4.2 Graphs of Linear Functions

In Unit 3 we graphed functions, so we already know how to graph linear (and proportional) functions. After a review of that, we'll discuss solving with a graph and comparing graphs.

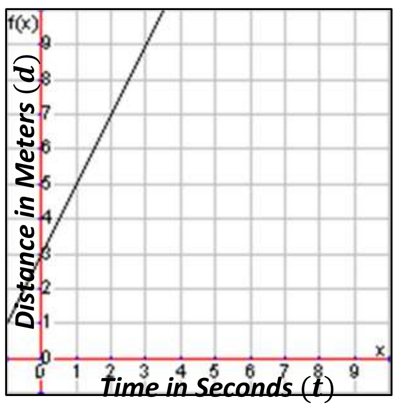
Graphing Linear Functions with Initial Values

If a linear function is given to us as an equation, we simply graph it using an x/y chart like we did previously in Unit 3.

Example 1: *Adam was given a 3 meter head start and runs at 2 m/s which is the equation* $d = 2t + 3$.

Notice in this case we don't have an x and y as the variable, but we know that the variable d is equivalent to the variable y since it is the dependent variable or output and the variable t is equivalent to the variable x since it is the independent variable or input. Thus we can graph as follows:

t	0	1	2	3
d	3	5	7	9



Example 2: *A company uses* 2 *bottles of ink to print* 3 *t-shirts and the machine uses one bottle to warm up.*

Let's start by defining the variables and writing an equation:

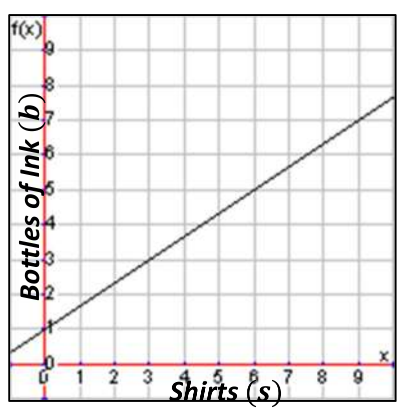
$b = bottles\ of\ ink\ used$

$s = shirts\ printed$

$b = \frac{2}{3}s + 1$

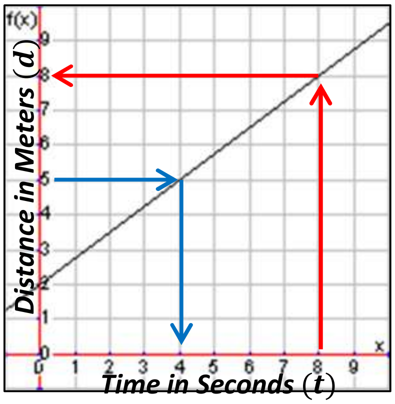
Now graph with an x/y chart as follows:

s	0	3	6
b	1	3	5



Solving Linear Situations with Graphs

If we are given a graph with an initial value, we can still solve linear problems. Consider the following graph and answer the question, "How far did the toy car travel after 8 seconds?"

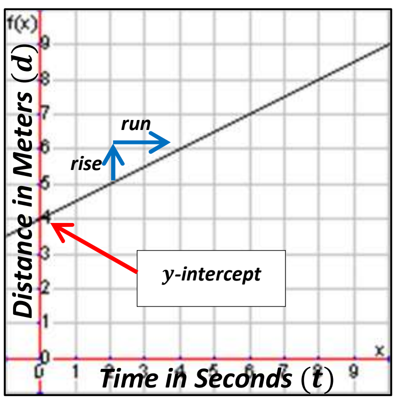


To solve using the graph, simply go to 8 seconds and move up to the line. How far did the car travel? It looks like the toy car has traveled 8 meters at that point.

Similarly we could ask a question such as, "How long did it take the toy car to travel 5 meters?" Go up to five meters and move over to the line. Looking straight down from that point we see that it took the toy car 4 seconds.

