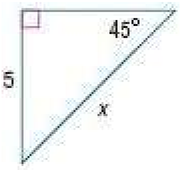
10-3 Special Right Triangles

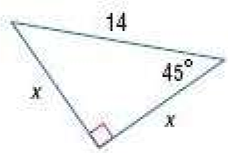
Find *x*.

1. 

SOLUTION:

In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

Therefore, since the side length (*l*) is 5, then $x = 5\sqrt{2}$.

2. 

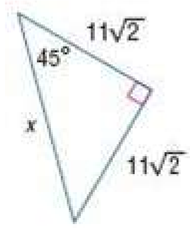
SOLUTION:

In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

Therefore, since the hypotenuse *(h)* is 14, then $x\sqrt{2} = 14$.

Solve for *x*.

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{14}{\sqrt{2}}$$
$$x = \frac{14}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= 7\sqrt{2}$$

3. 

SOLUTION:

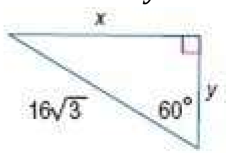
In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

Therefore, since $l = 11\sqrt{2}$, then $x = \left(11\sqrt{2}\right)\sqrt{2}$.

Simplify:

$$x = \left(11\sqrt{2}\right)\sqrt{2}$$
$$= 11 \cdot 2$$
$$= 22.$$

Find x and y.

4. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(2s)* and the length of the longer leg *l* is $\sqrt{3}$ times th length of the shorter leg ($s\sqrt{3}$).

The length of the hypotenuse is $16\sqrt{3}$, the shorter le is y, and the longer leg is *x*.

Therefore, $2y = 16\sqrt{3}$ *and* $x = \left(y\sqrt{3}\right)$.

Solve for y:

$$y = \frac{16\sqrt{3}}{2} = 8\sqrt{3}$$

Substitute and solve for *x:*

$$x = \left(8\sqrt{3}\right)\sqrt{3} = 24$$

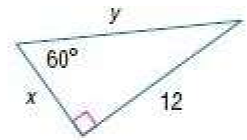
5. 

SOLUTION:

.In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(2s)* and the length of the longer leg *l* is $\sqrt{3}$ times th length of the shorter leg ($s\sqrt{3}$).

The length of the hypotenuse is *x*, the shorter leg is 7, and the longer leg is y.

Therefore, $x = 2(7) = 14$ *and* $y = (7)\sqrt{3} = 7\sqrt{3}$.

6. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(2s)* and the length of the longer leg *l* is $\sqrt{3}$ times th length of the shorter leg ($s\sqrt{3}$).

The length of the hypotenuse is y, the shorter leg is *x*, and the longer leg is 12.

Therefore, $x\sqrt{3} = 12$ *and* $y = 2x$.

Solve for *x*.

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{12}{\sqrt{3}}$$

$$x = \frac{12}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$= 4\sqrt{3}$$

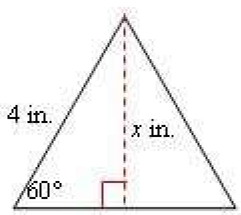
Substitute and solve for y:

$$y = 2\left(4\sqrt{3}\right) = 8\sqrt{3}.$$

7. **ART** Paulo is mailing an engraved plaque that is $3\frac{1}{4}$ inches high to the winner of a chess tournament. He has a mailer that is a triangular prism with 4-inch equilateral triangle bases as shown in the diagram. Will the plaque fit through the opening of the mailer? Explain.

SOLUTION:

If the plaque will fit, then the height of the plaque must be less than the altitude of the equilateral triangle. Draw the altitude of the equilateral triangle. Since the triangle is equilateral, the altitude will divide the triangle into two smaller congruent 30-60-90 triangles. Let *x* represent the length of the altitude and use the 30-60-90 Triangle Theorem to determine the value of *x*.



The hypotenuse is twice the length of the shorter leg *s*.

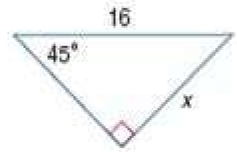
$$\begin{aligned} h &= 2s && \text{Theorem 8.9} \\ 4 &= 2s && h = 4 \\ 2 &= s && \text{Divide each side by 2.} \end{aligned}$$

Since the altitude is across from the 60º-angle, it is the longer leg.

$$\begin{aligned} \ell &= s\sqrt{3} && \text{Theorem 8.9} \\ x &= 2\sqrt{3} && s = 2 \\ x &\approx 3.5 && \text{Multiply using a calculator.} \end{aligned}$$

Since 3.25 < 3.5, the height of the plaque is less than the altitude of the equilateral triangle. Therefore, the plaque will fit through the opening of the mailer.

CCSS SENSE-MAKING Find *x*.

8. 

SOLUTION:

In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

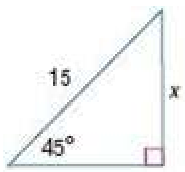
Therefore, since the hypotenuse is 16 and the legs are *x*, then $x\sqrt{2} = 16$.

Solve for *x*.

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{16}{\sqrt{2}}$$

$$x = \frac{16}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{16\sqrt{2}}{2} = 8\sqrt{2}$$

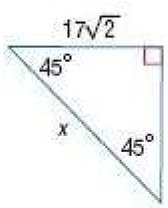
9. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Since the hypotenuse is 15 and the legs are x, then $x\sqrt{2}=15.$

Solve for x.

$$\frac{x\sqrt{2}}{\sqrt{2}}=\frac{15}{\sqrt{2}}$$
$$x=\frac{15}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$=\frac{15\sqrt{2}}{2}$$

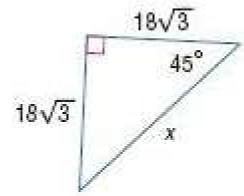
10. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Since the the legs are $17\sqrt{2}$, then the hypotenuse is $x=\left(17\sqrt{2}\right)\sqrt{2}$.

Simplify:

$$x=\left(17\sqrt{2}\right)\sqrt{2}$$
$$=17\cdot 2$$
$$=34$$

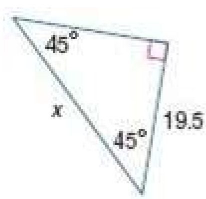
11. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the legs are $18\sqrt{3}$, the hypotenuse is $x=\left(18\sqrt{3}\right)\sqrt{2}$.

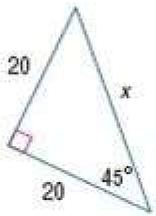
Simplify:

$$x=\left(18\sqrt{3}\right)\sqrt{2}$$
$$=18\cdot\sqrt{3}\cdot\sqrt{2}$$
$$=18\sqrt{6}$$

12. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the legs are 19.5, then the hypotenuse would be $x=(19.5)\sqrt{2}=19.5\sqrt{2}.$

13. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the legs are 20, then the hypotenuse would be $x=(20)\sqrt{2}=20\sqrt{2}$.

