

I can find domains and vertical asymptotes of rational functions.

- An asymptote is a line that the function gets closer & closer to but never touches. A vertical asymptote is an asymptote that is a vertical line, with equation  $y = \#$ .
- A rational function has a vertical asymptote at every  $x$ -value that makes the denominator 0. Find these by factoring the denominator.
- The domain is all real numbers except those that make the denominator 0. Write the domain simply as:  $x \neq \#, x \neq \#, \dots$

Example:

$$f(x) = \frac{x}{x^2 - x - 6}$$

factor the denominator:

$$f(x) = \frac{x}{(x-3)(x+2)}$$

find values of  $x$  that make the denominator 0, a.k.a. the roots of the denominator:

$$\text{either } x-3=0 \text{ or } x+2=0$$

$$x=-3 \text{ or } x=2$$

vertical asymptotes are at

$$x=-3 \text{ and } x=2$$

$$\text{domain: } x \neq -3, x \neq 2$$

I can find horizontal asymptotes of rational functions.

- A horizontal asymptote is an asymptote that is a horizontal line, with equation  $y = \#$ .
- If the degree of the numerator of the rational function is less than the degree of the denominator, the horizontal asymptote is  $y = 0$ .
- If the degree of the numerator is greater than the degree of the denominator, find the horizontal asymptote by putting the numerator and denominator in standard form and dividing the leading coefficients.

Example:

$$f(x) = \frac{2x^2 + 3}{(2x + 5)(2x - 5)} \quad \leftarrow \text{degree 2}$$

multiply out to get standard form:

$$\begin{array}{|c|c|c|} \hline & 2x & 5 \\ \hline 2x & 4x^2 & 10x \\ \hline -5 & -10x & -25 \\ \hline \end{array} = -25 + 4x^2$$

$$f(x) = \frac{2x^2 + 3}{-25 + 4x^2}$$

horizontal asymptote:

$$y = \frac{2}{-25} \quad \text{or} \quad y = -\frac{2}{25}$$