2.4 Two-Step Equations

Learning Objectives

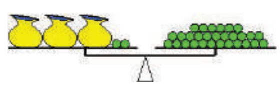
- Solve a two-step equation using addition, subtraction, multiplication, and division.
- Solve a two-step equation by combining like terms.
- Solve real-world problems using two-step equations.

Solve a Two-Step Equation

We have seen that in order to solve for an unknown variable we can isolate it on one side of the equal sign and evaluate the numbers on the other side. In this chapter we will expand our ability to do that, with problems that require us to combine more than one technique in order to solve for our unknown.

Example 1

Rebecca has three bags containing the same number of marbles, plus two marbles left over. She places them on one side of a balance. Chris, who has more marbles than Rebecca, added marbles to the other side of the balance. He found that with 29*marbles, the scales balanced. How many marbles are in each bag? Assume the bags weigh nothing.*



Solution

We know that the system balances, so the weights on each side must be equal. We can write an algebraic expression based on this equality. The unknown quantity, the number of marbles in each bag, will be our x. We can see that on the left hand scale we have three bags (each containing x marbles) and two extra marbles. On the right scale we have 29 marbles. The balancing of the scales is similar to the balancing of the following equation.

$$3x+2=29$$

Three bags plus two marbles **equals** 29 marbles

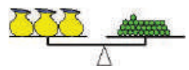
To solve for x we need to first get all the variables (terms containing an x) alone on one side. Look at the balance. There are no bags on the right. Similarly, there are no x terms on the right of our equation. We will aim to get all the constants on the right, leaving only the x on the left.

$$\begin{aligned} 3x+\cancel{2} &= 29 \\ \cancel{-2} &= -2 \qquad \text{Subtract 2 from both sides :} \\ \hline 3x &= 27 \\ \frac{\cancel{3}x}{\cancel{3}} &= \frac{27}{3} \qquad \text{Divide both sides by 3} \\ x &= 9 \end{aligned}$$

Solution

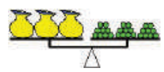
There are nine marbles in each bag.

We can do the same with the real objects as we have done with the equation. Our first action was to subtract two from both sides of the equals sign. On the balance, we could remove this number of marbles from each scale. Because we remove the same number of marbles from each side, we know the scales will still balance.



Next, we look at the left hand scale. There are three bags of marbles. To make our job easier, we divide the marbles on the right scale into three equal piles. You can see that there are nine marbles in each.

Three bags of marbles ***balances*** *three piles of nine marbles*



So each bag of marble balances nine marbles. Again you see we reach our solution:

Solution

Each bag contains nine marbles.

On the web: http://www.mste.uiuc.edu/pavel/java/balance/ has interactive balance beam activities!

Example 2

Solve $6(x+4) = 12$

Solution

This equation has the *xburied* in parentheses. In order to extract it we can proceed in one of two ways: we can either distribute the six on the left, or divide both sides by six to remove it from the left. Since the right hand side of the equation is a multiple of six, it makes sense to divide.

$$\begin{aligned}
6(x+4) &= 12 && \text{Divide both sides by } 6.\\
\frac{\cancel{6}(x+4)}{\cancel{6}} &= \frac{12}{6}\\
x+\cancel{4} &= 2 && \text{Subtract 4 from both sides.}\\
-\cancel{4} & -4\\
\hline
x &= -2
\end{aligned}$$

Solution

$$x = -2$$

Example 3

Solve $\frac{x-3}{5} = 7$

This equation has a fraction in it. It is always a good idea to get rid of fractions first.

$$\left(x - \frac{3}{5}\right) = 7$$

Solution:

$$\begin{aligned} \not{5}\left(\frac{x-3}{\not{5}}\right) &= 5 \cdot 7 && \text{Multiply both sides by 5} \\ x - 3 &= 35 \\ +3 &= +3 && \text{Add 3 to both sides} \\ \hline x &= 38 \end{aligned}$$

Solution

$$x = 38$$

Example 4

Solve $\frac{5}{9}(x+1) = \frac{2}{7}$

First, we will cancel the fraction on the left (making the coefficient equal to one) by multiplying by the reciprocal (the multiplicative inverse).

$$\frac{\not{9}}{\not{5}} \cdot \frac{\not{5}}{\not{9}}(x+1) = \frac{9}{5} \cdot \frac{2}{7}$$

$$\begin{aligned} x + 1 &= \frac{18}{35} && \text{Subtract1} \left(1 = \frac{35}{35}\right) \text{ from both sides.} \\ x &= \frac{18}{35} - \frac{35}{35} \\ x &= \frac{18-35}{35} \end{aligned}$$

Solution

$$x = -\frac{17}{35}$$

These examples are called **two-step equations,** as we need to perform two separate operations to the equation to isolate the variable.