Getting Equations from Graphs with Initial Values

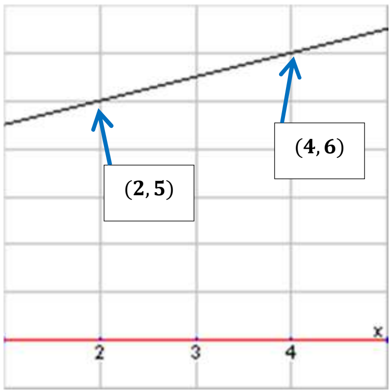
We can still get equations from graphs with initial values, but it will take the extra step of looking for the y-intercept on the graph.



First, find the slope using two points from the graph. Notice that on this graph we have a point on the line at $(2,5)$ and $(4,6)$. Those are not the only two nice integer value points on this line, but we only need two. Since the slope can be thought of as $\frac{rise}{run}$, we simply look for how far the line rises and runs between those two points.

It goes from 5 meters to 6 meters, so that is a rise of 1. It goes from 2 seconds to 4 seconds which means a run of 2. Therefore the rise over the run is $\frac{1}{2}$. This is our slope, rate of change, or lowest terms proportion ratio (whatever we want to call it).

We are still missing the y-intercept, or initial value, but we can clearly see it on the graph. The line crosses the y-axis at 4, so our initial value is 4. Now we can write the equation: $d = \frac{1}{2}t + 4$. What if we didn't have the whole graph, but only those two points to work with? Let's zoom in and find out.



We still have our two points and therefore can find the slope, but we can't see the initial value. However, since we know the slope and a point on the line, we can substitute those values in to find the initial value.

$$d = mt + b$$

Substitute the slope

$$d = \frac{1}{2}t + b$$

Substitute one point

$$6 = \frac{1}{2}(4) + b$$

$$6 = 2 + b$$

Solve for initial value

$$6 - 2 = 2 + b - 2$$

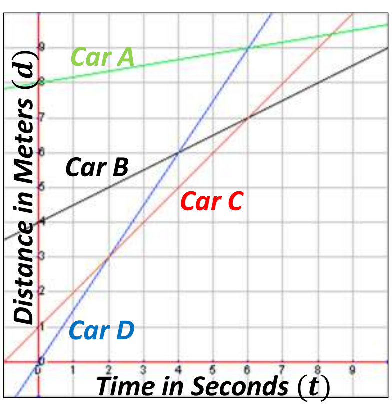
$$4 = b$$

Write the equation

$$d = \frac{1}{2}t + 4$$

Comparing Linear Function Graphs

We may need to compare graphs to other graphs or compare graphs to equations. Just as with equations, one of the main comparison points is the slope of the graph. For example, looking at the following graph, we could ask which toy race car is faster.



To compare these race cars, we need to know their speeds. The speed is a measure of distance over time, which is the slope or steepness of each line. So let's find the slope for each race car. After finding the rise and run for each car, we should come to the following values:

Car A: $\frac{1}{6}\ m/s$ **Car B:** $\frac{1}{2}\ m/s$ **Car C:** $1\ m/s$ **Car D:** $\frac{3}{2}\ m/s$

Based on this information, we see that Car D is the fastest car. We also know that Car A is the slowest. The curious thing is that Car A, even though it is the slowest, stays above most of the lines for the majority of the graph. This is because of the initial value for each car.

The initial value effectively is a head start for each of the cars. Notice the following initial values:

Car A: $8\ m$ **Car B:** $4\ m$ **Car C:** $1\ m$ **Car D:** $0\ m$

The reason the slowest car is ahead in the race for most of the graph is because of its tremendous head start. It received a whopping 8 meter head start! Also note that Car D did not have a head start and is therefore proportional.

We could also ask lots of other interesting questions now. For example, if the race were only 5 seconds long, which car would travel the farthest? Go to 5 seconds on the graph and move up. Which car is highest? Car A (again, due to the head start).