14. If a $45°-45°-90°$ triangle has a hypotenuse length of 9, find the leg length.

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the hypotenuse is 9, then $l\sqrt{2}=9$.

Solve for *x*.

$$\frac{l\sqrt{2}}{\sqrt{2}}=\frac{9}{\sqrt{2}}$$
$$l=\frac{9}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$=\frac{9\sqrt{2}}{2}$$

15. Determine the length of the leg of a $45°-45°-90°$ triangle with a hypotenuse length of 11.

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the hypotenuse is 11, $l\sqrt{2}=11$.

Solve for *x*.

$$\frac{l\sqrt{2}}{\sqrt{2}}=\frac{11}{\sqrt{2}}$$
$$l=\frac{11}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$=\frac{11\sqrt{2}}{2}$$

16. What is the length of the hypotenuse of a $45°-45°-90°$ triangle if the leg length is 6 centimeters?

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

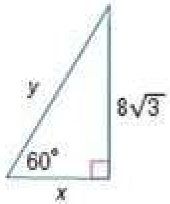
Therefore, since the legs are 6, the hypotenuse would be $h=(6)\sqrt{2}\approx 8.5$ cm.

17. Find the length of the hypotenuse of a $45°-45°-90°$ triangle with a leg length of 8 centimeters.

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent $(l=l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, since the legs are 8, the hypotenuse would be $h=(8)\sqrt{2}\approx 11.3$ cm.

Find x and y.

18. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$ and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The length of the hypotenuse is y, the shorter leg is x, and the longer leg is $8\sqrt{3}$.

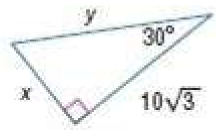
Therefore, $x\sqrt{3} = 8\sqrt{3}$ and $h = 2x$.

Solve for x:

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{\sqrt{3}}$$

$$x = 8$$

Then , to find the hypotenuse, $h = 2(8) = 16.$

19. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$ and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The length of the hypotenuse is 7, the shorter leg is x, and the longer leg is $10\sqrt{3}$.

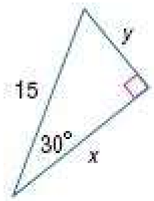
Therefore, $x\sqrt{3} = 10\sqrt{3}$ and $h = 2x$.

Solve for x:

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}}$$

$$x = 10$$

Then , to find the hypotenuse, $h = 2(10) = 20.$

20. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$ and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg $\left(l=s\sqrt{3}\right)$.

The length of the hypotenuse is 15, the shorter leg is y, and the longer leg is x.

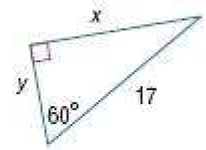
Therefore, $x=y\sqrt{3}$ *and* $15=2y$.

Solve for y:

$15=2y$
$\frac{15}{2}=y$

Substitute and solve for x:

$x=\left(\frac{15}{2}\right)\sqrt{3}$
$x=\frac{15\sqrt{3}}{2}$

21. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$ and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg $\left(l=s\sqrt{3}\right)$.

The length of the hypotenuse is 17, the shorter leg is y, and the longer leg is x.

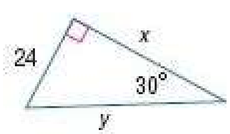
Therefore, $x=y\sqrt{3}$ *and* $17=2y$.

Solve for y:

$17=2y$
$\frac{17}{2}=y$

Substitute and solve for x:

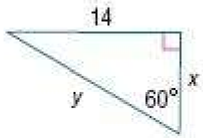
$x=\left(\frac{17}{2}\right)\sqrt{3}$
$x=\frac{17\sqrt{3}}{2}$

22. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$ and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg $\left(l=s\sqrt{3}\right)$.

The length of the hypotenuse is y, the shorter leg is 24, and the longer leg is x.

Therefore, $x=24\sqrt{3}$ *and* $y=2(24)=48$.

23. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(h=2s)* and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The length of the hypotenuse is y, the shorter leg is *x*, and the longer leg is *14*.

Therefore, $14 = x\sqrt{3}$ *and* $y = 2x$.

Solve for x:

$$\frac{14}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$$
$$x = \frac{14\sqrt{3}}{3}$$

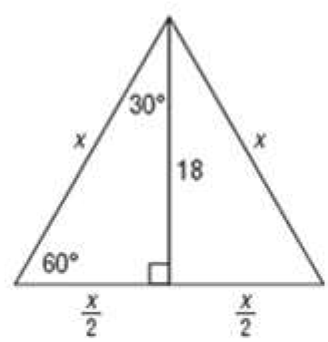
Substitute and solve for y:

$$y = 2\left(\frac{14\sqrt{3}}{3}\right) = \frac{28\sqrt{3}}{3}$$

24. An equilateral triangle has an altitude length of 18 feet. Determine the length of a side of the triangle.

SOLUTION:

Let *x* be the length of each side of the equilateral triangle. The altitude from one vertex to the opposite side divides the equilateral triangle into two 30°-60°-90° triangles.



In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(h=2s)* and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The length of the hypotenuse is *x*, the shorter leg is $\frac{x}{2}$, and the longer leg is 18.

Therefore, $18 = \left(\frac{x}{2}\right)\sqrt{3}$.

Solve for x:

$$\frac{18}{\sqrt{3}} = \frac{\left(\frac{x}{2}\right)\sqrt{3}}{\sqrt{3}}$$

$$\frac{18}{\sqrt{3}} = \frac{x}{2}$$

$$2 \cdot \frac{18}{\sqrt{3}} = \frac{x}{2} \cdot 2$$

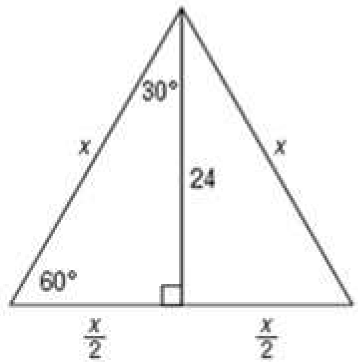
$$\frac{36}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$$

$$x = \frac{36\sqrt{3}}{3} = 12\sqrt{3} \approx 20.8\,ft.$$

25. Find the length of the side of an equilateral triangle that has an altitude length of 24 feet.

SOLUTION:

Let *x* be the length of each side of the equilateral triangle. The altitude from one vertex to the opposite side divides the equilateral triangle into two 30°-60°-90° triangles.



In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(h=2s)* and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The length of the hypotenuse is *x*, the shorter leg is $\frac{x}{2}$, and the longer leg is 24.

Therefore, $24 = \left(\frac{x}{2}\right)\sqrt{3}$.

Solve for *x*:

$$2 \cdot 24 = \left(\frac{x}{2}\right)\sqrt{3} \cdot 2$$

$$48 = x\sqrt{3}$$

$$\frac{48}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$$

$$\frac{48\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = x$$

$$x = \frac{48\sqrt{3}}{3} = 16\sqrt{3} \approx 27.7\ ft.$$

26. **CCSS MODELING** Refer to the beginning of the lesson. Each highlighter is an equilateral triangle with 9 cm sides. Will the highlighter fit in a 10 cm by 7 cm rectangular box? Explain.