Solve a Two-Step Equation by Combining Like Terms

When we look at linear equations we predominantly see two terms, those that contain the unknown variable as a factor, and those that dont. When we look at an equation that has an x on both sides, we know that in order to solve, we need to get all the $x-$terms on one side of the equation. This is called **combining like terms.** They are **like terms** because they contain the same variable (or, as you will see in later chapters, the same combination of variables).

Like Terms

- $17x, 12x, -1.2x$, and $\frac{17x}{9}$
- $3y, 19y$, and $\frac{y}{99}$
- $xy, 6xy$, and $0.0001xy$

Unlike Terms

- $3x$ and $2y$
- $12xy$ and $2x$
- $0.001x$ and 0.001

To add or subtract like terms, we can use the Distributive Property of Multiplication instead of addition and subtraction.

$$\begin{aligned}3x+4x&=(3+4)x=7x\\0.03xy-0.01xy&=(0.03-0.01)xy=0.02xy\\-y+16y-5y&=(-1+16-5)y=10y\\5z+2z-7z&=(5+2-7)z=0z=0\end{aligned}$$

To solve an equation with two or more like terms we need to combine them before we can solve for the variable.

Example 5

Solve $(x+5)-(2x-3)=6$

There are two like terms. The x and the $-2x$ (do not forget that the negativesign multiplies everything in the parentheses).

Collecting like terms means we write all the terms with matching variables together. We will then add, or subtract them individually. We pull out the x from the first bracket and the $-2x$ from the second. We then rewrite the equation collecting the like terms.

$$(x-2x)+(5-(-3))=6 \qquad \text{Combine like terms and constants.}$$

$$\begin{aligned}-x+\cancel{8}&=6\\ \cancel{-8}&=-8 &&\text{Subtract 8 from both sides}\\ \hline -x&=-2 &&\text{Mutliply both sides by }-1\text{ to get the variable by itself}\end{aligned}$$

Solution

$$x=2$$

Example 6

Solve $\frac{x}{2} - \frac{x}{3} = 6$

Solution

This problem involves fractions. Combining the variable terms will require dealing with fractions. We need to write all the terms on the left over a common denominator of six.

$$\frac{3x}{6} - \frac{2x}{6} = 6 \qquad \text{Next we combine the fractions.}$$
$$\frac{x}{6} = 6 \qquad \text{Multiply both sides by 6.}$$
$$x = 36$$

Solve Real-World Problems Using Two-Step Equations

When we are faced with real world problems the thing that gives people the most difficulty is going from a problem in words to an equation. First, look to see what the equation is asking. What is the **unknown** for which you have to solve? That will determine the quantity we will use for our **variable.** The text explains what is happening. Break it down into small, manageable chunks. Then, follow what is going on with our variable all the way through the problem.

Example 7

An emergency plumber charges \$65*as a call-out fee plus an additional* \$75 perhour per hour. He arrives at a house at 9 : 30 and works to repair a water tank. If the total repair bill is \$196.25*,* at what time was the repair completed?

In order to solve this problem, we collect the information from the text and convert it to an equation.

Unknown time taken in hours this will be our x

The bill is made up of two parts: a call out fee and a per-hour fee. The call out is a flat fee, and independent of x. The per-hour part depends on x. Lets look at how this works algebraically.