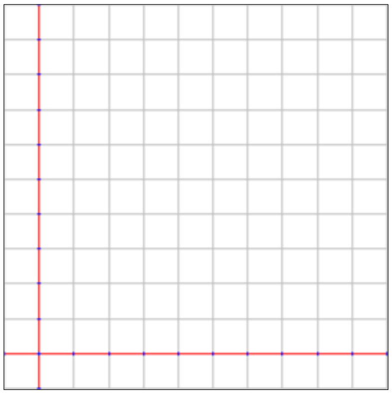
If the race were 10 meters long, which car would get to the finish line first? Go up to 10 meters and look over. Which car do you hit first? Car D finishes somewhere between 6 and 7 seconds.

If the race were 10 meters long, what order would the cars finish in? Go up to 10 meters and look over. Car D finishes first and then Car C, but we're not sure when Car A and B finish. It turns out that Car A and B would tie for last place at 12 seconds. Can you prove it?

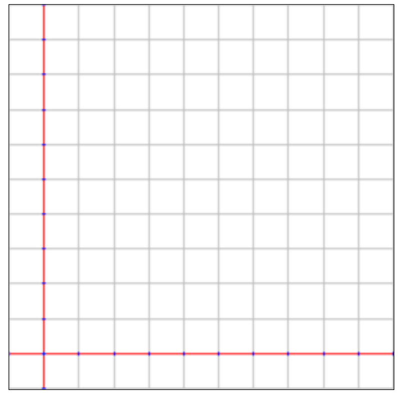
Lesson 4.2

Create a graph for each of the following linear situations or equations.

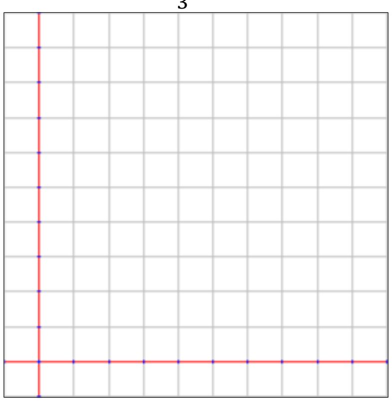
1. At Pizza Hut It costs \$8 for each large pizza plus \$5 for a delivery tip.



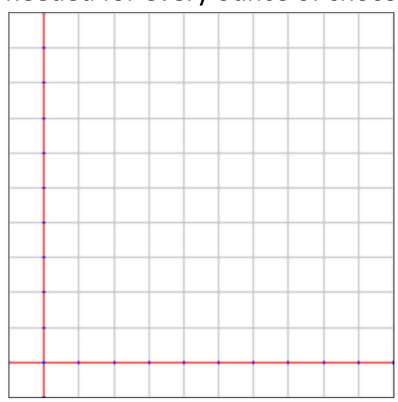
2. It costs \$40 per ticket to Six Flags plus \$100 for gas there and back.



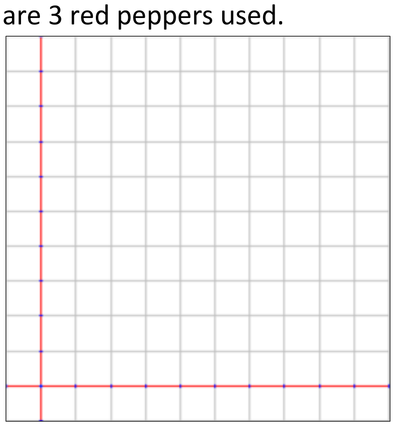
3. The number of lives (l) based on the number of levels completed (c) is determined by the following equation: $l = \frac{1}{3}c + 4$



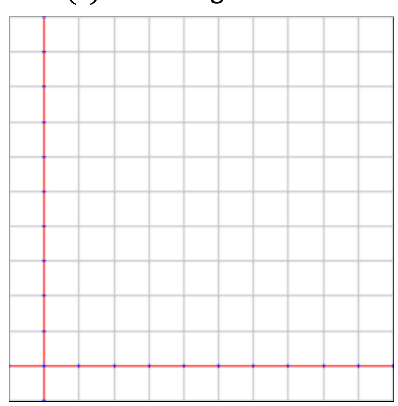
4. When making fudge, four ounces of sugar are needed for every ounce of chocolate.



