## CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy Chapter 9: Error and Series 9.3:

What you'll Learn About How to find the error of a series that does not alternate

Lagrange Error Bound/Taylors Inequality/Remainder Estimation Theorem

- Give the first term of the series for  $f(x) = e^x$  centered at x = 0P.(x) = 1
- Find the approximation for P(.1) = 4
- Find f(.1) = e = 1,105170918
- How accurate is the approximation.  $|f(x) P_1(x)| = .105170918$
- What is the value of the next term of the polynomial at x = .1

Give the first two terms of the series for  $f(x) = e^x$  centered at x = 0

- Find the approximation for P(.1) = [1 + .1] = [. ]
- Find f(.1) = [ 105170918
- How accurate is the approximation.  $|f(.1) P_1(.1)| = .005170918$
- What is the value of the next term of the polynomial at x = .1

Next term = 
$$\frac{x^2}{2!} = .005$$

Give the first three terms of the series for  $f(x) = e^x$  centered at x = 0

- $1 + x + \frac{x^2}{2}$ Find the approximation for P(.1) =  $[1 \cdot 1 + \frac{1}{2}]^2 = 1.105$
- Find f(.1) = 1.105170918

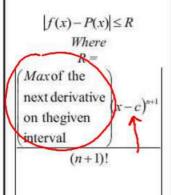
13/0)=1

Find 
$$f(.1) = [.[05110410]$$

How accurate is the approximation.  $|f(.1) - P_2(.1)| = .000170913$ 

What is the value of the next term of the polynomial at x = .1

$$\frac{25|\text{Pag}(x^3)|}{3!} = \text{next term} = \frac{|x^3|}{3!} \rightarrow \frac{1^3}{6} = \frac{.001}{6} = .000166666$$



Where x-c is the distance from the center

Where n is the order

We must build the next term a little bit bigger to have a good boundary for the error.

Remember, whenever you see this,  $|f(x)-P(x)| \le R$ , you are finding error bound

whenever you see this, f(x)-P(x), you are finding the actual error between the function and the approximation from the polynomial

Actual difference

Give the first 4 terms of the series for  $f(x)=e^x$  centered at x=0

Use Taylors Inequality to determine the error bound  $|f(x)-P(x)| \le R$ 

$$f'(x) = e^{x}$$

$$f'(x) = e^{x}$$

$$f''(x) = e^{x}$$

$$f''(x)$$

$$f(x) = \frac{1}{(1-x)^2} = (1-x)^{-2} \qquad f(0) = 1$$

$$f'(x) = +2(1-x)^{-3} \qquad f'(0) = 0$$

$$f''(x) = +6(1-x)^{-4} \qquad f''(0) = 0$$

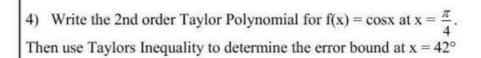
$$f'''(x) = 24(1-x)^{-5} \qquad f^{3}(0) = 24$$

$$f''(x) = 120(1-x)^{-6}$$

$$P_3(x) = 1 + 2x + \frac{6x^2}{2!} + \frac{24x^3}{3!}$$
Actual Difference

Find the Lagrange error bound f(x)-P(x) R or the series between  $0 \le x \le .2$ Build next form

$$x=0$$
 $120x^{4}$ 
 $120(.8)^{-6}x^{4} = \frac{12}{(.8)^{-6}}$ 



5) Write the 1st degree Taylor Polynomial for  $f(x) = \arcsin x$  at x = 0. Then use Taylors Inequality to determine the error bound at x = .2

Next Term
$$f'(x) = \frac{1}{\sqrt{1-x^2}} = (1-x^2) f'(0) = 1$$

$$f'(x) = \frac$$

9) Given that  $P_1(x) = x$  represents the first order polynomial for sinx centered at x = 0. Use the Lagrange Error Bound to find the error when  $|x| \leq .05$ f(x)= sinx ti(4) = COSX Build next term f"(x) = - sinx 14) Given that  $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$  represents the third order Taylor polynomial for ln(x) centered at x = 1. Use the Lagrange Error Bound to find the error when  $|x-1| \le 1$ 

## Summary of Error Bound

For an Alternating Series - Use the next term

For a series that is Not Alternating

- 1. Write down the formula for the next derivative.
- 2. Find the value of the next derivative at the ends of the interval and the center.
- 3. Whichever value is bigger is the value you use to build your error bound term

