x) = (x + 4)^6$
</td></tr>
<tr><td>$f(2) = 5$ and $f(-2) = 5$
$f(x) = \cos(x)$
</td><td>$g(2) = 3$ and $g(-2) = 5$
$g(x) = \sin(x)$
</td></tr>
</table>

Based on the following examples, come up with a definition of an **odd function**. Write your definitions on the whiteboard.

<table>
<tr><th>these functions are odd</th><th>these functions are not odd</th></tr>
<tr><td>$f(x) = x^3$
</td><td>$g(x) = \log_2(x)$
</td></tr>
</table>

these functions are odd	these functions are not odd
$f(x) = x$ 	$g(x) = x + 1$ 
$f(x) = -x^5$ 	$g(x) = x^3 + 3x - 7$ 
$f(x) = \frac{1}{x}$ 	$g(x) = 2x - 3$ 
$f(2) = 3$ and $f(-2) = -3$	$g(2) = 3$ and $g(-2) = 5$
$f(x) = \sin(x)$ 	$g(x) = \cos(x)$ 

Act 2: Thinking

Problem (1)**.** Let $f(x)$ be an even function, and let $f(2) = 14$. Find $f(-2)$.

Problem (2)**.** Let $g(x)$ be an odd function, with $g(3) = -1.5$. For what value of x will $g(x) = 1.5$?

Problem (3)**.** For what values of n is $f(x) = x^n$ an even function? For what values is it odd? Explain why.

Problem (4)**.** Let $f(x)$ be an odd function. Find $f(0)$.

Problem (5)**.** Find a function which is both odd and even.

Problem (6)**.** Algebraically prove that $f(x) = 2^x + 2^{-x}$ is an even function. That is, show that $f(x) = f(-x)$, aka explain why $2^x + 2^{-x} = 2^{(-x)} + 2^{-(-x)}$.

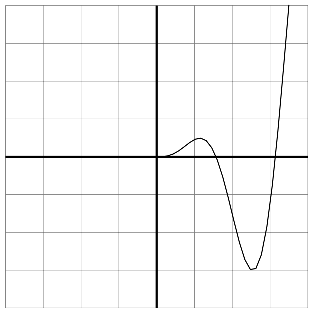
Problem (7)**.** Algebraically prove that $g(x) = 2^x - 2^{-x}$ is an odd function.

Problem (8)**.** Is the set of even functions closed under addition? Either explain why or give a counterexample.

Problem (9)**.** Is the set of odd functions closed under multiplication? Either explain why or give a counterexample.

Exit Ticket!

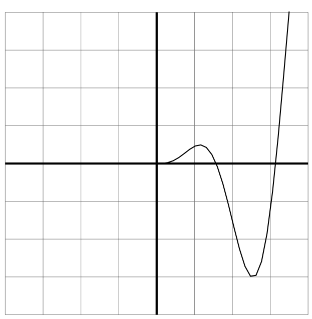
Problem. Let $f(x)$ be an odd function. Half of the graph is drawn below. Graph the rest of the function.



Problem. Algebraically show that $f(x) = x^4 - x^2 + 1$ is an even function.

Exit Ticket!

Problem. Let $f(x)$ be an odd function. Half of the graph is drawn below. Graph the rest of the function.



Problem. Algebraically show that $f(x) = x^4 - x^2 + 1$ is an even function.