5. For every 2 green peppers used in a salsa there are 3 red peppers used.

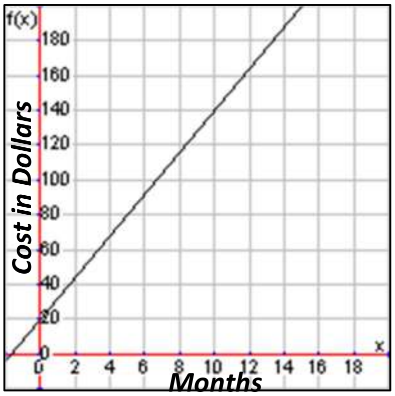


6. Mutant alien frogs from Zappax have a number of feet (f) based on the number of toes they are born with (t) according to the following equation: $f = \frac{1}{7}t$.

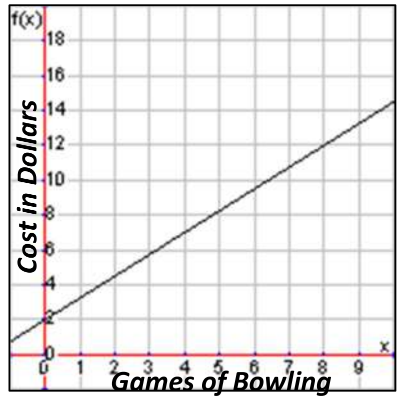


Use the given graph to solve the linear questions.

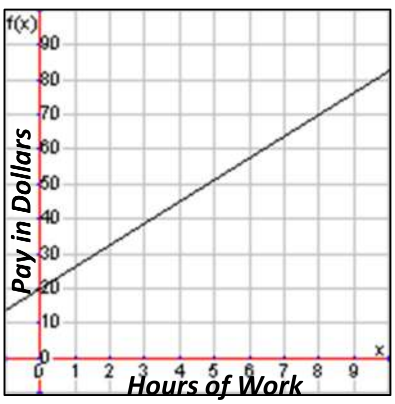
7. How much will it cost for ten months of internet service?



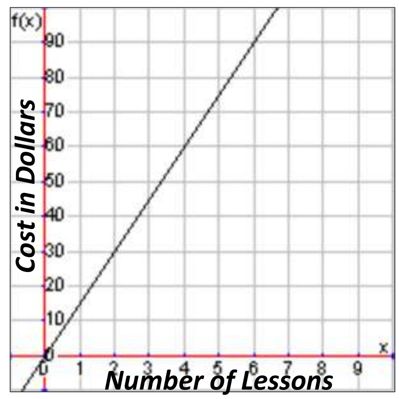
8. How many games of bowling can you play if you can spend $12?



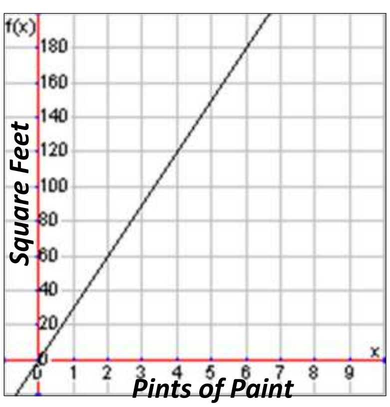
9. How many hours would you have to work to earn $70?



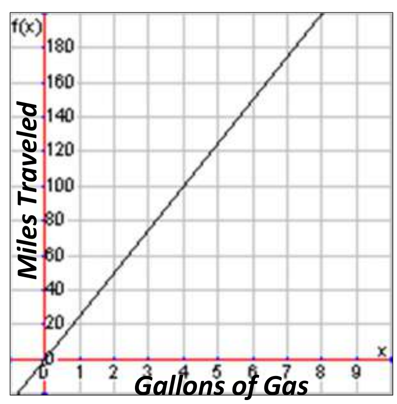
10. How much would it cost for four lessons?



