

Lesson 1: Testing, Testing

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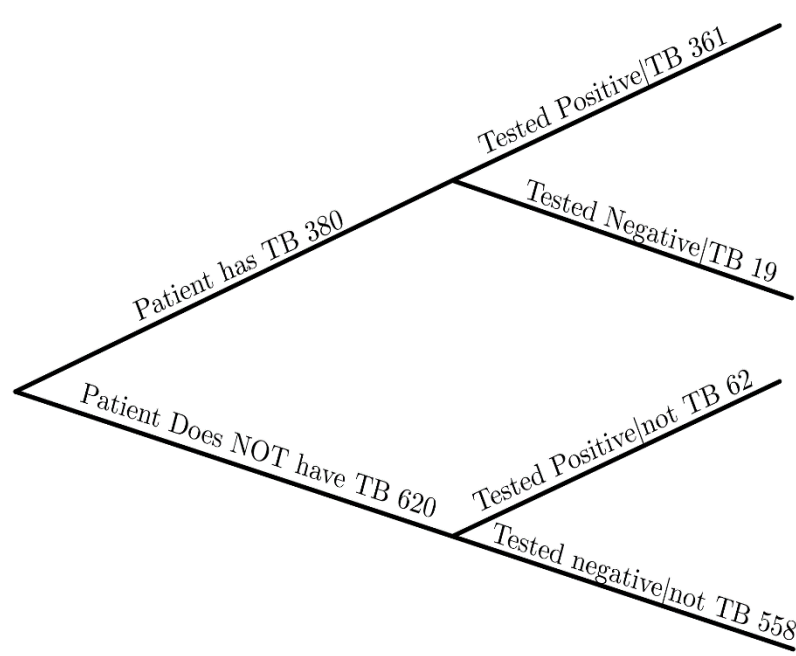
Develop Understanding

Learning Focus

- Interpret medical testing results using conditional probability.
- Use a tree diagram to find probabilities.
- How do I describe probabilities where one event seems related to another?

Open Up the Math: Launch, Explore, Discuss

Tuberculosis (TB) can be tested for in a variety of ways, including a skin test. If a person has tuberculosis antibodies, then they are considered to have TB. Below is a tree diagram representing data based on 1,000 people who have been given a skin test for tuberculosis.



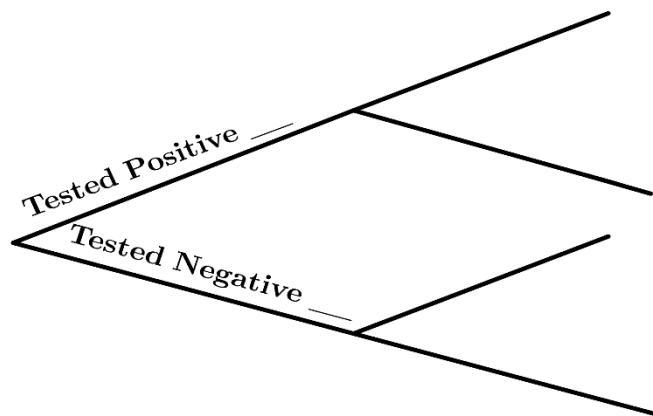
1. What do you notice about TB tests based on the tree diagram? What do you wonder?

Maybe you wondered about the notation used in statements like “Tested negative|TB.” The bar means “given that,” so this statement would be translated as “the number of patients who tested negative, given that they have TB.” We could even go one step further and write a statement about probability with this notation: $P(\textit{Tested negative}|TB) = \frac{19}{380}$.

2. How would you translate this statement? What does it mean about TB tests?

This is an example of conditional probability, which is the measure of an event, given that another event has occurred.

3. Write several other probability and conditional probability statements based on the tree diagram.
4. Now we’re going to change our perspective to see if we can gain new insights. We’re going to make the primary branches of the tree diagram the test results. It’s been started for you, so now you can finish it. You need to add what occurred in each branch and the number of people that it occurred to.



5. Write statements based on the diagram for the following probabilities:

a. $\frac{577}{1,000} = P(\text{_____})$

b. $\frac{62}{423} = P(\text{_____})$

c. $85\% = P(\text{_____})$

6. Find each probability and interpret the statement.

a. $P(\text{Have TB}) =$

What does this statement mean?

b. $P(\text{Don't have TB}) =$

What does this statement mean?

c. $P(\text{Tested negative}|\text{Don't have TB}) =$

What does this statement mean?

d. $P(\text{Tested positive}|\text{Don't have TB}) =$

What does this statement mean?

e. $P(\text{Have TB}|\text{Tested positive}) =$

What does this statement mean?

7. Find each probability and interpret the statement.

a. $P(\text{Tested negative}|\text{Has TB}) =$

What does this statement mean?

b. $P(\text{Has TB}|\text{Tested negative}) =$

What does this statement mean?

c. Explain why these two statements could have different probabilities. What is the difference in what these two statements tell us about the TB test?

Part of understanding the world around us is being able to analyze data and explain it to others.

8. Based on the probability statements that could be made from either tree diagram, what would you say to a friend regarding the validity of their results if they are testing for TB using a skin test and the result came back positive?

9. In this situation, explain the consequences of errors (having a test with incorrect results).

10. If a health test is not 100% certain, why might it be beneficial to have the results lean more toward a false positive, which means that the test indicates that you have the disease when you don't?

Ready for More?

1. Find these probabilities using the tree diagram:

a. $P(\text{Has TB and tests positive})$

b. $P(\text{Tests positive}|\text{Has TB})$

2. How are these probabilities different?

Takeaways

Conditional probability of A given B ,

Vocabulary

- **conditional probability**
- **false negative/positive**
- **tree diagram**

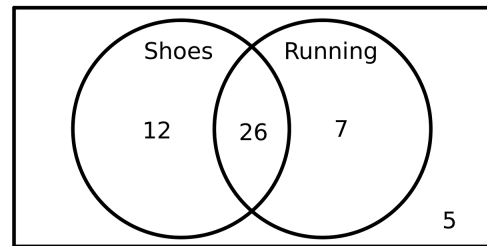
Bold terms are new in this lesson.

Lesson Summary

In this lesson, we learned about conditional probability, the probability of an event given that another event has occurred. We used basic probability statements along with conditional probability to analyze the effectiveness of a medical test and to consider the meaning of testing errors, false positives, and false negatives.

Retrieval

For problems 1–4, use the Venn diagram, which represents data collected about whether a group of people prefer wearing shoes or sandals and whether they prefer running or biking.



1. What does the overlapping section with the 26 in it represent?
2. Where on the Venn diagram do you find the numbers of people that prefer wearing sandals?
3. How many people prefer running?
4. How many total people are represented in the Venn diagram?
5. What is the probability of rolling the number 1 when rolling a standard six-sided die?
6. What is the probability of drawing an even number from a set of cards numbered 1 through 15?