SOLUTION:

Find the height of the highlighter.

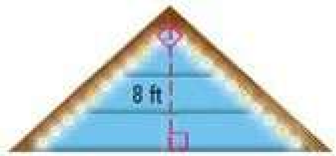
The altitude from one vertex to the opposite side divides the equilateral triangle into two 30°-60°-90° triangles.

Let *x* be the height of the triangle. Use special right triangles to find the height, which is the longer side of a 30°-60°-90° triangle.

The hypotenuse of this 30°-60°-90° triangle is 9, the shorter leg is $\frac{9}{2}$, which makes the height $\frac{9\sqrt{3}}{2}$, which is approximately 7.8 cm.

The height of the box is only 7 cm. and the height of the highlighter is about 7.8 cm., so it will not fit.

27. **EVENT PLANNING** Grace is having a party, and she wants to decorate the gable of the house as shown. The gable is an isosceles right triangle and she knows that the height of the gable is 8 feet. What length of lights will she need to cover the gable below the roof line?

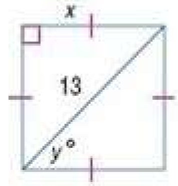


SOLUTION:

The gable is a 45°-45°-90° triangle. The altitude again divides it into two 45°-45°-90° triangles. The length of the leg (l) of each triangle is 8 feet.
In a 45°-45°-90° triangle, the legs l are congruent ($l=l$) and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

If $l = 8$, then $h = 8\sqrt{2}$. Since there are two hypotenuses that have to be decorated, the total length is $2\left(8\sqrt{2}\right) = 16\sqrt{2} \approx 22.6$ ft.

Find x and y.

28. 

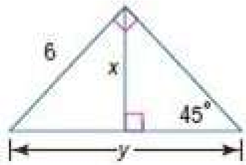
SOLUTION:

The diagonal of a square divides it into two 45°-45°-90° triangles. So, $y = 45$.
In a 45°-45°-90° triangle, the legs l are congruent ($l=l$) and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $\left(h=l\sqrt{2}\right)$.

Therefore, if the hypotenuse is 13, then $x\sqrt{2} = 13$.

Solve for x.

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{13}{\sqrt{2}}$$
$$x = \frac{13}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= \frac{13\sqrt{2}}{2}$$

29. 

SOLUTION:

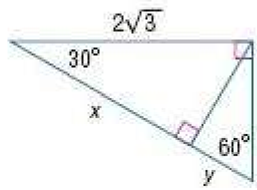
In a 45°-45°-90° triangle, the legs *l* are congruent (*l*=*l*) and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

Since *x* is a leg of a 45°-45°-90° triangle whose hypotenuse measures 6 units, then $x\sqrt{2} = 6$.

Solve for *x*.

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$
$$x = \frac{6}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= 3\sqrt{2}$$

Since y is the hypotenuse of a 45°-45°-90° triangle whose legs measure 6 units each, then $y = 6\sqrt{2}$.

30. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* (*h*=2*s*) and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

In one of the 30-60-90 triangles in this figure, the length of the hypotenuse is $2\sqrt{3}$, the shorter leg is *s*, and the longer leg is *x*.

Therefore, $x = s\sqrt{3}$ *and* $2\sqrt{3} = 2s$.

Solve for *s*:

$$2\sqrt{3} = 2s$$
$$s = \sqrt{3}$$

Substitute and solve for x:

$$x = \left(\sqrt{3}\right)\sqrt{3} = 3$$

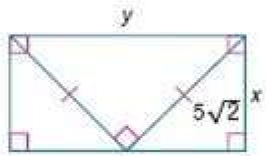
In a different 30-60-90 triangles in this figure, the length of the shorter leg is y and the longer leg is $\sqrt{3}$.

Therefore, $\sqrt{3} = y\sqrt{3}$.

Solve for y:

$$\frac{\sqrt{3}}{\sqrt{3}} = \frac{y\sqrt{3}}{\sqrt{3}}$$

$$y = 1$$

31. 

SOLUTION:
In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

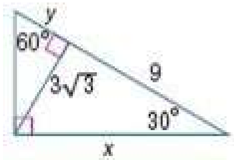
Since y is the hypotenuse of a 45°-45°-90° triangle whose each leg measures $5\sqrt{2}$ units, then $y = \left(5\sqrt{2}\right)\sqrt{2} = 10.$

Since *x* is a leg of a 45°-45°-90° triangle whose hypotenuse measures $5\sqrt{2}$ units, then $x\sqrt{2} = 5\sqrt{2}.$

Solve for x:

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}}$$

$x = 5$

32. 

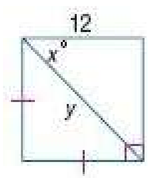
SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(h=2s)* and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

In one of the 30-60-90 triangles in this figure, the length of the hypotenuse is x, the shorter leg is $3\sqrt{3}$, and the longer leg is *9*.

Therefore, $x = 2\left(3\sqrt{3}\right) = 6\sqrt{3}$.

In a different 30-60-90 triangles in this figure, the length of the shorter leg is y, and the longer leg is $3\sqrt{3}$.

Therefore, $x = 2\left(3\sqrt{3}\right) = 6\sqrt{3}$.

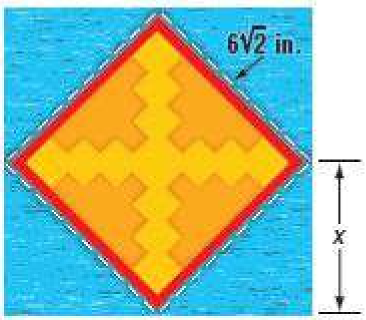
33. 

SOLUTION:
The diagonal of a square divides it into two 45°-45°-90°. Therefore, *x* = 45°.

In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)*and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg $\left(h = l\sqrt{2}\right)$.

Therefore, since the legs are 12, then the hypotenuse would be $y = 12\sqrt{2}.$

34. **QUILTS** The quilt block shown is made up of a square and four isosceles right triangles. What is the value of x? What is the side length of the entire quilt block?



SOLUTION:

In a 45°-45°-90° triangle, the legs l are congruent $(l = l)$ and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

Since the hypotenuse of the triangles formed by the diagonals of this square are each $6\sqrt{2}$, then $x\sqrt{2} = 6\sqrt{2}$.

Solve for x:

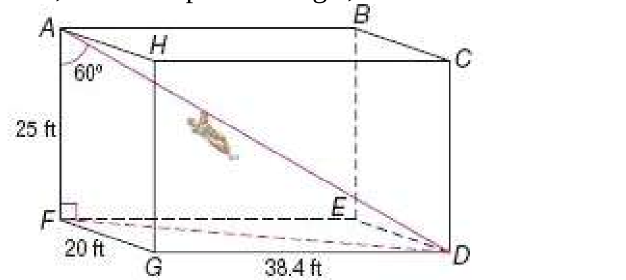
$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2}}$$

$$x = 6$$

Therefore, $x = 6$ inches.