11. How many pints of paint should you buy if you have to paint 120 square feet?



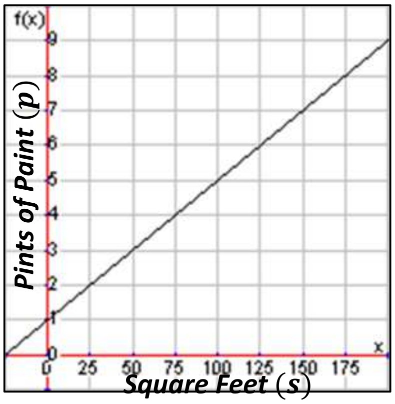
12. How many miles can you travel if you have four gallons of gas left in your tank?



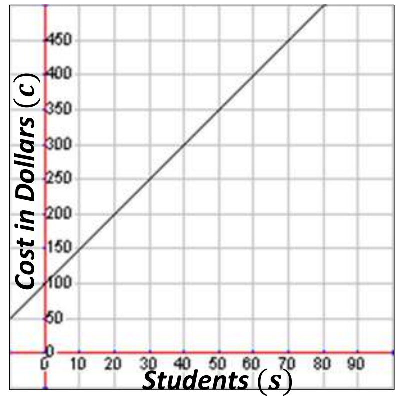
13. Which of the above graphs are proportional situations and how do you know?

Create an equation for the following linear graphs.

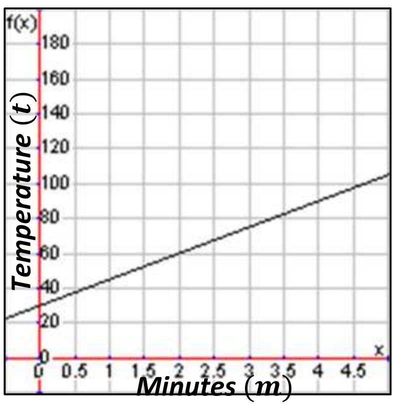
14. Number of pints of paint (p) needed for a certain number of square feet (s)



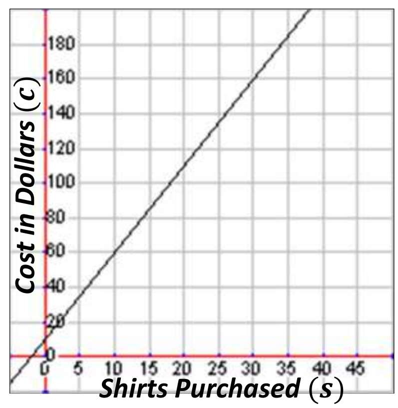
15. The cost (c) of a field trip based on the number of students (s) attending



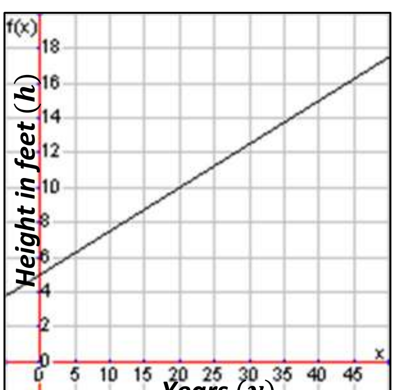
16. Temperature (t) of water per minute (m) of time on the stove



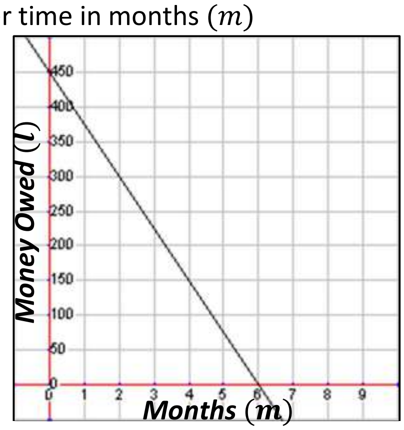
17. Cost (c) of an order depending on the number of shirts (s) purchased



