Taylor Polynomials and Taylor's Inequality <u>Calculus: 2<sup>nd</sup> Edition</u> by Dennis Berkey

1a. Find the 3rd order Taylor polynomial for f(x) = ln(x + 1) centered at x = 0.
b. Then find the Lagrange Error Bound when x = .2

2a. Find the 3rd order Taylor polynomial for  $f(x) = e^x$  centered at x = 0.

b. Then use Taylors Inequality to find  $f(.4) - P_3(.4) \le R$  at x = .4

3a. Find the 3rd order Taylor polynomial for  $f(x) = \sin x$  centered at  $x = \frac{\pi}{6}$ . b. Then use the Remainder Estimation Thm to find  $|f(x) - P_1(x)| \le R$  at  $x = 32^\circ$ 

4a. Find the 2nd order Taylor polynomial for  $f(x) = \cos x$  centered at  $x = \frac{\pi}{4}$ .

b. Then use the Remainder Estimation Thm to find  $f(x) - P_2(x) \le R$  at  $x = 42^\circ$ 

5a. Find the 3rd order Taylor polynomial for f(x) = arcsin x centered at x = 0.
b. Then find the Lagrange Error Bound when x = .2

6a. Find the 1st order Taylor polynomial for  $f(x) = \frac{\ln x}{x}$  centered at x = 1.

b. Then use Taylors Inequality to find  $f(1.2) - P_1(1.2) \le R$  at x = 1.2

7a. Find the 1st order Taylor polynomial for  $f(x) = xe^{-2x}$  centered at x = 0.

b. Then use Taylors Inequality to find 
$$f(.2) - P_3(.2) \le R$$
 at  $x = .2$ 

8*a*. Find the 1st order Taylor polynomial for  $f(x) = \sqrt{3 + x^2}$  centered at x = 1*b*. Then find the Lagrange Error Bound when x = 1.2 Determine a bound on the accuracy of the given approximation for the indicated range of x

9. 
$$\sin x \approx x$$
,  $|\mathbf{x}| < .05$   
10.  $\sin x \approx x - \frac{x^3}{3!}$ ,  $|\mathbf{x}| < .15$   
11.  $\cos x \approx \frac{1}{2} - \frac{\sqrt{3}}{2} \left( x - \frac{\pi}{3} \right)$ ,  $|\mathbf{x} - \frac{\pi}{3}| < .05$   
12.  $\tan x \approx 1 + 2 \left( x - \frac{\pi}{4} \right)$ ,  $|\mathbf{x} - \frac{\pi}{4}| < \frac{\pi}{36}$   
13.  $\sqrt[3]{1 + \mathbf{x}} \approx 1 + \frac{x}{3}$ ,  $|\mathbf{x}| < .025$   
14.  $\ln x \approx (\mathbf{x} - 1) - \frac{1}{2} (\mathbf{x} - 1)^2 + \frac{1}{3} (\mathbf{x} - 1)^3$ ,  $|\mathbf{x} - 1| < ..1$   
15.  $\sqrt{1 + \mathbf{x}} \approx 1 + \frac{x}{2}$ ,  $0 < \mathbf{x} < .02$