Here, x is half the length of each side of the entire quilt block. Therefore, the length of each side of the entire quilt block is 12 inches.

35. **ZIP LINE** Suppose a zip line is anchored in one corner of a course shaped like a rectangular prism. The other end is anchored in the opposite corner as shown. If the zip line makes a $60°$ angle with post $\overline{AF}$, find the zip line's length, AD.



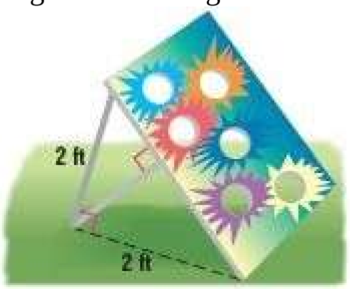
SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s (or $h = 2s$) and the length of the longer leg ℓ is $\sqrt{3}$ times the length of the shorter leg $\left(\text{or } \ell = s\sqrt{3}\right)$.

Since ΔAFD is a 30-60-90 triangle, the length of the hypotenuse $\overline{AD}$ of ΔAFD is twice the length of th shorter leg $\overline{AF}$.

Therefore, $AD = 2(25)$ or 50 feet.

36. **GAMES** Kei is building a bean bag toss for the school carnival. He is using a 2-foot back support that is perpendicular to the ground 2 feet from the front of the board. He also wants to use a support that is perpendicular to the board as shown in the diagram. How long should he make the support?



SOLUTION:

The ground, the perpendicular support, and the board form an isosceles right triangle.The support that is perpendicular to the board is the altitude to the hypotenuse of the triangle.

In a 45-45-90 right triangle, the hypotenuse is $\sqrt{2}$ times the length of the legs *(l)*, which are congruent to each other, or $h = l\sqrt{2}$.

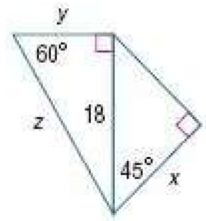
Therefore, since the legs are 2, then $h = 2\sqrt{2}$.

The altitude to the base of the isosceles triangle bisects the base. To find the altitude, we can use the sides of the right triangle with a leg of $\sqrt{2}$ ft and hypotenuse 2 ft long.

Use the Pythagorean theorem to find the altitude:

$$hypotenuse^2 = leg^2 + altitude^2$$
$$(2)^2 = \left(\sqrt{2}\right)^2 + altitude^2$$
$$4 = 2 + altitude^2$$
$$2 = altitude^2$$
$$altitude = \sqrt{2} \approx 1.4 \text{ ft.}$$

37. Find *x*, *y*, and *z*



SOLUTION:

In a 45°-45°-90° triangle, the legs *l* are congruent *(l=l)* and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg or $h = l\sqrt{2}$.

x is the length of each leg of a 45°-45°-90° triangle whose hypotenuse measures 18 units, therefore the hypotenuse would be $x\sqrt{2} = 18$.

Solve for *x*:

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{18}{\sqrt{2}}$$
$$x = \frac{18}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= 9\sqrt{2}$$

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s* *(h=2s)* and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg $\left(l = s\sqrt{3}\right)$.

The hypotenuse is z, the longer leg is 18, and the shorter leg is y. Therefore, $18 = y\sqrt{3}\ and\ z = 2y$.

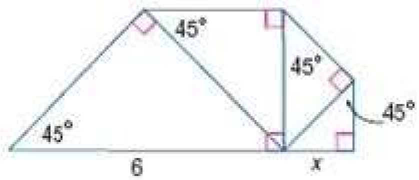
Solve for y:

$$\frac{18}{\sqrt{3}} = \frac{y\sqrt{3}}{\sqrt{3}}$$
$$\frac{18\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = y$$
$$y = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$$

Substitute and solve for z:

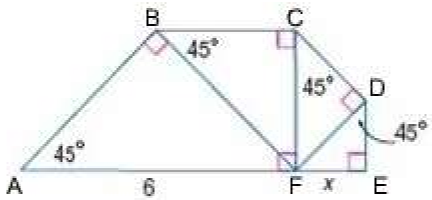
$$z = 2\left(6\sqrt{3}\right) = 12\sqrt{3}$$

38. Each triangle in the figure is a $45° - 45° - 90°$ triangle. Find x.



SOLUTION:

In a 45°-45°-90° triangle, the legs l are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.



$\overline{FD}$ is the hypotenuse of a 45°-45°-90° triangle whose each leg measures x. Therefore,

$$FD = x\sqrt{2}.$$

$\overline{FC}$ is the hypotenuse of a 45°-45°-90° triangle whose each leg measures $x\sqrt{2}$. Therefore,

$$\begin{aligned} FC &= \left(x\sqrt{2}\right)\sqrt{2} \\ &= x\left(\sqrt{2}\right)\left(\sqrt{2}\right) \\ &= 2x \end{aligned}$$

$\overline{FB}$ is the hypotenuse of a 45°-45°-90° triangle whose each leg measures $2x$. Therefore,

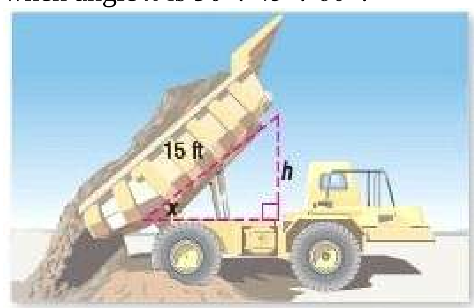
$$\begin{aligned} FB &= (2x)\sqrt{2} \\ &= 2x\sqrt{2} \end{aligned}$$

$\overline{FA}$ is the hypotenuse of a 45°-45°-90° triangle whose each leg measures $2x\sqrt{2}$. Therefore,

$$\begin{aligned} FA &= \left(2x\sqrt{2}\right)\sqrt{2} \\ &= 2\left(\sqrt{2}\sqrt{2}\right)x \\ &= 2(2)x \\ &= 4x \end{aligned}$$

Since FA = 6 units, then $4x$ = 6 and $x = \frac{3}{2}$.

39. **CCSS MODELING** The dump truck shown has a 15-foot bed length. What is the height of the bed h when angle x is 30°? 45°? 60°?



SOLUTION:

In a 45°-45°-90° triangle, the legs l are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg.

When x is $30°$, h is the shorter leg of a 30°-60°-90° triangle whose hypotenuse is 15 ft.
Therefore, $2h = 15 \text{ and } h = 7.5$ ft.

