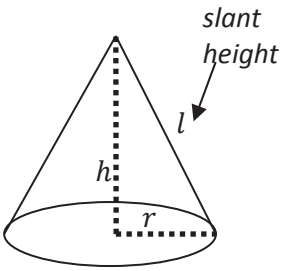
# 8.3 3D Applications of the Pythag. Theorem

Now that we know how to use the Pythagorean Theorem to find either a missing leg or missing hypotenuse, we can move this concept into three-dimensional concepts. Let's look at some examples.

## Regular Cones and Pyramids

In both cones and pyramids we can use the Pythagorean Theorem to find the height or slant height. We can also find the radius in a cone or the side length of the base of a pyramid. First look at this cone.




The height and radius are fairly obvious, but the slant height might be new vocabulary. The slant height, usually referred to as $l$ in problems, is the height from the outer bottom edge of the cone up to the tip. It is not the actual height because it is not perpendicular to the base. Notice that each of these three variables form a right triangle. Therefore if we know two of them we can find the other one.

For example, assume that $r = 5\ in.$ and $h = 12\ in.$ What is $l$, or the slant height, in inches?

$$a^2 + b^2 = c^2$$

$$r^2 + h^2 = l^2$$

$$5^2 + 12^2 = l^2$$

$$25 + 144 = l^2$$

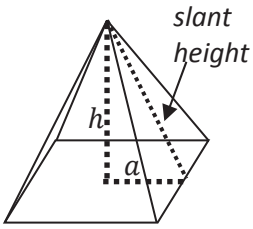
$$169 = l^2$$

$$\sqrt{169} = \sqrt{l^2}$$

$$13\ in. = l$$

Why is this useful? Knowing each dimension allows us to find the volume or surface area of the shape. For example, the formula for the surface of a cone is $SA = \pi r^2 + \pi rl$. Now that we know $l = 13\ in.$ we can find the surface area is $SA = 25\pi + 65\pi = 90\pi\ in^2$.

Let's look at a pyramid missing its height.




Assume that the base of the pyramid (bottom) is a square with side length of $12\ cm.$ and the slant height is $10\ cm.$ What is the height of the pyramid? Since we have the side length of the square, we only need half of that to form the short leg of our right triangle, or $6\ cm.$ This gives us the following:

$$a^2 + b^2 = c^2$$

$$a^2 + h^2 = l^2$$

$$6^2 + h^2 = 10^2$$

$$36 + h^2 = 100$$

$$-36 \qquad -36$$

$$h^2 = 64$$

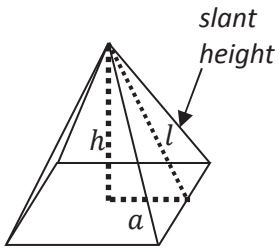
$$\sqrt{h^2} = \sqrt{64}$$

$$h = 8\ cm.$$

Again, we can now answer further questions about this shape like finding the volume. The volume of a regular pyramid uses the formula $V = \frac{1}{3}Bh$ where $B$ is the area of the base shape (the square). This means that the volume is $\frac{1}{3}(144)(8) = 384\ cm^3$.

## Lesson 8.3

***Use the picture below to find information about the pyramid with a square base in problems 1-14. Round your answers to three decimal places if necessary.***




1. The pyramid has a square base that is $70\ ft$ on each side. The slant height is $37\ ft$. What is $h$, the height of the pyramid?

2. The pyramid has a square base that is $120\ in$ on each side. The slant height is $61\ in$. What is $h$, the height of the pyramid?

3. The pyramid has a square base that is $50\ m$ on each side. The slant height is $30\ m$. What is $h$, the height of the pyramid?

4. The pyramid has a square base that is $14\ cm$ on each side. The slant height is $25\ cm$. What is $h$, the height?

5. The pyramid has a square base that is $14\ cm$ on each side. The height is $24\ cm$. What is $l$, the slant height?

6. The pyramid has a square base that is $24\ ft$ on each side. The height is $5\ ft$. What is $l$, the slant height?

7. The pyramid has a square base that is $70\ mm$ on each side. The height is $10\ mm$. What is $l$, the slant height?

8. The pyramid has a square base that is $26\ ft$ on each side. The height is $82\ ft$. What is $l$, the slant height?

9. The height of the pyramid is $15\ cm$, and the slant height is $39\ cm$. Find the value of $a$ in the diagram.

10. The height of the pyramid is $80\ in$, and the slant height is $82\ in$. Find the value of $a$ in the diagram.

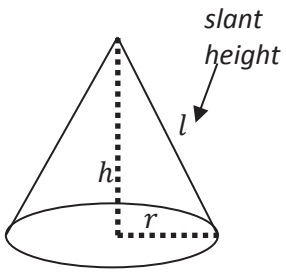
11. The slant height is $17\ ft$ and the height is $8\ ft$. What is $s$, the side length of the base?