\$65 as a call-out fee	65
Plus an additional \$75 per hour	$+75x$

So the bill, made up from the call out fee plus the per hour charge times the hours taken creates the following equation.

$$\text{Total Bill} = 65 + 75x$$

Lastly, we look at the final piece of information. The total on the bill was \$196.25. So our final equation is:

$$196.25 = 65 + 75x$$

We solve for x:

$$\begin{aligned} 196.25 &= \cancel{65} + 75x \\ -65 &= \cancel{-65} \\ \hline 131.25 &= 75x \end{aligned}$$

To isolate x first subtract 65 from both sides :

Divide both sides by 75

$$\frac{131.25}{75} = x = 1.75$$

The time taken was one and three quarter hours.

Solution

The repair job was completed at 11:15AM.

Example 8

When Asia was young her Daddy marked her height on the door frame every month. Asias Daddy noticed that between the ages of one and three, he could predict her height (in inches) by taking her age in months, adding 75inches and multiplying the result by one-third. Use this information to determine the following

a) Write an equation linking her predicted height, h, *with her age in months*, m.

b) Determine her predicted height on her second birthday.

c) Determine at what age she is predicted to reach three feet tall.

a) To convert the text to an equation, first determine the type of equation we have. We are going to have an equation that links **two variables.** Our unknown will change, depending on the information we are given. For example, we could solve for height given age, or solve for age given height. However, the text gives us a way to determine **height.** Our equation will start with $h =$.

Next we look at the text.

$$(m+75) \qquad \text{Take her age in months, and add75.}$$

$$\frac{1}{3}(m+75) \qquad \text{Multiply the result by one-third.}$$

Solution

Our full equation is $h = \frac{1}{3}(m+75)$.

b) To determine the prediction of Asias height on her second birthday, we substitute $m = 24(2 \text{ years} = 24 \text{ months})$ into our equation and solve for h.

$$h = \frac{1}{3}(24+75) \qquad \text{Combine terms in parentheses.}$$

$$h = \frac{1}{3}(99) \qquad \text{Multiply.}$$

$$h = 33$$

Solution

Asias height on her second birthday was predicted to be 33 inches .

c) To determine the predicted age when she reached three feet, substitute $h = 36$ into the equation and solve for m.

$$\begin{aligned} 36 &= \frac{1}{3}(m+75) && \text{Multiply both sides by 3.} \\ 108 &= m+75 && \text{Subtract 75 from both sides.} \\ 33 &= m \end{aligned}$$

Solution

Asia was predicted to be 33 months old when her height was three feet.

Example 9

To convert temperatures in Fahrenheit to temperatures in Celsius follow the following steps: Take the temperature in Fahrenheit and subtract 32. *Then divide the result by* 1.8*and this gives temperature in degrees Celsius.*

a) Write an equation that shows the conversion process.

b) Convert 50*degrees Fahrenheit to degrees Celsius.*

c) Convert 25*degrees Celsius to degrees Fahrenheit.*

d) Convert -40*degrees Celsius to degrees Fahrenheit.*

a) The text gives the process to convert Fahrenheit to Celsius. We can write an equation using two variables. We will use f for temperature in Fahrenheit, and c for temperature in Celsius. Follow the text to see it work.

$$\begin{aligned} &(F-32) && \text{Take the temperature in Fahrenheit and subtract 32.} \\ &\frac{F-32}{1.8} && \text{Then divide the result by 1.8.} \\ C = &\frac{F-32}{1.8} && \text{This gives temperature in degrees Celsius.} \end{aligned}$$

In order to convert from one temperature scale to another, simply substitute in for the **known** temperature and solve for the **unknown.**

b) To convert 50degrees Fahrenheit to degrees Celsius substitute $F = 50$ into the equation.

$$\begin{aligned} C &= \frac{50-32}{1.8} && \text{Evaluate numerator.} \\ C &= \frac{18}{1.8} && \text{Perform division operation.} \end{aligned}$$

Solution

$C = 10$, so 50 degrees Fahrenheit is equal to 10 degrees Celsius.

ci) To convert 25 degrees Celsius to degrees Fahrenheit substitute $C = 25$ into the equation:

$25 = \frac{F-32}{1.8}$	Multiply both sides by 1.8
$45 = F - 32$	
$+32 = +32$	Add 32 to both sides.
$77 = F$	

Solution

25 degrees Celsius is equal to 77 degrees Fahrenheit.

d) To convert -40 degrees Celsius to degrees Fahrenheit substitute $C = -40$ into the equation.