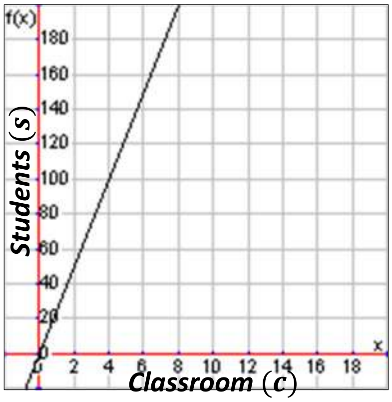
18. A tree's height (h) based on the number of years (y) since being transplanted



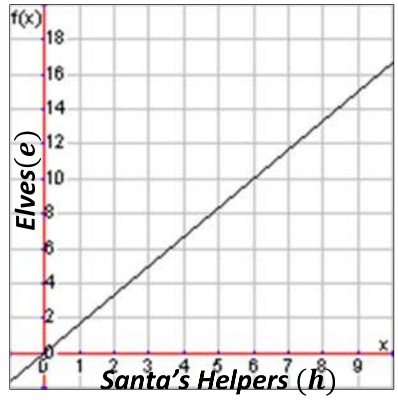
19. Money you owe on your loan (l) for your first car over time in months (m)



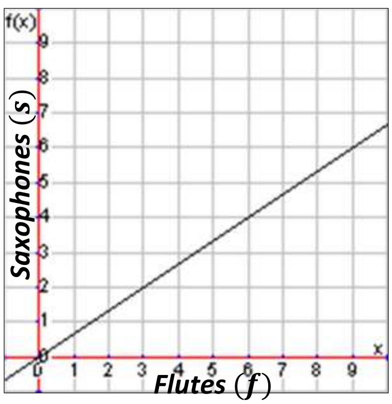
20. Number of students (s) in every classroom (c)



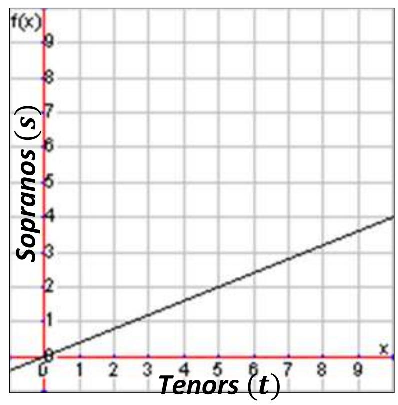
21. Number of elves (e) for Santa's helpers (h)



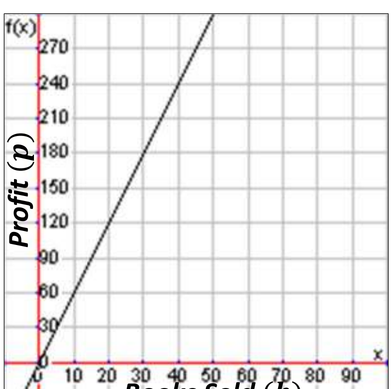
22. Number of saxophones (s) compared to the number of flutes (f) in an orchestra



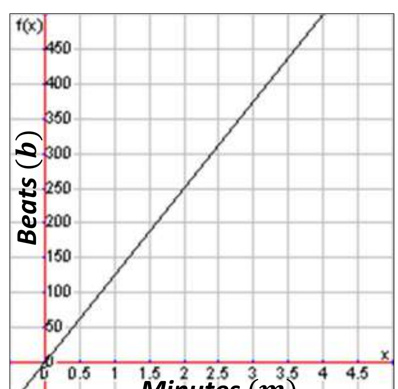
23. Number of sopranos (s) compared to the number of tenors (t) in a choir.