When x is $45°$, h is the length of each leg of a 45°-45°-90° triangle whose hypotenuse is 15 ft.
Therefore, $h\sqrt{2} = 15.$

Solve for h:

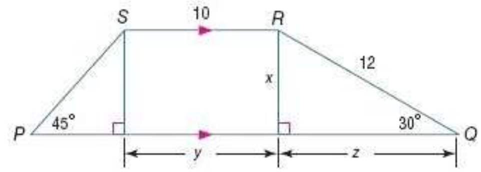
$$\frac{h\sqrt{2}}{\sqrt{2}} = \frac{15}{\sqrt{2}}$$
$$h = \frac{15}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= 7.5\sqrt{2}$$
$$\approx 10.6 \text{ ft}$$

When x is $60°$, h is the longer leg of a 30°-60°-90° triangle whose hypotenuse is 15 ft. The shorter leg is half the hypotenuse.

Therefore,

$$h = \left(\frac{15}{2}\right)\sqrt{3}$$
$$= 7.5\sqrt{3}$$
$$\approx 13$$

40. Find x, y, and z, and the perimeter of trapezoid *PQRS*.



SOLUTION:

x is the shorter leg of a 30°-60°-90° triangle whose hypotenuse is 12.
Therefore, $2x = 12$ or $x = 6$.

z is the longer leg of the 30-60-90 triangle.
Therefore, $z = 6\sqrt{3}$.

The two parallel bases and the perpendicular sides form a rectangle. Since opposite sides of a rectangle are congruent, then
$y = 10$ and the side opposite x ,a leg of the 45°-45°-90° triangle, is also 6.

$\overline{PS}$ is the hypotenuse of a 45°-45°-90° triangle whose each leg measures 6 units.
Therefore, $PS = 6\sqrt{2}$

To find the perimeter of the trapezoid, find the sum of all of the sides:

$$PQ = 6 + 10 + 6\sqrt{3} = 16 + 6\sqrt{3}$$
$$QR = 12$$
$$RS = 10$$
$$PS = 6\sqrt{2}$$

Therefore,

$$P = \left(16 + 6\sqrt{3}\right) + (12) + (10) + \left(6\sqrt{2}\right)$$
$$= 38 + 6\sqrt{2} + 6\sqrt{3}$$
$$\approx 56.9 \text{ units}$$

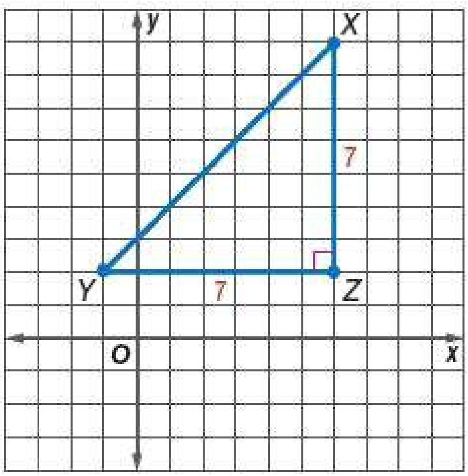
41. **COORDINATE GEOMETRY** ΔXYZ is a $45°$-$45°$-$90°$ triangle with right angle *Z*. Find the coordinates of *X* in Quadrant I for *Y*(–1, 2) and *Z*(6, 2).

SOLUTION:

$\overline{YZ}$ is one of the congruent legs of the right triangle and YZ=7 units. So, the point *X* is also 7 units away from *Z*.

Find the point *X* in the first quadrant, 7 units away from *Z* such that $\overline{XZ} \perp \overline{YZ}$.

Therefore, the coordinates of *X* is (6, 9).



42. **COORDINATE GEOMETRY** ΔEFG is a $30°$-$60°$-$90°$ triangle with $m\angle F = 90°$. Find the coordinates of *E* in Quadrant III for *F*(–3, –4) and *G* (–3, 2). $\overline{FG}$ is the longer leg.

SOLUTION:

The side $\overline{FG}$ is the longer leg of the 30°-60°-90° triangle and it is 6 units long and $(EF)\sqrt{3} = 6.$

Solve for EF:

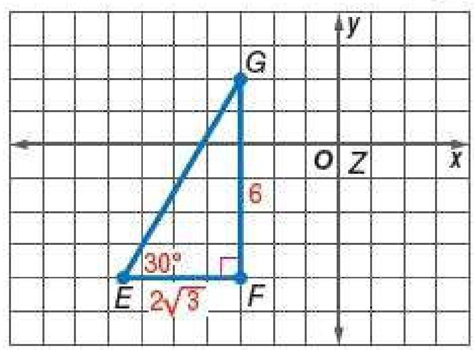
$$\frac{(EF)\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$$

$$EF = \frac{6 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$$EF = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Find the point *E* in the third quadrant, $2\sqrt{3}$ units away (approximately 3.5 units) from *F* such that $\overline{FG} \perp \overline{EF}$.

Therefore, the coordinates of *E* is ($-3 - 2\sqrt{3}$, –4).



43. **COORDINATE GEOMETRY** ΔJKL is a $45°$-$45°$-$90°$ triangle with right angle *K*. Find the coordinates of *L* in Quadrant IV for *J*(–3, 5) and *K*(–3, –2).

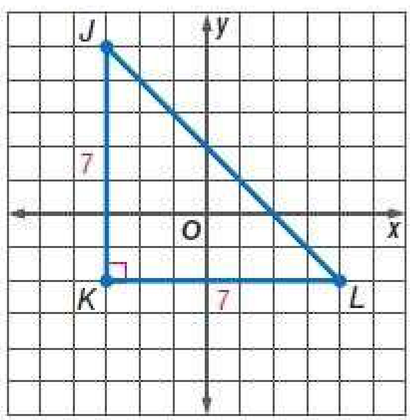
SOLUTION:

The side $\overline{JK}$ is one of the congruent legs of the right triangle and it is 7 units.

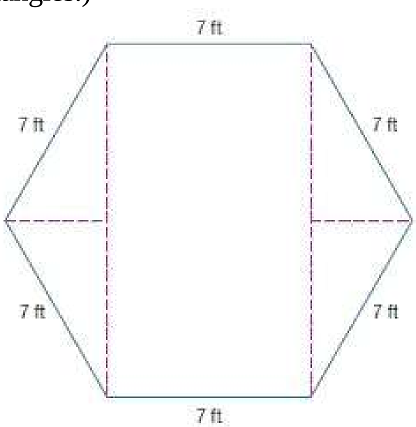
Therefore, the point *L* is also 7 units away from *K*.

Find the point *L* in the Quadrant IV, 7 units away from *K*, such that $\overline{JK} \perp \overline{KL}$.

Therefore, the coordinates of *L* is (4, –2).