$-40 = \frac{F-32}{1.8}$	Multiply both sides by 1.8.
$-72 = F - 32$	
$+32 = +32$	Add 32 to both sides.
$-40 = F$	

Solution

-40 degrees Celsius is equal to -40 degrees Fahrenheit.

Lesson Summary

- Some equations require more than one operation to solve. Generally it, is good to go from the outside in. If there are parentheses around an expression with a variable in it, cancel what is outside the parentheses first.
- Terms with the same variable in them (or no variable in them) are **like terms. Combine like terms** (adding or subtracting them from each other) to simplify the expression and solve for the unknown.

Review Questions

1. Solve the following equations for the unknown variable.
   (a) $1.3x - 0.7x = 12$
   (b) $6x - 1.3 = 3.2$
   (c) $5x - (3x + 2) = 1$
   (d) $4(x + 3) = 1$
   (e) $5q - 7 = \frac{2}{3}$
   (f) $\frac{3}{5}x + \frac{5}{2} = \frac{2}{3}$
   (g) $s - \frac{3s}{8} = \frac{5}{6}$
   (h) $0.1y + 11 = 0$
   (i) $\frac{5q-7}{12} = \frac{2}{3}$
   (j) $\frac{5(q-7)}{12} = \frac{2}{3}$
   (k) $33t - 99 = 0$
   (l) $5p - 2 = 32$
2. Jade is stranded downtown with only \$10 to get home. Taxis cost \$0.75 per mile, but there is an additional \$2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money. Determine how many miles she can ride.

3. Jasmins Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs \$150 dollars for the afternoon, and the food will cost \$3.00 per person. Andrew, Jasmins Dad, has a budget of \$300 . Write an equation to help him determine the maximum number of guests he can invite.

Review Answers

1. (a) $x = 20$
   (b) $x = 0.75$
   (c) $x = 1.5$
   (d) $x = -2.75$
   (e) $q = \frac{23}{15}$
   (f) $= -\frac{55}{18}$
   (g) $s = \frac{4}{3}$
   (h) $y = -110$
   (i) $q = 3$
   (j) $q = \frac{43}{5}$
   (k) $t = 3$
   (l) $p = \frac{34}{5}$
2. $0.75x + 2.35 = 10$; $x = 10.2$ miles
3. $3x + 150 = 300$; $x = 50$ guests

2.5 Multi-Step Equations

Learning Objectives

- Solve a multi-step equation by combining like terms.
- Solve a multi-step equation using the distributive property.
- Solve real-world problems using multi-step equations.

Solving Multi-Step Equations by Combining Like Terms

We have seen that when we solve for an unknown variable, it can be a simple matter of moving terms around in one or two steps. We can now look at solving equations that take several steps to isolate the unknown variable. Such equations are referred to as **multi-step equations**.

In this section, we will simply be combining the steps we already know how to do. Our goal is to end up with all the constants on one side of the equation and all of the variables on the other side. We will do this by collecting like terms. Dont forget, like terms have the same combination of variables in them.

Example 1

Solve $\frac{3x+4}{3} - 5x = 6$

This problem involves a fraction. Before we can combine the variable terms we need to deal with it. Lets put all the terms on the left over a common denominator of three.

$$\frac{3x+4}{3} - \frac{15x}{3} = 6 \qquad \text{Next we combine the fractions.}$$
$$\frac{3x+4-15x}{3} = 6 \qquad \text{Combine like terms.}$$
$$\frac{4-12x}{3} = 6 \qquad \text{Multiply both sides by 3.}$$
$$4-12x = 18 \qquad \text{Subtract 4 from both sides.}$$
$$-12x = 14 \qquad \text{Divide both sides by } -12$$
$$\frac{-12}{-12}x = -\frac{14}{12}$$

Solution

$x = -\frac{7}{6}$

Solving Multi-Step Equations Using the Distributive Property

You have seen in some of the examples that we can choose to divide out a constant or distribute it. The choice comes down to whether on not we would get a fraction as a result. We are trying to simplify the expression. If we can divide out large numbers without getting a fraction, then we avoid large coefficients. Most of the time, however, we will have to distribute and then collect like terms.

Example 2