24. Amount of profit (p) based on the number of books sold (b)



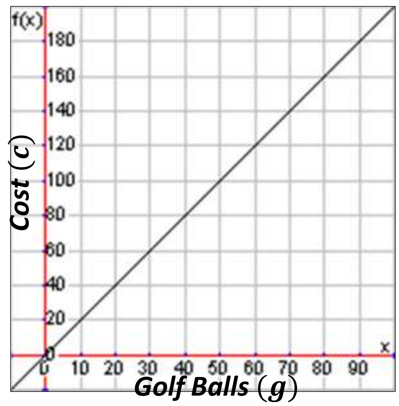
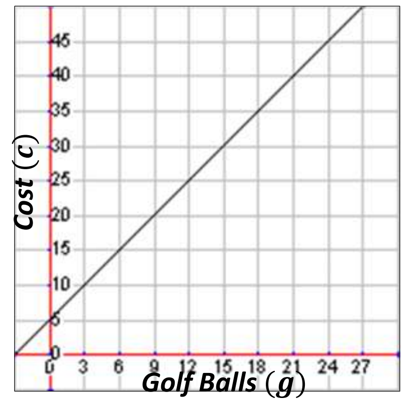
25. Number of beats (b) per minute (m) in a hip-hop song



26. Which of the graphs from problems 14 to 25 are proportional and how do you know?

Answer the following questions comparing linear function equations, graphs and descriptions.

Various golf ball manufacturers offer deals for packs of golf balls. Here is the information about the total cost (c) *for golf balls* (g) *including shipping costs.*

Callaway Charges a fee of $10 for shipping and $5 for 3 golf balls	**Nike** Cost is modeled by the equation $c = \frac{5}{2}g + 5$
Titleist 	**Top-Flight** 

27. Which manufacturer has the cheapest cost per golf ball, and how do you know?

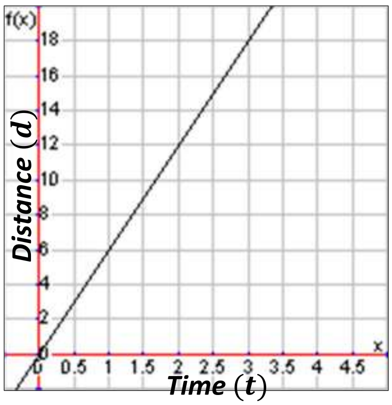
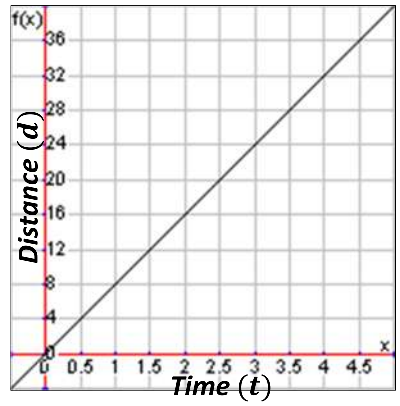
28. Which manufacturer has the cheapest shipping fee, and how do you know?

29. How many golf balls could you buy at each company for $200? Which manufacture would give you the most golf balls for that amount of money?

30. Which manufacturer would be the cheapest if you wanted to buy 30 golf balls?

Answer the following questions comparing proportional function equations, graphs and descriptions.

Scientists are studying how location affects the speed of a bottlenose dolphin. Here is the information about the distance (d) *in kilometers a dolphin traveled in terms of time* (t) *in hours.*

Dolphin in Gulf of Mexico Swims 11 kilometers in 2 hours	**Dolphin in Mediterranean Sea** Distance is modeled by the equation $d = \frac{35}{4}t$
Dolphin in Indian Ocean 	**Dolphin in North Atlantic Ocean** 

31. Which location has the fastest dolphin?

32. Which location has the slowest dolphin?

33. How far could each dolphin travel in 4 hours? Which location has the dolphin that went the farthest?

34. How long would it take each dolphin to swim 100 kilometers? Which location has the dolphin that finished in the shortest amount of time?