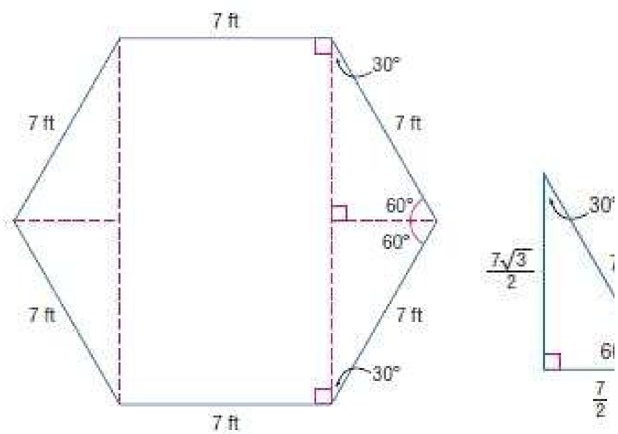


44. **EVENT PLANNING** Eva has reserved a gazebo at park for a party. She wants to be sure that there will enough space for her 12 guests to be in the gazebo at same time. She wants to allow 8 square feet of area each guest. If the floor of the gazebo is a regular hex and each side is 7 feet, will there be enough room for and her friends? Explain. (Hint: Use the Polygon Inte Angle Sum Theorem and the properties of special rig triangles.)



SOLUTION:

A regular hexagon can be divided into a rectangle an congruent right angles as shown. The length of the hypotenuse of each triangle is 7 ft. By the Polygon In Angle Theorem, the sum of the interior angles of a h is (6 – 2)180 = 720. Since the hexagon is a regular he each angle is equal to $\frac{720}{6} = 120.$ Therefore, each tri in the diagram is a 30°-60°-90° triangle, and the lengt the shorter and longer legs of the triangle are $\frac{7}{2}$ ft and $\frac{7\sqrt{3}}{2}$ ft.



The total area is the sum of the four congruent triangl the rectangle of sides $7 \text{ ft and } 7\sqrt{3} \text{ ft}.$ Therefore, tot is

When planning a party with a stand-up buffet, a host allow 8 square feet of area for each guest.
Divide the area by 8 to find the number of guests that be accommodated in the gazebo.

$\frac{127.3}{8} \approx 16$

So, the gazebo can accommodate about 16 guests. W and her friends, there are a total of 13 at the party, so will all fit.

45. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate ratios in right triangles.
a. GEOMETRIC Draw three similar right triangles with a $50°$ angle. Label one triangle *ABC* where angle *A* is the right angle and *B* is the $50°$ angle. Label a second triangle *MNP* where *M* is the right angle and N is the $50°$ angle. Label the third triangle

XYZ where *X* is the right angle and *Y* is the $50°$ angle.

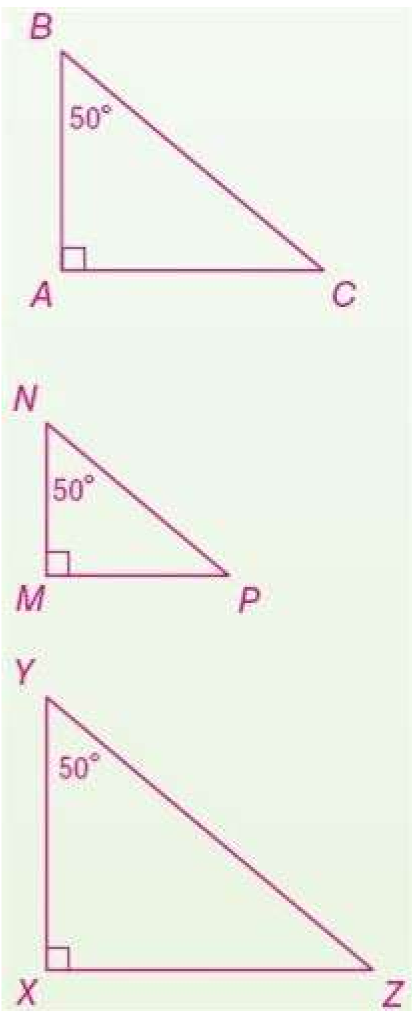
b. TABULAR Copy and complete the table below.

Triangle		Length				Ratio	
ABC	*AC*		*BC*			$\frac{AC}{BC}$	
MNP	*MP*		*NP*			$\frac{MP}{NP}$	
XYZ	*XZ*		*YZ*			$\frac{XZ}{YZ}$	

c. VERBAL Make a conjecture about the ratio of the leg opposite the $50°$ angle to the hypotenuse in any right triangle with an angle measuring $50°$.

SOLUTION:

a. It is important that you use a straightedge and a protractor when making these triangles, to ensure accuracy of measurement. Label each triangle as directed. Sample answers:



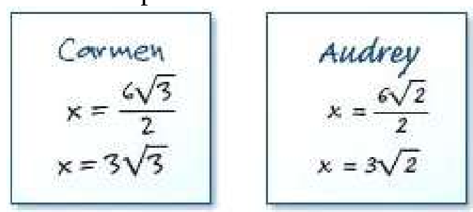
b. Using a metric ruler, measure the indicated lengths and record in the table, as directed. Measure in centimeters.

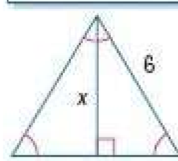
Triangle		Length				Ratio	
ABC	*AC*	2.4 cm	*BC*	3.2 cm		$\frac{AC}{BC}$	1.3
MNP	*MP*	1.7 cm	*NP*	2.2 cm		$\frac{MP}{NP}$	1.3
XYZ	*XZ*	3.0 cm	*YZ*	3.9 cm		$\frac{XZ}{YZ}$	1.3

c. What observations can you make, based on patterns you notice in the table? Pay special attention to the ratio column.

Sample answer: The ratios will always be the same.

46. **CCSS CRITIQUE** Carmen and Audrey want to find *x* in the triangle shown. Is either of them correct? Explain.





SOLUTION:

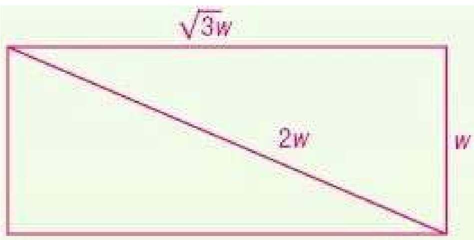
Carmen; Sample answer: Since the three angles of the larger triangle are congruent, it is an equilateral triangle. Therefore, the right triangles formed by the altitude are $30°\text{-}60°\text{-}90°$ triangles. The hypotenuse is 6, so the shorter leg is $\frac{6}{2}$, or 3, and the longer leg *x* is $3\sqrt{3}$.

47. **OPEN ENDED** Draw a rectangle that has a diagonal twice as long as its width. Then write an equation to find the length of the rectangle.

SOLUTION:

The diagonal of a rectangle divides it into two right triangles. If the diagonal (hypotenuse) is twice the shorter side of the rectangle, then it forms a 30-60-90 triangle. Therefore, the longer side of the rectangle would be $\sqrt{3}$ times the shorter leg.

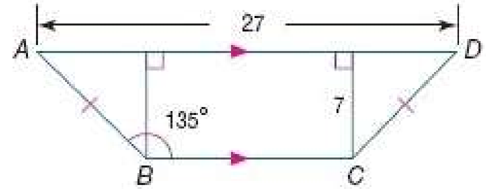
Sample answer:



Let ℓ represent the length.

