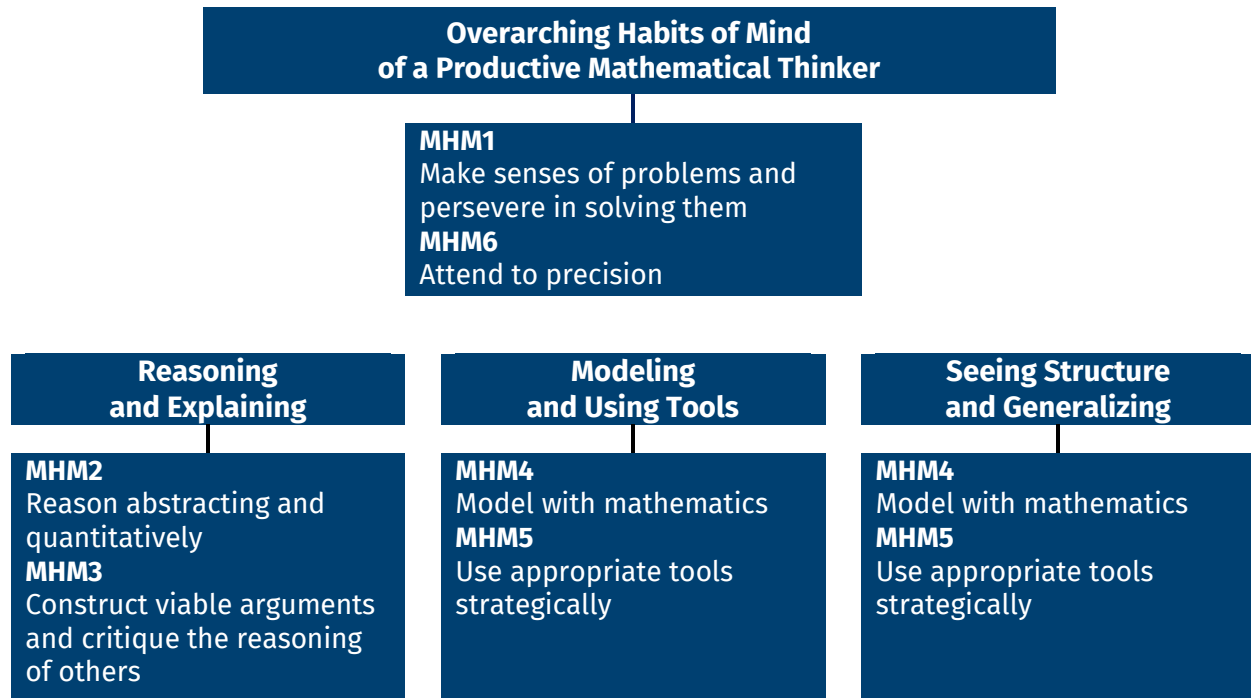# Kindergarten

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker
MHM1 Make senses of problems and persevere in solving them **MHM6** Attend to precision

Reasoning and Explaining	Modeling and Using Tools	Seeing Structure and Generalizing
MHM2 Reason abstracting and quantitatively **MHM3** Construct viable arguments and critique the reasoning of others	**MHM4** Model with mathematics **MHM5** Use appropriate tools strategically	**MHM4** Model with mathematics **MHM5** Use appropriate tools strategically

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B
West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples.

They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Policy 2520.2B
West Virginia College- and Career-Readiness Standards for Mathematics

Kindergarten

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in kindergarten will focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space.   Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. The skill progressions begin in kindergarten as foundational understanding of numeracy. The following chart represents the mathematical understandings that will be developed in kindergarten:

Counting and Cardinality	Operations and Algebraic Thinking
• Count objects to tell how many there are by ones and by tens. • Write numbers from 0 to 20. • Compare two groups of objects to tell which group, if either, has more; compare two written numbers to tell which is greater. • Group pennies.	• Understand addition as putting together and adding to. • Understand subtraction as taking apart and taking from. • Add and subtract very small numbers quickly and accurately (e.g., 3 + 1).
Number and Operations in Base Ten	**Measurement and Data**
• Act out addition and subtraction word problems and draw diagrams to represent them. • Add with a sum of 10 or less; subtract from a number 10 or less; and solve addition and subtraction word problems. • Group objects by tens and ones. (1 group of 10 and 3 ones makes 13)	• Describe and compare objects as longer, shorter, larger, smaller, etc. • Classify objects and count the number of objects in each category. (e.g., Identify coins and sort them into groups of 5s or 10s.)
Geometry	
• Name shapes correctly regardless of orientation or size (e.g., a square oriented as a "diamond" is still a square).	

<u>Numbering of Standards</u>
The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Counting and Cardinality	
Know number names and the count sequence.	Standards 1-3
Count to tell the number of objects.	Standards 4-5
Compare numbers.	Standards 6-7
Operations and Algebraic Thinking	
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	Standard 8-12
Number and Operations in Base Ten	
Work with numbers 11-19 to gain foundations for place value.	Standard 13
Measurement and Data	
Describe and compare measurable attributes.	Standards 14-15
Classify objects and count the number of objects in each category.	Standard 16
Geometry	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)	Standards 17-19
Analyze, compare, create, and compose shapes	Standards 20-22

Counting and Cardinality

Cluster	Know number names and the count sequence.
M.K.1	Count to 100 by ones and by tens.
M.K.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
M.K.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Cluster	Count to tell the number of objects.
M.K.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
M.K.5	Count to answer questions (e.g., "How many?") about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Cluster	Compare numbers.
M.K.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies).
M.K.7	Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking

Cluster	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
M.K.8	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations.
M.K.9	Solve addition and subtraction word problems and add and subtract within 10 by using objects or drawings to represent the problem.
M.K.10	Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M.K.11	For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.
M.K.12	Fluently add and subtract within 5.

Number and Operations in Base Ten

Cluster	Work with numbers 11-19 to gain foundations for place value.
M.K.13	Compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones (one ten) and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Cluster	Describe and compare measurable attributes.
M.K.14	Describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object.
M.K.15	Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.

Cluster	Classify objects and count the number of objects in each category.
M.K.16	Classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10. (e.g., Identify coins and sort them into groups of 5s or 10s.)

Geometry

Cluster	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
M.K.17	Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to.
M.K.18	Correctly name shapes regardless of their orientations or overall size.
M.K.19	Through the use of real-life objects, identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Cluster	Analyze, compare, create and compose shapes.
M.K.20	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners"), and other attributes (e.g., having sides of equal length). Instructional Note: Student focus should include real-world shapes.
M.K.21	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.