12. The slant height is $10\ cm$ and the height is $8\ cm$. What is $s$, the side length of the base?

13. The slant height is $26\ mm$ and the height is $10\ mm$. What is $s$, the side length of the base?

14. The slant height is $50\ ft$ and the height is $32\ ft$. What is $s$, the side length of the base?

***Use the picture below to find information about the pyramid in problems 15-26. Round your answers to three decimal places if necessary.***




15. The cone has a radius of $12\ cm$ and a height of $5\ cm$. What is $l$, the slant height of the cone?

16. The cone has a radius of $15\ mm$ and a height of $8\ mm$. What is $l$, the slant height of the cone?

17. The cone has a radius of $24\ in$ and a height of $70\ in$. What is $l$, the slant height of the cone?

18. The cone has a radius of $40\ cm$ and a height of $42\ cm$. What is $l$, the slant height of the cone?

19. The cone has a radius of $30\ ft$ and a slant height of $34\ ft$. What is $h$, the height of the cone?

20. The cone has a radius of $33\ m$ and a slant height of $65\ m$. What is $h$, the height of the cone?

21. The cone has a radius of $16\ in$ and a slant height of $20\ in$. What is $h$, the height of the cone?

22. The cone has a radius of $30\ cm$ and a slant height of $50\ cm$. What is $h$, the height of the cone?

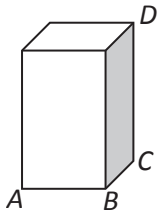
23. The cone has a height of $16\ cm$ and a slant height of $65\ cm$. What is $r$, the radius of the cone?

24. The cone has a height of $48\ ft$ and a slant height of $50\ ft$. What is $r$, the radius of the cone?

25. The cone has a height of $4\ in$ and a slant height of $6\ in$. What is $r$, the radius of the cone?

26. The cone has a height of $14\ cm$ and a slant height of $55\ cm$. What is $r$, the radius of the cone?

***Use the picture below to find lengths of segments in the rectangular prism in problems 27-38. Round your answers to three decimal places if necessary.***




27. The length of $\overline{AB}$ is $6\ ft$ and the length of $\overline{BC}$ is $8\ ft$. Find the length of $\overline{AC}$.

28. The length of $\overline{AB}$ is $40\ mm$ and the length of $\overline{BC}$ is $42\ mm$. Find the length of $\overline{AC}$.

29. The length of $\overline{AB}$ is $23\ cm$ and the length of $\overline{BC}$ is $70\ cm$. Find the length of $\overline{AC}$.

30. The length of $\overline{AB}$ is $7\ in$ and the length of $\overline{BC}$ is $7\ in$. Find the length of $\overline{AC}$.

31. The length of $\overline{AC}$ is $13\ mm$ and the length of $\overline{DC}$ is $84\ mm$. Find the length of $\overline{AD}$.

32. The length of $\overline{AC}$ is $5\ ft$ and the length of $\overline{DC}$ is $12\ ft$. Find the length of $\overline{AD}$.

33. The length of $\overline{AC}$ is $11\ mm$ and the length of $\overline{DC}$ is $30\ mm$. Find the length of $\overline{AD}$.

34. The length of $\overline{AC}$ is $5\ in$ and the length of $\overline{DC}$ is $4\ in$. Find the length of $\overline{AD}$.

35. The length of $\overline{AB}$ is $4\ ft$, the length of $\overline{BC}$ is $3\ ft$ and the length of $\overline{DC}$ is $12\ ft$. Find the length of $\overline{AD}$.

36. The length of $\overline{AB}$ is $12\ cm$, the length of $\overline{BC}$ is $5\ cm$ and the length of $\overline{DC}$ is $84\ cm$. Find the length of $\overline{AD}$.

37. The length of $\overline{AB}$ is $2\ ft$, the length of $\overline{BC}$ is $3\ ft$ and the length of $\overline{DC}$ is $10\ ft$. Find the length of $\overline{AD}$.

38. The length of $\overline{AB}$ is $6\ mm$ the length of $\overline{BC}$ is $8\ mm$ and the length of $\overline{DC}$ is $50\ mm$. Find the length of $\overline{AD}$.