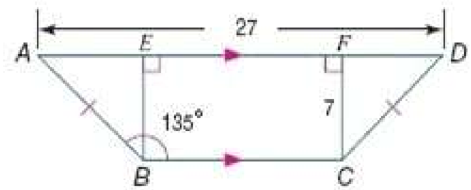
$\ell^2 + w^2 = (2w)^2; \ell^2 = 3w^2; \ell = w\sqrt{3}$.

48. **CHALLENGE** Find the perimeter of quadrilateral *ABCD*.



SOLUTION:

The triangles on either side are congruent to each other as one of the angles is a right angle and the hypotenuses are congruent. (HL)



$m\angle ABC = 135$

Quadrilateral *BCFE* is a rectangle, so

$m\angle CBE = 90$.

$m\angle ABE = 135 - 90 = 45$

So, the triangles are 45°-45°-90° special triangles.

The opposite sides of a rectangle are congruent, so, *BE* = 7.

Each leg of the 45°-45°-90° triangle is 7 units, so the hypotenuse, $AB = CD = 7\sqrt{2}$ units.

BC = *EF* = *AD* – (*AE* + *FD*) = 27 – (7 + 7) = 13.

Therefore, the perimeter of the quadrilateral *ABCD* is

$27 + 7\sqrt{2} + 13 + 7\sqrt{2} = 40 + 14\sqrt{2} \approx 59.8$ units.

49. **REASONING** The ratio of the measure of the angles of a triangle is 1:2:3. The length of the shortest side is 8. What is the perimeter of the triangle?

SOLUTION:

The sum of the measures of the three angles of a triangle is 180. Since the ratio of the measures of angles is 1:2:3, let the measures be x, $2x$, and $3x$. Then, $x + 2x + 3x = 180$. So, $x = 30$. That is, it is a 30°-60°-90° triangle. In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg.
Here, the shorter leg measures 8 units. So, the longer leg is $8\sqrt{3} \approx 13.9$ units and the hypotenuse is 2(8) = 16 units long. Therefore, the perimeter is about 8 + 13.9 + 16 = 37.9 units.

50. **WRITING IN MATH** Why are some right triangles considered *special*?

SOLUTION:

Sample answer: Once you identify that a right triangle is special or has a 30°, 60°, or 45° angle measure, you can solve the triangle without the use of a calculator.

51. If the length of the longer leg in a $30°$ - $60°$ - $90°$ triangle is $5\sqrt{3}$, what is the length of the shorter leg?

A 3
B 5
C $5\sqrt{2}$
D 10

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg.
So, the length of shorter leg if the longer leg is $5\sqrt{3}$ units, is

$$\frac{5\sqrt{3}}{\sqrt{3}} = 5.$$

Therefore, the correct choice is B.

52. **ALGEBRA** Solve $\sqrt{5-4x} - 6 = 7$.

F –44
G –41
H 41
J 44

SOLUTION:

Add 6 to both sides.

$$\sqrt{5-4x} - 6 + 6 = 7 + 6$$
$$\sqrt{5-4x} = 13$$

Square both sides.

$$\left(\sqrt{5-4x}\right)^2 = (13)^2$$
$$5 - 4x = 169$$

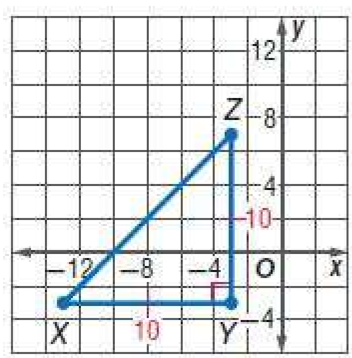
Solve for x.
$-4x = 164$
$x = -41$

Therefore, the correct choice is G.

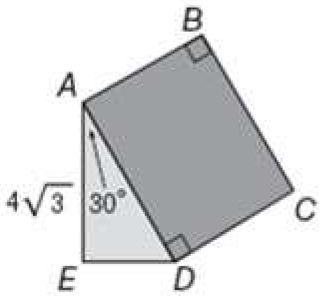
53. **SHORT RESPONSE** ΔXYZ is a $45°$ - $45°$ - $90°$ triangle with right angle Y. Find the coordinates of X in Quadrant III for $Y(-3, -3)$ and $Z(-3, 7)$.

SOLUTION:

$YZ = 10$ and is one of the congruent legs of the right triangle. Therefore, X is also 10 units away from Y. Find the point X in Quadrant III, 10 units away from Y, such that $\overline{YZ} \perp \overline{XY}$. Therefore, the coordinates of X is (–13, –3).



54. **SAT/ACT** In the figure, below, square *ABCD* is attached to ΔADE as shown. If $m\angle EAD$ is $30°$ and *AE* is equal to $4\sqrt{3}$, then what is the area of square *ABCD*?



A $8\sqrt{3}$
B 16
C 64
D 72
E $64\sqrt{2}$

SOLUTION:

ΔADE is a 30°-60°-90° triangle and $\overline{AE}$ is the longer side.

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s*, and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg.
So, the length of each side of the square which same as the length of the hypotenuse is $\left(\frac{4\sqrt{3}}{\sqrt{3}}\right)2 = 8$ units.

Then the area of the square is 8(8) = 64 sq. units. Therefore, the correct choice is C.

55. **SPORTS** Dylan is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 9 feet. What length of plywood does Dylan need for the ramp?

SOLUTION:

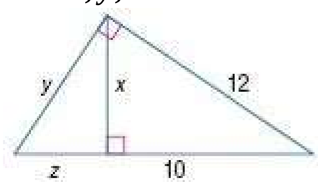
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The length of the base is 12 ft and the height is 9 ft. Let *x* be the length of the plywood required. So, $x^2 = 12^2 + 9^2$.

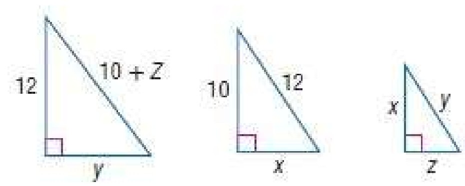
$$x = \sqrt{144 + 81}$$
$$= \sqrt{225}$$
$$= 15$$

Therefore, the length of the plywood required will be 15 ft.

Find *x*, *y*, and *z*.

56. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}}$$

Solve for z:

$$\frac{10+z}{12} = \frac{12}{10}$$
$$10(10+z) = 144$$
$$100 + 10z = 144$$
$$10z = 44$$
$$z = 4.4$$

$$\frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

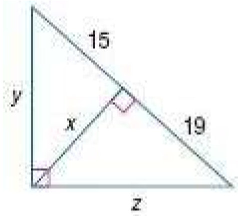
Set up a proportion to solve for y, using the value of z.

$$\frac{10+z}{y} = \frac{y}{z}$$
$$\frac{10+4.4}{y} = \frac{y}{4.4}$$
$$\frac{14.4}{y} = \frac{y}{4.4}$$
$$y^2 = 63.36$$
$$y \approx 8.0$$

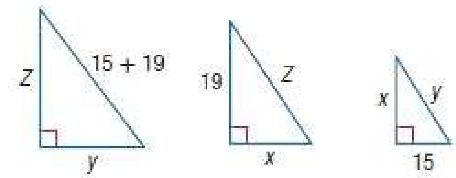
By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments and the length of this altitude is the geometric mean between the lengths of these two segments.

$$\frac{\text{longer leg}}{\text{shorter leg}} = \frac{\text{longer leg}}{\text{shorter leg}}$$

$$\frac{10}{x} = \frac{x}{z}$$
$$\frac{10}{x} = \frac{x}{4.4}$$
$$x^2 = 44$$
$$x \approx 6.6$$

57. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{hypotenuse}{shorter\ leg} = \frac{hypotenuse}{shorter\ leg}$$ and
$$\frac{hypotenuse}{longer\ leg} = \frac{hypotenuse}{longer\ leg}$$

Solve for y and z:

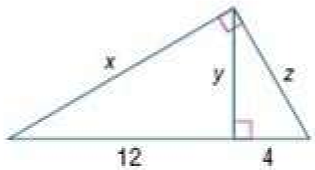
$$\frac{34}{y} = \frac{y}{15} \text{ and } \frac{34}{z} = \frac{z}{19}$$

$$y^2 = 510 \qquad z^2 = 646$$
$$y = \sqrt{510} \text{ and } z = \sqrt{646}$$
$$\approx 22.6 \qquad \approx 25.4$$

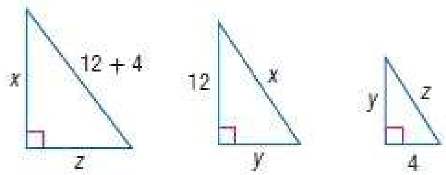
Solve for x:

$$\frac{longer\ leg}{shorter\ leg} = \frac{longer\ leg}{shorter\ leg}$$

$$\frac{15}{x} = \frac{x}{19}$$
$$x^2 = 285$$
$$x \approx 16.9$$

58. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}} \quad \frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

Set up a proportion and solve for x and z:

$$\frac{16}{x} = \frac{x}{12} \quad \text{and} \quad \frac{16}{z} = \frac{z}{4}.$$
$$x^2 = 192 \qquad z^2 = 64$$
$$x = \sqrt{192} \quad \text{and} \quad z = \sqrt{64}$$
$$= 8\sqrt{3} \approx 13.9 \qquad = 8$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for y:

$$\frac{12}{y} = \frac{y}{4}$$
$$y^2 = 48$$
$$y = 4\sqrt{3} \approx 6.9$$

Find the measures of the angles of each triangle.

59. The ratio of the measures of the three angles is 2:5:3.

SOLUTION:

The sum of the measures of the three angles of a triangle is 180.

Since the ratio of the measures of angles is 2:5:3, let the measures be 2*x*, 5*x*, and 3*x*.

Then, 2*x* + 5*x* + 3*x* = 180. Solve for *x*:

$$2x + 5x + 3x = 180$$
$$10x = 180$$
$$x = 18$$

Therefore, the measures of the angles are 2(18) =36, 5(18)= 90, and 3(18)=54.

60. The ratio of the measures of the three angles is 6:9:10.

SOLUTION:

The sum of the measures of the three angles of a triangle is 180.

Since the ratio of the measures of angles is 6:9:10, let the measures be 6*x*, 9*x*, and 10*x*.

6*x* + 9*x* + 10*x* = 180. Solve for *x*:

$$6x + 9x + 10x = 180$$
$$25x = 180$$
$$x = 7.2$$

Therefore, the measures of the angles are 26(7.2) =43.2, 9(7.2)= 64.8, and 10(7.2)=72.

61. The ratio of the measures of the three angles is 5:7:8.

SOLUTION:

The sum of the measures of the three angles of a triangle is 180.

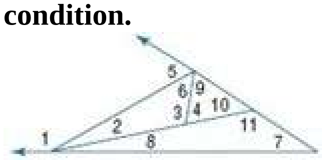
Since the ratio of the measures of angles is 5:7:8, let the measures be 5*x*, 7*x*, and 8*x*.

$5x + 7x + 8x = 180$. Solve for *x*:

$$5x + 7x + 8x = 180$$
$$20x = 180$$
$$x = 9$$

Therefore, the measures of the angles are 5(9)= 45, 7(9)= 63, and 8(9)= 72.

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



62. measures less than $m\angle 5$

SOLUTION:

By the Exterior Angle Inequality Theorem, the measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. So,

$m\angle 2 < m\angle 4, m\angle 7 < m\angle 4, m\angle 8 < m\angle 4$, and $m\angle 10 < m\angle 4$.

63. measures greater than $m\angle 6$

SOLUTION:

By the Exterior Angle Inequality Theorem, the measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. So,

$m\angle 1 > m\angle 6, m\angle 4 > m\angle 6$, and $m\angle 11 < m\angle 6$.

64. measures greater than $m\angle 10$

SOLUTION:

By the Exterior Angle Inequality Theorem, the measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. So,

$m\angle 3 > m\angle 10$, and $m\angle 5 < m\angle 10$.

65. measures less than $m\angle 11$

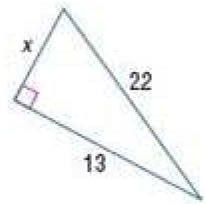
SOLUTION:

By the Exterior Angle Inequality Theorem, the measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. So,

$m\angle 2 < m\angle 11, m\angle 6 < m\angle 11, m\angle 9 < m\angle 11, m\angle 8 < m\angle 11$, and $m\angle 7 < m\angle 11$.

Find *x*.

66.

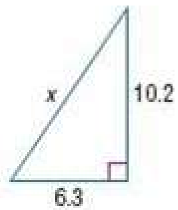


SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The length of the hypotenuse is 22 and the lengths of the legs are 13 and *x*. You can use the Pythagorean Theorem to solve for *x*.

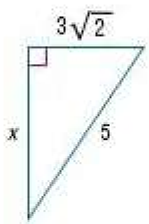
$$x^2 + 13^2 = 22^2$$
$$x^2 + 169 = 484$$
$$x^2 = 315$$
$$x = \sqrt{315}$$
$$x = 3\sqrt{35} \approx 17.7$$

67. 

SOLUTION:
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The length of the hypotenuse is *x* and the lengths of the legs are 10.2 and 6.3. You can use the Pythagorean Theorem to solve for *x*.

$$10.2^2 + 6.3^2 = x^2$$
$$104.04 + 36.69 = x^2$$
$$x^2 = 143.73$$
$$x = \sqrt{143.73}$$
$$x \approx 12.0$$

68. 

SOLUTION:
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.
The length of the hypotenuse is 5 and the lengths of the legs are $3\sqrt{2}$ and *x*. You can use the Pythagorean Theorem to solve for *x*:

$$x^2 + \left(3\sqrt{2}\right)^2 = 5^2$$
$$x^2 + 18 = 25$$
$$x^2 = 7$$
$$x = \sqrt{7}$$